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HYERS–ULAM STABILITY OF THE FIRST ORDER LINEAR DIFFERENTIAL EQUATION FOR BANACH SPACE–VALUED HOLOMORPHIC MAPPINGS

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(*communicated by Th. Rassias*)

Abstract. Let Ω be a convex open set of $\mathbb{C}$, and let X be a complex Banach space. Suppose that $p\colon\Omega\to\mathbb{C}$ and $q\colon\Omega\to X$ are holomorphic. We give sufficient conditions in order that the first order linear differential equation $f'(z)+p(z)f(z)+q(z)=0$ for X-valued holomorphic mapping $f\colon\Omega\to X$ has the Hyers-Ulam stability.

1. Introduction

It seems that the stability problem of functional equations had been first raised by S. M. Ulam (cf. [18, Chapter VI]). "For what metric groups G is it true that an ε-automorphism of G is necessarily near to a strict automorphism? (An ε-automorphism of G means a transformation f of G into itself such that $\rho(f(x\cdot y),f(x)\cdot f(y))<\varepsilon$ for all $x,y\in G$.)"

D. H. Hyers [6] gave an affirmative answer to the problem as follows.

THEOREM A. *Suppose that E_1 and E_2 are two real Banach spaces and $f\colon E_1\to E_2$ is a mapping. If there exists $\varepsilon\geqslant 0$ such that*

$$\|f(x+y)-f(x)-f(y)\|\leqslant\varepsilon$$

for all $x,y\in E_1$, then the limit

$$T(x)=\lim_{n\to\infty}\frac{f(2^nx)}{2^n}$$

exists for each $x\in E_1$, and $T\colon E_1\to E_2$ is the unique additive mapping such that

$$\|f(x)-T(x)\|\leqslant\varepsilon$$

for all $x\in E_1$. If, in addition, the mapping $\mathbb{R}\ni t\mapsto f(tx)$ is continuous for each fixed $x\in E_1$, then T is linear.

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This result is called the *Hyers-Ulam stability* of the *additive* Cauchy equation $g(x+y)=g(x)+g(y)$. Here we note that Hyers [6] calls any solution of this equation a “linear" function or transformation. Hyers considered only *bounded* Cauchy difference $f(x+y)-f(x)-f(y)$. T. Aoki [2] introduced unbounded one and generalized a result [6, Theorem 1] of Hyers obtaining the stability of additive mapping. Th.M. Rassias [13], who independently introduced the unbounded Cauchy difference, was the first to prove the stability of the linear mapping between Banach spaces. The concept of the Hyers-Ulam-Rassias stability was originated from Rassias' paper [13] for the stability of the linear mapping. Rassias [13] generalized Hyers's Theorem as follows:

THEOREM B. *Suppose that E_1 and E_2 are two real Banach spaces and $f: E_1 \to E_2$ is a mapping. If there exist $\varepsilon \geqslant 0$ and $0 \leqslant p < 1$ such that*

$$\|f(x+y)-f(x)-f(y)\| \leqslant \varepsilon(\|x\|^p + \|y\|^p)$$

for all $x, y \in E_1$, then there is a unique additive mapping $T: E_1 \to E_2$ such that

$$\|f(x) - T(x)\| \leqslant \frac{2\varepsilon}{|2-2^p|}\|x\|^p$$

for all $x \in E_1$. If, in addition, the mapping $\mathbb{R} \ni t \mapsto f(tx)$ is continuous for each fixed $x \in E_1$, then T is linear.

