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APPLICATIONS OF SRIVASTAVA–ATTIYA OPERATOR TO THE CLASSES OF STRONGLY STARLIKE AND STRONGLY CONVEX FUNCTIONS

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Abstract. Srivastava-Attiya operator is used to define some new subclasses of strongly starlike and strongly convex functions of order β and type α in the open unit disk $\mathbb{U}$. For each of these new function classes, several inclusion relationships are established. Some interesting corollaries and applications of the results presented here are also discussed.

1. Introduction

Let $\mathscr{A}$ denote the class of functions $f(z)$ normalized by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are *analytic* in the *open unit* disk $\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}$. A function $f \in \mathscr{A}$ is said to be in the class $\mathscr{S}^*(\alpha)$ of starlike functions of order α in $\mathbb{U}$, if it satisfies the following inequality:

$$\Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha \qquad (z \in \mathbb{U},\ 0 \leqslant \alpha < 1), \tag{1.2}$$

and is said to be in the class $\mathscr{K}(\alpha)$ of convex functions of order α in $\mathbb{U}$, if it satisfies the following inequality:

$$\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha \qquad (z \in \mathbb{U},\ 0 \leqslant \alpha < 1). \tag{1.3}$$

On the other hand, a function $f(z) \in \mathscr{A}$ is said to be in the class of strongly starlike functions of order β and type α, denoted by $\mathscr{S}_s^*(\alpha,\beta)$, if it satisfies the following inequality:

$$\left|\arg\left(\frac{zf'(z)}{f(z)} - \alpha\right)\right| < \frac{\pi}{2}\beta \qquad (z \in \mathbb{U};\ 0 \leqq \alpha < 1;\ 0 < \beta \leqq 1), \tag{1.4}$$

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and is said to be in a corresponding class $\mathscr{K}_c(\alpha,\beta)$ of strongly convex functions of order β and type α, if

$$\left|\arg\left(1+\frac{zf''(z)}{f'(z)}-\alpha\right)\right| < \frac{\pi}{2}\,\beta \qquad (z\in\mathbb{U};\ 0\leqq\alpha<1;\ 0<\beta\leqq 1). \tag{1.5}$$

It is obvious that $f(z)\in\mathscr{K}_c(\alpha,\beta)$, if and only if $zf'(z)\in\mathscr{S}_s^*(\alpha,\beta)$. We also observe that

$$\mathscr{S}_s^*(0,\beta)=\mathscr{S}_s^*(\beta) \qquad \text{and} \qquad \mathscr{K}_c(0,\beta)=\mathscr{K}_c(\beta),$$

where for $0<\beta\leqq 1$, $\mathscr{S}_s^*(\beta)$ and $\mathscr{K}_c(\beta)$ are, respectively, the classes of strongly starlike functions of order β and strongly convex functions of order β in $\mathbb{U}$. Furthermore, we have the following relationships:

$$\mathscr{S}_s^*(\alpha,1)=\mathscr{S}^*(\alpha) \qquad \text{and} \qquad \mathscr{K}_c(\alpha,1)=\mathscr{K}(\alpha)$$

The generalized Hurwitz-Lerch Zeta function $\phi(z,s,a)$ is defined by (*cf., e.g.,* [9, p. 121 *et seq.*]):

$$\phi(z,s,a) \;=\; \sum_{n=0}^{\infty}\frac{z^n}{(a+n)^s} \tag{1.6}$$

$$(a\in\mathbb{C}/\mathbb{Z}_0^-;\ s\in\mathbb{C}\ \text{ when }\ |z|<1;\ \Re(s)>1\ \text{ when }\ |z|=1),$$

contains, as its special cases, well-known functions as the Riemann and Hurwitz (or generalized) Zeta function, Lerch Zeta function, the Polylogarithmic function and the Lipschitz-Lerch Zeta function. One may refer to the Srivastava and Choi [9] (see also, [8]) for further details and references to these functions.

