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AN ITERATIVE SCHEME FOR A SYSTEM OF QUASI VARIATIONAL INEQUALITIES

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Abstract. In this paper, we consider a new system of quasi variational inequalities involving two different operators. Using the projection technique, we suggest and analyze a new iterative method for solving this system of quasi variational inequalities. We also prove the convergence of this iterative method under some mild conditions. As a special case, our results include the results of Huang and Noor [6] for solving system of variational inequalities.

1. Introduction

Variational inequalities theory has emerged as a fascinating branch of applicable mathematics with a wide range of applications in industry, finance and applied sciences. Variational inequalities have been generalized and extended in several directions using some novel and innovative techniques. A useful and significant generalization of the variational inequalities is called the quasi variational inequality where the underlying convex set also depends upon the solution implicitly or explicitly. It is well known that a wide class of problems arising in pure and applied sciences can be studied by quasi variational inequalities, see [1–14,16] and the references therein.

Inspired and motivated by the ongoing research in this field, Huang and Noor [6] have considered and studied a system of variational inequalities involving two different nonlinear operators. In this paper, we introduce a system of quasi variational inequalities involving two different operators and study the convergence analysis of the iterative method under some mild conditions. Since the system of quasi variational inequalities includes the system of variational inequalities, quasi variational inequality and variational inequalities as special cases, results proved in this paper continue to hold for these problems. In this respect, our results can be viewed as a refinement of the previous known results.

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2. Basic Results

Let K be a nonempty closed and convex set in a real Hilbert space, whose inner product and norm are denoted by $\langle \cdot, \cdot \rangle$ and $\|.\|$ respectively. Let $T_1, T_2 : K \longrightarrow K$ be nonlinear operators. Let $K(x, y)$ be a nonempty closed convex-valued set in H.

We consider the problem of finding $x^*, y^* \in K(x^*, y^*)$ such that

$$\langle \rho T_1(y^*, x^*) + x^* - y^*, x - x^* \rangle \geqslant 0, \quad \forall x \in K(y^*, x^*) \tag{1}$$

$$\langle \eta T_2(x^*, y^*) + y^* - x^*, x - y^* \rangle \geqslant 0, \quad \forall x \in K(x^*, y^*) \tag{2}$$

which is called the system of quasi variational inequalities (SQVID), where $\rho > 0, \quad \eta > 0$ are constants.

For $T_1 = T_2 = T$, SQVID is equivalent to finding $x^*, y^* \in K(x^*, y^*)$ such that

$$\langle \rho T(y^*, x^*) + x^* - y^*, x - x^* \rangle \geqslant 0, \quad \forall x \in K(y^*, x^*)$$
$$\langle \eta T(x^*, y^*) + y^* - x^*, x - y^* \rangle \geqslant 0, \quad \forall x \in K(x^*, y^*)$$

which is called the system of quasi variational inequalities(SQVI) and appears to be a new one.

Note that, if $K(x^*, y^*) = K(y^*, x^*) = K$, the nonempty closed and convex set in H, then SQVID reduces to the following system of variational inequalities of finding $x^*, y^* \in K$ such that

$$\langle \rho T_1(y^*, x^*) + x^* - y^*, x - x^* \rangle \geqslant 0, \quad x \in K,$$
$$\langle \eta T_2(x^*, y^*) + y^* - x^*, x - y^* \rangle \geqslant 0, \quad x \in K,$$

which is known as the system of variational inequalities involving two different nonlinear operators (SVID). This system of variational inequalities (SVID) was considered and introduced by Huang and Noor [6].

In brief, for appropriate and suitable choice of the operators T_1, T_2 and the convex-valued set K, one can obtain a number of new and previous known problems from the SQVID as special cases, which have been considered in [1-14,16]. This clearly shows that the SQVID is quite general and unifying one and has important applications in various branches of pure and applied sciences.

LEMMA 2.1. *For a given $z \in H$, $u \in K$ satisfies the inequality*

$$\langle u - z, v - u \rangle \geqslant 0, \quad \forall v \in K,$$

if and only if $u \in K$ satisfies the relation $u = P_K z$, where P_K is a projection from K onto H.