This result is, what is called, *the Hyers-Ulam-Rassias stability* of the additive Cauchy equation $g(x+y)=g(x)+g(y)$. The result of Hyers is just the case where $p=0$. So, the result of Rassias is a generalization to the case where $0 \leqslant p < 1$: It should be mentioned that it allows Cauchy difference to be unbounded. During the 27th International Symposium on Functional Equations, Rassias raised the problem whether a similar result holds for $1 \leqslant p$. Z. Gajda [3, Theorem 2] proved that Theorem B is valid for $1 < p$; In the same paper [3, Example], he also gave an example to show that a similar result to the above does not hold for $p=1$. Later, Th.M. Rassias and P. Šemrl [14, Theorem 2] gave another counter example for $p=1$. Note that if $p<0$, then $\|0\|^p$ is obviously meaningless. However, if we assume that $\|0\|^p$ means ∞, then the proof given in [13] also works for $x \neq 0$. Moreover, with minor changes in the proof, we see that the result is also valid for $p<0$. Thus, the Hyers-Ulam-Rassias stability of the additive Cauchy equation holds for all $p \in \mathbb{R} \setminus \{1\}$.

It seems that Alsina and Ger [1] are the first who considered the Hyers-Ulam stability of differential equations. They remarked that the Hyers-Ulam stability of the differential equation $y' = y$ holds: If $\varepsilon \geqslant 0$ and if f is a differentiable function on an open interval I into $\mathbb{R}$ with $|f'(t) - f(t)| \leqslant \varepsilon$ for all $t \in I$, then there exists a differentiable function $g: I \to \mathbb{R}$ such that $g'(t) = g(t)$ and $|f(t) - g(t)| \leqslant 3\varepsilon$ for all $t \in I$. Many authors generalize the result of Alsina and Ger (cf. [5, 9, 10, 11, 12, 15, 16]). The first and third authors with G. Hirasawa [11] considered the first order linear differential equation $f'(z) + h(z)f(z) = 0$ for entire functions $f(z)$ and $h(z)$. Then they proved the Hyers-Ulam stability of $f'(z) + h(z)f(z) = 0$. Let Ω be a convex open set of $\mathbb{C}$ and X a complex Banach space, and let $h: \Omega \to \mathbb{C}$ be a holomorphic function. In this paper, we prove the Hyers-Ulam stability of the X-valued differential equation $f'(z)+p(z)f(z)+q(z)=0$.

2. Main results

Let Ω be an open set of $\mathbb{C}$, and let X be a complex Banach space. A mapping $f:\Omega\to X$ is said to be *holomorphic* if and only if

$$\lim_{w\to z}\frac{f(w)-f(z)}{w-z}$$

exists in the norm-topology of X for each $z\in\Omega$. It is well-known that $f:\Omega\to X$ is holomorphic if and only if $\phi\circ f:\Omega\to\mathbb{C}$ is holomorphic (as a complex-valued function) for each $\phi\in X^*$, the dual space of X. It follows that complex analysis is valid for X-valued holomorphic mappings. We will denote by $H(\Omega,X)$ the set of all holomorphic mappings $f:\Omega\to X$. In short, $H(\Omega)\overset{\text{def}}{=}H(\Omega,\mathbb{C})$. Let $f\in H(\Omega,X)$. Just for the sake of simplicity, we will consider the case where $0\in\Omega$. For $z\in\Omega$, we will write $\int_0^z f(\zeta)d\zeta$ for $\int_0^1 zf(zt)\,dt$, the integral of f over the path γ defined by

$$\gamma(t)=zt\qquad(\forall t\in[0,1]).$$

We associate to each $p\in H(\Omega)$ a function $\tilde{p}$ defined by

$$\tilde{p}(z)=\exp\int_0^z p(\zeta)\,d\zeta\qquad(\forall z\in\Omega).$$

If Ω is convex, then we see that $\tilde{p}\in H(\Omega)$ with

$$\tilde{p}'(z)=p(z)\tilde{p}(z)\qquad(\forall z\in\Omega).$$

LEMMA 2.1. *Let Ω be a convex open set of $\mathbb{C}$ with $0\in\Omega$. Let X be a complex Banach space and $p\in H(\Omega)$. For $f,q,u\in H(\Omega,X)$, each of the following conditions are equivalent.*

(i) $f'(z)+p(z)f(z)+q(z)=u(z)$ *for all* $z\in\Omega$.