Srivastava and Attiya in [8] (see also, [7]), introduced the following family of linear operator:

$$J_{\mu,b}\ :\ \mathscr{A}\longrightarrow\mathscr{A},$$

defined by

$$J_{\mu,b}(f)(z) \;=\; G_{\mu,b}(z)*f(z) \qquad (z\in\mathbb{U};\ b\in\mathbb{C}/\mathbb{Z}_0^-;\ \mu\in\mathbb{C};\ f\in\mathscr{A}), \tag{1.7}$$

where * denote the Hadamard product (or convolution) of analytic functions and function $G_{\mu,b}$ is given by

$$\begin{aligned} G_{\mu,b}(z) \;:=&\; (1+b)^{\mu}[\phi(z,\mu,b)\;-\;b^{-\mu}] \\ =&\; z+\sum_{n=2}^{\infty}\left(\frac{b+1}{b+n}\right)^{\mu} z^n \qquad (z\in\mathbb{U}). \end{aligned} \tag{1.8}$$

Now using (1.8) in (1.7), we get

$$J_{\mu,b}(f)(z) \;=\; z+\sum_{n=2}^{\infty}\left(\frac{b+1}{b+n}\right)^{\mu} a_n z^n \qquad (z\in\mathbb{U};\ f\in\mathscr{A}). \tag{1.9}$$

For $f(z) \in \mathscr{A}$ and $z \in \mathbb{U}$, Srivastava and Attiya in [8] showed that

$$J_{0,b}(f)(z) := f(z), \tag{1.10}$$

$$J_{1,0}(f)(z) = \int_0^z \frac{f(t)}{t}\, dt := A(f)(z), \tag{1.11}$$

$$J_{1,\gamma}(f)(z) = \frac{1+\gamma}{z^\gamma} \int_0^z t^{\gamma-1} f(t)\, dt := \mathscr{I}_\gamma(f)(z) \quad (\gamma > -1), \tag{1.12}$$

$$J_{\sigma,1}(f)(z) = z + \sum_{n=2}^{\infty} \left(\frac{2}{n+1}\right)^\sigma a_n\, z^n := I^\sigma(f)(z) \quad (\sigma > 0), \tag{1.13}$$

where $A(f)$ and $\mathscr{I}_\gamma$ are the integral operators introduced by Alexandor [1] and Bernardi [2], respectively, and $I^\sigma(f)$ is the Jung-Kim-Srivastava integral operator [4] closely related to some multiplier transformation studied by Fleet [3] (see also, [5]).

By applying the definition (1.9), Srivastava and Attiya obtained the following relation [8, p. 210, Eq. (24)]:

$$zJ'_{\mu+1,b}(f)(z) = (b+1)\, J_{\mu,b}(f)(z) - b\, J_{\mu+1,b}(f)(z). \tag{1.14}$$

Using the linear operator $J_{\mu,b}$, we now introduce the following subclasses of $\mathscr{A}$:

$$\mathscr{S}_s^*(\mu,b,\alpha,\beta) := \left\{ f : f(z) \in \mathscr{A},\ J_{\mu,b}(f)(z) \in \mathscr{S}_s^*(\alpha,\beta) \text{ and } \frac{zJ'_{\mu,b}(f)(z)}{J_{\mu,b}(f)(z)} \neq \alpha\ (z \in \mathbb{U}) \right\} \tag{1.15}$$

and

$$\mathscr{K}_c(\mu,b,\alpha,\beta) := \left\{ f : f(z) \in \mathscr{A},\ J_{\mu,b}(f)(z) \in \mathscr{K}_c(\alpha,\beta) \text{ and } \frac{\left(zJ'_{\mu,b}(f)(z)\right)'}{J'_{\mu,b}(f)(z)} \neq \alpha\ (z \in \mathbb{U}) \right\}. \tag{1.16}$$

It is obvious from the definitions (1.15) and (1.16) that

$$f(z) \in \mathscr{K}_c(\mu,b,\alpha,\beta) \iff zf'(z) \in \mathscr{S}_s^*(\mu,b,\alpha,\beta).$$

2. Main Results

In order to derive our main results, we shall need the following lemmas.