Lemma 2.1 plays an important role in obtaining our results. Using Lemma 2.1, one can easily know that finding the solution (x^*, y^*) of the system of quasi variational inequalities (SQVID) is equivalent to find x^*, y^* such that

$$x^* = P_{K(y^*,x^*)}[y^* - \rho T_1(y^*, x^*)],$$
$$y^* = P_{K(x^*,y^*)}[x^* - \eta T_2(x^*, y^*)].$$

DEFINITION 2.1. A mapping $T: K \to H$ is called r-strongly monotonic if for all $x, y \in K$, there exists a constant $r > 0$, such that

$$\langle Tx - Ty, x - y \rangle \geqslant r||x - y||^2.$$

DEFINITION 2.2. A mapping $T: K \to H$ is called γ-cocoercive if for all $x, y \in K$, there exists a constant $\gamma > 0$, such that

$$\langle Tx - Ty, x - y \rangle \geqslant \gamma||Tx - Ty||^2.$$

DEFINITION 2.3. A mapping $T: K \to H$ is called relaxed (γ, r)-cocoercive if for all $x, y \in K$, there exists constants $\gamma > 0, r > 0$, such that

$$\langle Tx - Ty, x - y \rangle \geqslant -\gamma||Tx - Ty||^2 + r||x - y||^2.$$

Clearly the class of the relaxed (γ, r)-cocoercive mappings in Definition 2.3 is the most general class than the class of r-strongly monotonic mappings in Definition 2.1 and γ-cocoercive mappings in Definition 2.2.

DEFINITION 2.4. A mapping $T: K \to H$ is called μ-Lipschitzian if for all $x, y \in K$, there exists a constant $\mu > 0$, such that

$$||Tx - Ty|| \leqslant \mu||x - y||.$$

LEMMA 2.2. *(see [15]). Suppose* $\{\delta_n\}_{n=0}^{\infty}$ *is a nonnegative sequence satisfying the following inequality:*

$$\delta_{n+1} \leqslant (1 - \lambda_n)\delta_n + \sigma_n, \ \textit{for all } n \geqslant 0,$$

with $\lambda_n \in [0, 1]$, $\sum_{n=0}^{\infty} \lambda_n = \infty$, *and* $\sigma_n = o(\lambda_n)$. *Then* $\lim_{n\to\infty} \delta_n = 0$.

In order to consider the convergence analysis of the iterative methods, we need the following assumption, which is mainly due to Noor [9,10].

ASSUMPTION 2.1. *The projection operator* $P_{K(x,y)}$ *satisfies the following condition*

$$\|P_{K(x,y)}(w) - P_{K(u,v)}(w)\| \leqslant \nu\|x - u\|, \quad \forall x, y, u, v, w \in H,$$

where $\nu > 0$ *is a constant.*

REMARK 2.1. We remark that Assumption 2.1 is true for the special case, $K(x, y) = m(x) + K,$ which appears in many important applications (see [7]), where m is a point-to-point mapping and K is a closed convex set in $H.$ It is well known that

$$P_{K(x,y)}(w) = m(x) + P_K[w - m(x)].$$

If m is a Lipschitz continuous with a constant $\tilde{\nu} > 0,$ then

$$\begin{aligned}
&\|P_{K(x,y)}(w) - P_{K(u,v)}(w)\| \\
= \ & \|m(x) - m(u) + P_K[w - m(x)] - P_K[w - m(u)]\| \\
\leqslant \ & \|m(x) - m(u)\| + \|P_K[w - m(x)] - P_K[w - m(u)]\| \\
\leqslant \ & 2\|m(x) - m(u)\| \leqslant 2\tilde{\nu}\|x - u\|,
\end{aligned}$$

which shows that Assumption 2.1 is true for $\nu = 2\tilde{\nu} > 0.$

3. Iterative Algorithms and Convergence

In this section, we suggest and investigate a class of iterative methods for solving the system of quasi variational inequalities(SQVID) using the projection technique. We also consider the convergence analysis of the iterative method under some mild and suitable conditions.

Using Lemma 2.1, we can suggest the following iterative method for SQVID.

ALGORITHM 3.1. For arbitrarily chosen initial points $x_0, y_0,$ compute the sequence $\{x_n\}$ and $\{y_n\}$ by

$$x_{n+1} = (1-a_n)x_n + a_n P_{K(y_n,x_n)}[y_n - \rho T_1(y_n,x_n)], \tag{3}$$

$$y_{n+1} = (1-b_n)x_{n+1} + b_n P_{K(x_{n+1},y_n)}[x_{n+1} - \eta T_2(x_{n+1},y_n)], \tag{4}$$

where $a_n, b_n \in [0,1]$ for all $n \geqslant 0$ and $\rho > 0, \quad \eta > 0$ are constants.

For $a_n \equiv 1 = b_n$ for all $n \geqslant 0$, then Algorithm 3.1 reduces to the following iterative algorithm.