(ii) $f(z)=\dfrac{1}{\tilde{p}(z)}\left\{f(0)+\displaystyle\int_0^z\tilde{p}(\zeta)(u(\zeta)-q(\zeta))\,d\zeta\right\}$ *for all* $z\in\Omega$.

Proof. $(i)\Rightarrow(ii)$ Since $\tilde{p}f\in H(\Omega,X)$, we have

$$\begin{aligned}\{\tilde{p}(z)f(z)\}'&=\tilde{p}'(z)f(z)+\tilde{p}(z)f'(z)=\tilde{p}(z)\{p(z)f(z)+f'(z)\}\\&=\tilde{p}(z)(u(z)-q(z))\end{aligned}$$

for each $z\in\Omega$. It follows that

$$\tilde{p}(z)f(z)-\tilde{p}(0)f(0)=\int_0^z\tilde{p}(\zeta)(u(\zeta)-q(\zeta))\,d\zeta\qquad(\forall z\in\Omega).$$

Since $\tilde{p}(0) = 1$, we have

$$f(z) = \frac{1}{\tilde{p}(z)} \left\{ f(0) + \int_0^z \tilde{p}(\zeta)(u(\zeta) - q(\zeta))\, d\zeta \right\} \qquad (\forall z \in \Omega).$$

$(ii) \Rightarrow (i)$ By a simple calculation, we have

$$\begin{aligned} f'(z) &= \frac{1}{\tilde{p}^2(z)} \left\{ \tilde{p}^2(z)(u(z) - q(z)) - \tilde{p}(z) f(z) \tilde{p}'(z) \right\} \\ &= u(z) - q(z) - p(z) f(z) \end{aligned}$$

for each $z \in \Omega$, and so $f'(z) + p(z)f(z) + q(z) = u(z)$ $(\forall z \in \Omega)$. □

THEOREM 2.2. *Let Ω be a convex open set of $\mathbb{C}$ with $0 \in \Omega$. Let X be a complex Banach space and $q \in H(\Omega, X)$. Suppose that $p \in H(\Omega)$ satisfies that*

$$C_p \stackrel{\text{def}}{=} \sup_{z \in \Omega} \frac{1}{|\tilde{p}(z)|} \left| \int_0^z |\tilde{p}(\zeta)|\, d\zeta \right| < \infty.$$

For each $\varepsilon \geqslant 0$ and $f \in H(\Omega, X)$ satisfying

$$\|f'(z) + p(z)f(z) + q(z)\| \leqslant \varepsilon \qquad (\forall z \in \Omega), \tag{1}$$

there exists $g \in H(\Omega, X)$ such that

$$g'(z) + p(z)g(z) + q(z) = 0 \qquad (\forall z \in \Omega)$$

and that

$$\|f(z) - g(z)\| \leqslant C_p \varepsilon \qquad (\forall z \in \Omega).$$

Proof. Let $\varepsilon \geqslant 0$ and $f \in H(\Omega, X)$ satisfy (1). Set, for each $z \in \Omega$, $u(z) = f'(z) + p(z)f(z) + q(z)$. We see from Lemma 2.1 that

$$f(z) = \frac{1}{\tilde{p}(z)} \left\{ f(0) + \int_0^z \tilde{p}(\zeta)(u(\zeta) - q(\zeta))\, d\zeta \right\} \qquad (\forall z \in \Omega). \tag{2}$$

Set, for each $z \in \Omega$,

$$g(z) = \frac{1}{\tilde{p}(z)} \left\{ f(0) - \int_0^z \tilde{p}(\zeta) q(\zeta)\, d\zeta \right\}.$$