LEMMA 1. *(see [6]) Let a function $p(z)$ be analytic in $\mathbb{U}$ with*

$$p(0) = 1 \qquad \textit{and} \qquad p(z) \neq 0 \quad (z \in \mathbb{U}).$$

If there exists a point $z_0 \in \mathbb{U}$ such that

$$|\arg(p(z))| < \frac{\pi}{2}\beta \quad (|z| < |z_0|) \quad \textit{and} \quad |\arg(p(z_0))| = \frac{\pi}{2}\beta \quad (0 < \beta \leqq 1), \tag{2.1}$$

then

$$\frac{z_0 p'(z_0)}{p(z_0)} = ik\beta, \tag{2.2}$$

where

$$k \geqq \frac{1}{2}\left(a+\frac{1}{a}\right) \qquad \textit{when} \qquad \arg\left(p(z_0)\right)=\frac{\pi}{2}\,\beta, \tag{2.3}$$

$$k \leqq -\frac{1}{2}\left(a+\frac{1}{a}\right) \qquad \textit{when} \qquad \arg\left(p(z_0)\right)=-\frac{\pi}{2}\,\beta, \tag{2.4}$$

and

$$\left(p(z_0)\right)^{\frac{1}{\beta}} = \pm\, ia \qquad (a>0).$$

LEMMA 2. *(see, [8, p. 210]) Let* $f \in \mathscr{A}$, $z,\ t_j \in \mathbb{U}\ (j=1,...,n)$, $n \in \mathbb{N}$ *and* $b \in \mathbb{C}/\mathbb{Z}_0^-$, *we have*

$$J_{2,0}(f)(z) =: \int_0^z \frac{1}{t_1}\int_0^{t_1} \frac{f(t_2)}{t_2}\, dt_2\, dt_1, \tag{2.5}$$

$$J_{n,0}(f)(z) =: \int_0^z \frac{1}{t_1}\int_0^{t_1} \frac{1}{t_2}\int_0^{t_2} \cdots \frac{1}{t_{n-1}}\int_0^{t_{n-1}} \frac{f(t_n)}{t_n}\, dt_n\, dt_{n-1} \ldots dt_1, \tag{2.6}$$

$$J_{2,b}(f)(z) =: \frac{(1+b)^2}{z^b}\int_0^z \frac{1}{t_1}\int_0^{t_1} t_2^{b-1}\, f(t_2)\, dt_2\, dt_1, \tag{2.7}$$

$$J_{n,b}(f)(z) =: \frac{(1+b)^n}{z^b}\int_0^z \frac{1}{t_1}\int_0^{t_1} \frac{1}{t_2}\int_0^{t_2} \cdots \frac{1}{t_{n-1}}\int_0^{t_{n-1}} t_n^{b-1}\, f(t_n)\, dt_n\, dt_{n-1} \ldots dt_1. \tag{2.8}$$

Theorem 1 below gives our first main inclusion relationship.

THEOREM 1. *Let* $f \in \mathscr{A}$. *Suppose also that*

$$\mu \in \mathbb{C}, \quad 0 \leqq \alpha < 1, \quad 0 < \beta \leqq 1 \quad \textit{and} \quad b > -\alpha.$$

Then

$$\mathscr{S}_s^*(\mu,b,\alpha,\beta) \subset \mathscr{S}_s^*(\mu+1,b,\alpha,\beta).$$

proof. Let $f \in \mathscr{S}_s^*(\mu,b,\alpha,\beta)$. Then, upon setting

$$p(z) = \frac{1}{1-\alpha}\left(\frac{zJ'_{\mu+1,b}(f)(z)}{J_{\mu+1,b}(f)(z)} - \alpha\right) \qquad (z \in \mathbb{U}), \tag{2.9}$$

we see that the function $p(z)$ is analytic in $\mathbb{U}$, with $p(0)=1$ and $p(z)\neq 0$ for $z \in \mathbb{U}$.