ALGORITHM 3.2. For arbitrarily chosen initial points $x_0, y_0,$ compute the sequence $\{x_n\}$ and $\{y_n\}$ by

$$x_{n+1} = P_{K(y_n,x_n)}[y_n - \rho T_1(y_n,x_n)],$$

$$y_{n+1} = P_{K(x_{n+1},y_n)}[x_{n+1} - \eta T_2(x_{n+1},y_n)],$$

For $K(y,x) = K(x,y) \equiv K,$ the convex set, Algorithm 3.1 reduces to the following iterative method for solving SVID, which is mainly due to Huang and Noor [6].

ALGORITHM 3.3. For arbitrarily chosen initial points $x_0, y_0,$ compute the sequence $\{x_n\}$ and $\{y_n\}$ by

$$x_{n+1} = (1-a_n)x_n + a_n P_K[y_n - \rho T_1(y_n,x_n)],$$

$$y_{n+1} = (1-b_n)x_{n+1} + b_n P_K[x_{n+1} - \eta T_2(x_{n+1},y_n)],$$

where $a_n, b_n \in [0,1]$ for all $n \geqslant 0.$

For $T_1 = T_2 = T,$ Algorithm 3.1 reduces to:

ALGORITHM 3.4. For arbitrarily chosen initial points $x_0, y_0,$ compute the sequence $\{x_n\}$ and $\{y_n\}$ by

$$x_{n+1} = (1-a_n)x_n + a_n P_{K(y_n,x_n)}[y_n - \rho T(y_n,x_n)],$$

$$y_{n+1} = (1-b_n)x_{n+1} + b_n P_{K(x_{n+1},y_n)}[x_{n+1} - \eta T(x_{n+1},y_n)],$$

where $a_n, b_n \in [0,1]$ for all $n \geqslant 0.$ Algorithm 3.4 appears to be new one for SQVI.

In a similar way, for suitable and appropriate choice of the operators T_1, T_2 and the convex-valued set $K(x,y),$ one can obtain several new and known algorithms from

Algorithm 3.1. This clearly shows that Algorithm 3.1 is quite general and includes the previous methods as special cases.

We now consider the convergence criteria of Algorithm 3.1 and this is the main motivation of our main result.

THEOREM 3.1. *Let $K(x,y)$ be a nonempty closed convex subset of a real Hilbert space H and let $(x^*,\ y^*)$ be the solution of SQVID. Let $T_1 : K\times K \to H$ be relaxed (γ_1, r_1)-cocoercive and μ_1-Lipschitzian in the first variable, and $T_2 : K\times K\to H$ be relaxed (γ_2, r_2)-cocoercive and μ_2-Lipschitzian in the first variable with conditions*

$$\left|\rho - \frac{r_1-\gamma_1\mu_1^2}{\mu_1^2}\right| < \frac{\sqrt{(r_1-\gamma_1\mu_1^2)^2-\mu_1^2(2\nu-\nu^2)}}{\mu_1^2} \tag{5}$$

$$r_1 > \gamma_1\mu_1^2+\mu_1\sqrt{\nu(2-\nu)}, \quad \nu\in(0,1), \tag{6}$$

and

$$\left|\eta - \frac{r_2-\gamma_2\mu_2^2}{\mu_2^2}\right| < \frac{\sqrt{(r_2-\gamma_1\mu_2^2)^2-\mu_2^2(2\nu-\nu^2)}}{\mu_2^2} \tag{7}$$

$$r_2 > \gamma_2\mu_2^2+\mu_2\sqrt{\nu(2-\nu)}, \quad \nu\in(0,1). \tag{8}$$

Let $a_n, b_n \in [0,1]$, $\sum_{n=0}^{\infty} a_n = \infty$, and $\lim_{n\to\infty} b_n = 1$. If Assumption 2.1 holds with a constant $\nu\in(0,1)$, then for arbitrarily chosen initial points $x_0, y_0 \in K$, x_n and y_n obtained from Algorithm 3.1 converge strongly to x^* and y^* respectively.