Then $g \in H(\Omega, X)$ satisfying

$$g'(z) + p(z)g(z) + q(z) = 0 \qquad (\forall z \in \Omega)$$

by Lemma 2.1. Moreover, since $\|u(z)\| \leqslant \varepsilon$ for all $z \in \Omega$, it follows from (2) that

$$\|f(z) - g(z)\| = \frac{1}{|\tilde{p}(z)|}\left\|\int_0^z \tilde{p}(\zeta)u(\zeta)\,d\zeta\right\| \leqslant \frac{\varepsilon}{|\tilde{p}(z)|}\left|\int_0^z |\tilde{p}(\zeta)|\,d\zeta\right|$$

for each $z \in \Omega$. We thus conclude that

$$\|f(z) - g(z)\| \leqslant \varepsilon \sup_{z\in\Omega} \frac{1}{|\tilde{p}(z)|}\left|\int_0^z |\tilde{p}(\zeta)|\,d\zeta\right| = C_p\varepsilon$$

for each $z \in \Omega$, and the proof is complete. □

THEOREM 2.3. *Let Ω be a convex open set of $\mathbb{C}$ with $0 \in \Omega$. Let X be a complex Banach space, $q \in H(\Omega, X)$ and $p \in H(\Omega)$. Suppose that there exists $\lambda \in \partial\Omega$, the boundary of Ω, such that*

$$D_p(\lambda) \stackrel{\text{def}}{=} \sup_{z\in\Omega} \frac{1}{|\tilde{p}(z)|}\left|\int_\lambda^z |\tilde{p}(\zeta)|\,d\zeta\right| < \infty,$$

where

$$\int_\lambda^z |\tilde{p}(\zeta)|\,d\zeta = \lim_{a\searrow 0}\int_a^1 |\tilde{p}(\lambda + t(z-\lambda))|(z-\lambda)\,dt.$$

For each $\varepsilon \geqslant 0$ and $f \in H(\Omega, X)$ satisfying (1) there exists $g \in H(\Omega, X)$ such that

$$g'(z) + p(z)g(z) + q(z) = 0 \qquad (\forall z \in \Omega)$$

and that

$$\|f(z) - g(z)\| \leqslant D_p(\lambda)\varepsilon \qquad (\forall z \in \Omega).$$

Proof. Suppose that there exists $\lambda \in \partial\Omega$ such that

$$D_p(\lambda) = \sup_{z\in\Omega} \frac{1}{|\tilde{p}(z)|}\left|\int_\lambda^z |\tilde{p}(\zeta)|\,d\zeta\right| < \infty.$$

Let $\varepsilon \geqslant 0$ and $f \in H(\Omega, X)$ satisfy (1). Set, for each $z \in \Omega$, $v(z) = f'(z) + p(z)f(z) + q(z)$. Then $\|v(z)\| \leqslant \varepsilon$ for all $z \in \Omega$. By Lemma 2.1, we have

$$f(z) = \frac{1}{\tilde{p}(z)}\left\{f(0) + \int_0^z \tilde{p}(\zeta)(v(\zeta) - q(\zeta))\,d\zeta\right\} \tag{3}$$

for all $z \in \Omega$. Since Ω is convex, so is $\bar{\Omega}$, the closure of Ω. We have $\lambda + t(z-\lambda) \in \Omega$ for each $z \in \Omega$ and $0 < t \leqslant 1$. Note, by the hypothesis, that the integral of $|\tilde{p}|$ over the path $[\lambda, z]$ exists for each $z \in \Omega$. Since $\tilde{p}v \in H(\Omega, X)$, it follows from the Cauchy theorem that

$$\int_0^z \tilde{p}(\zeta)v(\zeta)\,d\zeta = \int_0^\lambda \tilde{p}(\zeta)v(\zeta)\,d\zeta + \int_\lambda^z \tilde{p}(\zeta)v(\zeta)\,d\zeta \tag{4}$$

for all $z \in \Omega$. Set, for each $z \in \Omega$,

$$g(z) = \frac{1}{\tilde{p}(z)}\left\{f(0) + \int_0^\lambda \tilde{p}(\zeta)v(\zeta)\,d\zeta - \int_0^z \tilde{p}(\zeta)q(\zeta)\,d\zeta\right\}. \tag{5}$$