Using the identity (1.14) in (2.9), and differentiating with respect to z, we get

$$\frac{zJ'_{\mu,b}(f)(z)}{J_{\mu,b}(f)(z)} - \alpha = (1-\alpha)p(z) + \frac{(1-\alpha)zp'(z)}{b+\alpha+(1-\alpha)p(z)}. \tag{2.10}$$

Suppose now that there exists a point $z_0 \in \mathbb{U}$ such that the conditions (2.1) to (2.4) of Lemma 1 are satisfied. Thus, if $\arg\left(p(z_0)\right) = \frac{\pi}{2}\,\beta$ for $z_0 \in \mathbb{U}$, then

$$\begin{aligned}\frac{z_0 J'_{\mu,b}(f)(z_0)}{J_{\mu,b}(f)(z_0)} - \alpha &= (1-\alpha)p(z_0)\left(1 + \frac{z_0 p'(z_0)/p(z_0)}{b+\alpha+(1-\alpha)p(z_0)}\right)\\ &= (1-\alpha)\,a^{\beta} e^{\frac{i\pi\beta}{2}}\left(1 + \frac{ik\beta}{b+\alpha+(1-\alpha)a^{\beta}e^{\frac{i\pi\beta}{2}}}\right)\end{aligned}$$

This implies that

$$\begin{aligned}&\arg\left(\frac{z_0 J'_{\mu,b}(f)(z_0)}{J_{\mu,b}(f)(z_0)} - \alpha\right)\\ &= \frac{\pi\beta}{2} + \arg\left(1 + \frac{ik\beta}{b+\alpha+(1-\alpha)a^{\beta}e^{\frac{i\pi\beta}{2}}}\right)\\ &= \frac{\pi\beta}{2} + \tan^{-1}\left(\frac{k\beta\left(b+\alpha+(1-\alpha)a^{\beta}\cos\frac{\pi\beta}{2}\right)}{(b+\alpha)^2+(1-\alpha)^2a^{2\beta}+2(b+\alpha)(1-\alpha)a^{\beta}\cos\frac{\pi\beta}{2}+k\beta(1-\alpha)a^{\beta}\sin\frac{\pi\beta}{2}}\right)\\ &\geqq \frac{\pi\beta}{2} \qquad \left(\text{since} \quad k \geqq \frac{1}{2}\left(a+\frac{1}{a}\right) \geqq 1 \text{ and } z_0 \in \mathbb{U}\right). \qquad (2.11)\end{aligned}$$

Thus, (2.11) would contradict our assumption that $f(z) \in \mathscr{S}_s^*(\mu,b,\alpha,\beta)$.

On the other hand, if we set $\arg\left(p(z_0)\right) = -\frac{\pi}{2}\,\beta$, then it can *similarly* be shown that

$$\arg\left(\frac{z_0 J'_{\mu,b}(f)(z_0)}{J_{\mu,b}(f)(z_0)} - \alpha\right) \leqq -\frac{\pi}{2}\,\beta \qquad \left(\text{since} \quad k \leqq \frac{1}{2}\left(a+\frac{1}{a}\right) \leqq 1 \text{ and } z_0 \in \mathbb{U}\right),$$

which again contradicts the assumption that $f(z) \in \mathscr{S}_s^*(\mu,b,\alpha,\beta)$.

Hence the function $p(z)$ defined by (2.9) satisfies the following inequality:

$$\left|\arg\left(p(z)\right)\right| < \frac{\pi}{2}\,\beta \qquad (z \in \mathbb{U}),$$

which implies that $f(z) \in \mathscr{S}_s^*(\mu,b,\alpha,\beta)$. This completes the proof of Theorem 1. □

We next prove the following inclusion relationships.