Proof. Let x^*, y^* be a solution of the SQVID. Then

$$x^* = (1-a_n)x^* + a_nP_{K(y^*,x^*)}[y^*-\rho T_1(y^*,x^*)] \tag{9}$$

$$y^* = (1-b_n)y^* + b_nP_{K(x^*,y^*)}[x^*-\eta T_2(x^*,y^*)] \tag{10}$$

From (3), (9), and Assumption 2.1, we have

$$\begin{aligned}
&||x_{n+1}-x^*||\\
=\ & ||(1-a_n)x_n + a_nP_{K(y_n,x_n)}[y_n-\rho T_1(y_n,x_n)]\\
&-(1-a_n)x^* - a_nP_{K(y^*,x^*)}[y^*-\rho T_1(y^*,x^*)]||\\
\leqslant\ & (1-a_n)||x_n-x^*||\\
&+a_n||P_{K(y_n,x_n)}[y_n-\rho T_1(y_n,x_n)] - P_{K(y^*,x^*)}[y^*-\rho T_1(y^*,x^*)]||\\
\leqslant\ & (1-a_n)||x_n-x^*||\\
&+a_n||P_{K(y^*,x^*)}[y_n-\rho T_1(y_n,x_n)] - P_{K(y^*,x^*)}[y^*-\rho T_1(y^*,x^*)]||\\
&+a_n||P_{K(y_n,x_n)}[y_n-\rho T_1(y_n,x_n)] - P_{K(y^*,x^*)}[y_n-\rho T_1(y_n,x_n)]||\\
\leqslant\ & (1-a_n)||x_n-x^*|| + a_n||[y_n-\rho T_1(y_n,x_n)] - [y^*-\rho T_1(y^*,x^*)]||\\
&+\nu\|y_n-y^*\|\\
=\ & (1-a_n)||x_n-x^*|| + a_n||y_n-y^*-\rho[T_1(y_n,x_n)-T_1(y^*,x^*)]||\\
&+\nu\|y_n-y^*\|.
\end{aligned} \tag{11}$$

From the relaxed (γ_1, r_1)-cocoercive and μ_1-Lipschitzian definition in the first variable on T_1, we have

$$\begin{aligned} & ||y_n - y^* - \rho[T_1(y_n, x_n) - T_1(y^*, x^*)]||^2 \\ = \ & ||y_n - y^*||^2 - 2\rho\langle T_1(y_n, x_n) - T_1(y^*, x^*), y_n - y^*\rangle + \rho^2||T_1(y_n, x_n) - T_1(y^*, x^*)||^2 \\ \leqslant \ & ||y_n - y^*||^2 - 2\rho[-\gamma_1||T_1(y_n, x_n) - T_1(y^*, x^*)||^2 + r_1||y_n - y^*||^2] \\ & +\rho^2||T_1(y_n, x_n) - T_1(y^*, x^*)||^2 \\ \leqslant \ & ||y_n - y^*||^2 + 2\rho\gamma_1\mu_1^2||y_n - y^*||^2 - 2\rho r_1||y_n - y^*||^2 + \rho^2\mu_1^2||y_n - y^*||^2 \\ = \ & [1 + 2\rho\gamma_1\mu_1^2 - 2\rho r_1 + \rho^2\mu_1^2]||y_n - y^*||^2. \end{aligned} \tag{12}$$

From (11) and (12), it follows that

$$\begin{aligned} \|x_{n+1} - x^*\| \leqslant \ & (1 - a_n)\|x_n - x^*\| \\ & +a_n \left\{ \sqrt{1 + 2\rho\gamma_1\mu_1^2 - 2\rho r_1 + \rho^2\mu_1^2} + \nu \right\} \|y_n - y^*\| \\ = \ & (1 - a_n)\|x_n - x^*\| + a_n\theta_1\|y_n - y^*\|, \end{aligned} \tag{13}$$

where

$$\theta_1 = \sqrt{1 + 2\rho\gamma_1\mu_1^2 - 2\rho r_1 + \rho^2\mu_1^2]^{1/2}} + \nu.$$

From (5) and (6), we see that $\theta_1 < 1$.

Similarly, from the relaxed (γ_2, r_2)-cocoercive and μ_2-Lipschitzian definition in the first variable on T_2, we obtain