It follows from Lemma 2.1 that

$$g'(z) + p(z)g(z) + q(z) = 0 \qquad (\forall z \in \Omega).$$

By (3), (4) and (5), we have

$$\begin{aligned} \|f(z) - g(z)\| &= \frac{1}{|\tilde{p}(z)|} \left\| \int_\lambda^z \tilde{p}(\zeta) v(\zeta)\, d\zeta \right\| \\ &\leqslant \varepsilon \sup_{z\in\Omega} \frac{1}{|\tilde{p}(z)|} \left| \int_\lambda^z |\tilde{p}(\zeta)|\, d\zeta \right| = D_p(\lambda)\varepsilon \qquad (\forall z \in \Omega) \end{aligned}$$

since $\|v(z)\| \leqslant \varepsilon$ for all $z \in \Omega$. This completes the proof. □

EXAMPLE 2.1. Let Ω be a bounded convex open set of $\mathbb{C}$, say $|z| \leqslant M$ for all $z \in \Omega$. If $0 \in \Omega$ and $p \in H(\Omega)$ is bounded, then we see that

$$C_p = \sup_{z\in\Omega} \frac{1}{|\tilde{p}(z)|} \left| \int_0^z |\tilde{p}(\zeta)|\, d\zeta \right| < \infty.$$

In fact, if $p \in H(\Omega)$ is bounded, say $|p(z)| \leqslant K$ for all $z \in \Omega$, then we have

$$\left| \int_0^z p(\zeta)\, d\zeta \right| \leqslant \int_0^1 |p(zt)|\, |z|\, dt \leqslant KM \qquad (\forall z \in \Omega),$$

and so

$$|\tilde{p}(z)| = \exp\left(\mathrm{Re} \int_0^z p(\zeta)\, d\zeta \right) \geqslant e^{-KM} \qquad (\forall z \in \Omega). \tag{6}$$

On the other hand, since $|\tilde{p}(z)| \leqslant e^{KM}$ $(\forall z \in \Omega)$, we have

$$\left| \int_0^z |\tilde{p}(z)|\, d\zeta \right| \leqslant Me^{KM} \qquad (\forall z \in \Omega). \tag{7}$$

It follows from (6) and (7) that

$$C_p = \sup_{z\in\Omega} \frac{1}{|\tilde{p}(z)|} \left| \int_0^z |\tilde{p}(\zeta)|\, d\zeta \right| \leqslant Me^{2KM} < \infty.$$

EXAMPLE 2.2. Let $\Omega = \{z \in \mathbb{C} : |z| < 1\}$. We consider $p \in H(\Omega)$ defined by $p(z) = 1/(z+1)$ $(\forall z \in \Omega)$. Then $\log(z+1)$ $(z \in \Omega)$ is well-defined so that

$$\tilde{p}(z) = \exp \int_0^z p(\zeta)\, d\zeta = e^{\log(z+1)} = z + 1 \qquad (\forall z \in \Omega). \tag{8}$$

We thus obtain

$$\begin{aligned} \left| \int_{-1}^z |\tilde{p}(\zeta)|\, d\zeta \right| &= \lim_{a \searrow 0} \left| \int_a^1 |\tilde{p}(-1 + t(z+1))|(z+1)\, dt \right| \qquad (9) \\ &= \lim_{a \searrow 0} \left| \int_a^1 |t(z+1)|(z+1)\, dt \right| \\ &= |z+1|^2 \int_0^1 t\, dt = \frac{|z+1|^2}{2} \end{aligned}$$

for each $z \in \Omega$. It follows from (8) and (9) that

$$D_p(-1) = \sup_{z\in\Omega} \frac{1}{|\tilde{p}(z)|} \left| \int_{-1}^{z} |\tilde{p}(\zeta)|\, d\zeta \right| = \sup_{z\in\Omega} \frac{|z+1|}{2} = 1.$$

Here, we notice that

$$D_p(\lambda) = \sup_{z\in\Omega} \frac{1}{|\tilde{p}(z)|} \left| \int_{\lambda}^{z} |\tilde{p}(\zeta)|\, d\zeta \right| = \infty \tag{10}$$

for each $\lambda \in \partial\Omega \setminus \{-1\}$, and that

$$C_p = \sup_{z\in\Omega} \frac{1}{|\tilde{p}(z)|} \left| \int_{0}^{z} |\tilde{p}(\zeta)|\, d\zeta \right| = \infty. \tag{11}$$

First, we prove that (10) holds. To do this, take $\lambda \in \partial\Omega \setminus \{-1\}$ arbitrarily. For each $n \in \mathbb{N}$ there exists $z_n \in \Omega$ such that

$$|\lambda + 1| \geqslant |\lambda - z_n| \quad \text{and} \quad |z_n + 1| < \frac{1}{n}. \tag{12}$$

We have, for each $n \in \mathbb{N}$, that

$$\begin{aligned}
\left| \int_{\lambda}^{z_n} |\tilde{p}(\zeta)|\, d\zeta \right| &= \lim_{a \searrow 0} \left| \int_{a}^{1} |\tilde{p}(\lambda + t(z_n - \lambda))|(z_n - \lambda)\, dt \right| \\
&= \lim_{a \searrow 0} \left| \int_{a}^{1} |\lambda + 1 + t(z_n - \lambda)|(z_n - \lambda)\, dt \right| \\
&\geqslant |z_n - \lambda| \int_{0}^{1} ||\lambda + 1| - t|z_n - \lambda||\, dt.
\end{aligned}$$

By (12), we have

$$\begin{aligned}
\int_{0}^{1} ||\lambda + 1| - t|z_n - \lambda||\, dt &= \int_{0}^{1} (|\lambda + 1| - t|z_n - \lambda|)\, dt \\
&= |\lambda + 1| - \frac{|z_n - \lambda|}{2},
\end{aligned}$$

and so

$$\left| \int_{\lambda}^{z_n} |\tilde{p}(\zeta)|\, d\zeta \right| \geqslant |z_n - \lambda| \left(|\lambda + 1| - \frac{|z_n - \lambda|}{2} \right). \tag{13}$$

Another application of (12) to (13) yields

$$\left| \int_{\lambda}^{z_n} |\tilde{p}(\zeta)|\, d\zeta \right| \geqslant |z_n - \lambda| \frac{|z_n - \lambda|}{2} = \frac{|z_n - \lambda|^2}{2}.$$

It follows that

$$\frac{1}{|\tilde{p}(z_n)|} \left| \int_{\lambda}^{z_n} |\tilde{p}(\zeta)|\, d\zeta \right| \geqslant \frac{|z_n - \lambda|^2}{2|z_n + 1|} \to \infty \qquad \text{as} \quad n \to \infty$$

since $z_n \to -1$ as $n \to \infty$. This proves (10) for each $\lambda \in \partial\Omega \setminus \{-1\}$.

Finally, we prove that (11) holds. In fact, since $|z| < 1$, we have, for each $z \in \Omega$, that

$$\begin{aligned} \left| \int_0^z |\tilde{p}(\zeta)| \, d\zeta \right| &= |z| \int_0^1 |\tilde{p}(tz)| \, dt = |z| \int_0^1 |tz+1| \, dt \\ &\geqslant |z| \int_0^1 ||tz|-1| \, dt = |z| \int_0^1 (1-|z|t) \, dt \\ &= |z| \left(1 - \frac{|z|}{2}\right). \end{aligned} \tag{14}$$

It follows from (8) and (14) that

$$C_p = \sup_{z\in\Omega} \frac{1}{|\tilde{p}(z)|} \left| \int_0^z |\tilde{p}(\zeta)| \, d\zeta \right| = \sup_{z\in\Omega} \frac{|z|(2-|z|)}{2|z+1|} = \infty.$$

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