$$\begin{aligned} & ||x_{n+1} - x^* - \eta[T_2(x_{n+1}, y_n) - T_2(x^*, y^*)]||^2 \\ = \ & ||x_{n+1} - x^*||^2 - 2\eta\langle T_2(x_{n+1}, y_n) - T_2(x^*, y^*), x_{n+1} - x^*\rangle \\ & +\eta^2||T_2(x_{n+1}, y_n) - T_2(x^*, y^*)||^2 \\ \leqslant \ & ||x_{n+1} - x^*||^2 - 2\eta[-\gamma_2||T_2(x_{n+1}, y_n) - T_2(x^*, y^*)||^2 + r_2||x_{n+1} - x^*||^2] \\ & +\eta^2||T_2(x_{n+1}, y_n) - T_2(x^*, y^*)||^2 \\ = \ & ||x_{n+1} - x^*||^2 + 2\eta\gamma_2||T_2(x_{n+1}, y_n) - T_2(x^*, y^*)||^2 - 2\eta r_2||x_{n+1} - x^*||^2 \\ & +\eta^2||T_2(x_{n+1}, y_n) - T_2(x^*, y^*)||^2 \\ \leqslant \ & ||x_{n+1} - x^*||^2 + 2\eta\gamma_2\mu_2^2||x_{n+1} - x^*||^2 - 2\eta r_2||x_{n+1} - x^*||^2 \\ & +\eta^2\mu_2^2||x_{n+1} - x^*||^2 \\ = \ & [1 + 2\eta\gamma_2\mu_2^2 - 2\eta r_2 + \eta_2^2\mu^2]||x_{n+1} - x^*||^2. \end{aligned} \tag{14}$$

Hence from (4), (10), (14), and Assumption 2.1, it yields that

$$\begin{aligned}
&||y_{n+1} - y^*|| \\
\leqslant\ & (1-b_n)\|x_{n+1} - x^*\| + (1-b_n)||y^* - x^*|| \\
&+ b_n\|P_{K(x_{n+1},y_n)}[x_{n+1} - \eta T_2(x_{n+1},y_n)] - P_{K(x^*,y^*)}[x^* - \eta T_2(x^*,y^*)]|| \\
\leqslant\ & (1-b_n)\|x_{n+1} - x^*\| + (1-b_n)||y^* - x^*|| \\
&+ b_n||P_{K(x_{n+1},y_n)}[x_{n+1} - \eta T_2(x_{n+1},y_n)] - P_{K(x_{n+1},y_n)}[x^* - \eta T_2(x^*,y^*)]|| \\
&+ b_n||P_{K(x_{n+1},y_n)}[x^* - \eta T_2(x^*,y^*)] - P_{K(x^*,y^*)}[x^* - \eta T_2(x^*,y^*)]|| \\
=\ & (1-b_n)\|x_{n+1} - x^*\| + (1-b_n)||y^* - x^*|| \\
&+ b_n||x_{n+1} - x^* - \eta[T_2(x_{n+1},y_n) - T_2(x^*,y^*)]|| \\
&+ b_n \nu\|x_{n+1} - x^*\| \\
\leqslant\ & (1-b_n)\|x_{n+1} - x^*\| + b_n\theta_2\|x_{n+1} - x^*\| + (1-b_n)||y^* - x^*|| \\
=\ & (1 - b_n(1-\theta_2))\,\|x_{n+1} - x^*\| + (1-b_n)||y^* - x^*|| \\
\leqslant\ & \|x_{n+1} - x^*\| + (1-b_n)||y^* - x^*||, \qquad (15)
\end{aligned}$$

where

$$\theta_2 = \left[\sqrt{1 + 2\eta\gamma_2\mu_2^2 - 2\eta r_2 + \eta^2\mu_2^2} + \nu\right].$$

From (7) and (8), we know that $\theta_2 < 1$.

Combining (13)-(15), we have

$$\begin{aligned}
||x_{n+1} - x^*|| &\leqslant (1-a_n)||x_n - x^*|| + a_n\theta_1||y_n - y^*|| \\
&\leqslant (1-a_n)||x_n - x^*|| + a_n\theta_1[||x_n - x^*|| + (1-b_{n-1})||y^* - x^*||] \\
&= [1 - a_n(1-\theta_1)]||x_n - x^*|| + a_n\theta_1(1-b_{n-1})||y^* - x^*||.
\end{aligned}$$

Since $(1-\theta_1) \in (0,1]$, $\sum_{n=0}^{\infty} a_n(1-\theta_1) = \infty$, and $a_n\theta_1(1-b_{n-1})||y^* - x^*|| = o(a_n)$, then by Lemma 2.2, $\lim_{n\to\infty} ||x_n - x^*|| = 0$. The result $\lim_{n\to\infty} ||y_n - y^*|| = 0$ is from (15). This completes the proof. □

REMARK. Algorithm 3.1 extends the main results of [6] from the solvability of the system of variational inequalities (SVID) to the solvability of the system of quasi variational inequalities (SQVID). Moreover, since SQVID includes SVID, quasi variational inequality and variational inequalities as special cases, results proved in this paper hold for these problems and give a refinement of the previous results [6,8,9,10,11,14,16] to the more general relaxed (γ, r)-cocoercive mappings.

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