THEOREM 2. *Let $f \in \mathscr{A}$. Suppose also that*

$$\mu \in \mathbb{C}, \quad 0 \leqq \alpha < 1, \quad 0 < \beta \leqq 1 \quad \textit{and} \quad b > -\alpha.$$

Then

$$\mathscr{K}_c(\mu,b,\alpha,\beta) \subset \mathscr{K}_c(\mu+1,b,\alpha,\beta).$$

Proof. We observe from Theorem 1 that

$$\begin{aligned} f(z) \in \mathscr{K}_c(\mu, b, \alpha, \beta) &\iff zf'(z) \in \mathscr{S}_s^*(\mu, b, \alpha, \beta) \\ &\implies zf'(z) \in \mathscr{S}_s^*(\mu+1, b, \alpha, \beta) \\ &\iff f(z) \in \mathscr{K}_c(\mu+1, b, \alpha, \beta), \end{aligned}$$

which establishes Theorem 2. □

THEOREM 3. *Let* $f \in \mathscr{A}$. *Suppose also that*

$$b \in \mathbb{C}/\mathbb{Z}_0^-, \quad \mu \in \mathbb{C}, \quad 0 \leqq \alpha < 1, \quad 0 < \beta \leqq 1 \quad \textit{and} \quad c > -\alpha,$$

and that

$$\frac{z\big(J_{\mu,b}\ \mathscr{I}_c(f)\big)'(z)}{J_{\mu,b}\ \mathscr{I}_c(f)(z)} \neq \alpha \qquad (z \in \mathbb{U}).$$

Then

$$f(z) \in \mathscr{S}_s^*(\mu, b, \alpha, \beta) \implies \mathscr{I}_c f(z) \in \mathscr{S}_s^*(\mu, b, \alpha, \beta).$$

Proof. We begin by assuming that $f(z) \in \mathscr{S}_s^*(\mu, b, \alpha, \beta)$ and defining a function $q(z)$ by

$$q(z) = \frac{1}{1-\alpha}\left(\frac{z\big(J_{\mu,b}\ \mathscr{I}_c(f)\big)'(z)}{J_{\mu,b}\ \mathscr{I}_c(f)(z)} - \alpha\right) \qquad (z \in \mathbb{U}), \tag{2.12}$$

where $q(z)$ is analytic in $\mathbb{U}$, with $q(0) = 1$ and $q(z) \neq 0$ for $z \in \mathbb{U}$.

It can easily be verified from (1.9) and (1.12) that

$$z\big(J_{\mu,b}\ \mathscr{I}_c(f)\big)'(z) = (c+1)J_{\mu,b}(f)(z) - c\, J_{\mu,b}\ \mathscr{I}_c(f)(z). \tag{2.13}$$

Thus, by using (2.13) and (2.12), we find that

$$\frac{zJ'_{\mu,b}(f)(z)}{J_{\mu,b}(f)(z)} - \alpha = (1-\alpha)q(z) + \frac{(1-\alpha)zq'(z)}{c+\alpha+(1-\alpha)q(z)}.$$

The remaining part of the proof of the Theorem 3 is similar to that of Theorem 1 and so we omit it. □

From Theorem 3, we easily see the following result.

THEOREM 4. *Under the parametric constraints stated with Theorem 3, let*

$$f(z) \in \mathscr{A} \qquad \textit{and} \qquad \frac{\Big(z\big(J_{\mu,b}\ \mathscr{I}_c(f)\big)'\Big)'(z)}{\big(J_{\mu,b}\ \mathscr{I}_c(f)\big)'(z)} \neq \alpha \qquad (z \in \mathbb{U}).$$

Then

$$f(z) \in \mathscr{K}_c(\mu, b, \alpha, \beta) \implies \mathscr{I}_c(f)(z) \in \mathscr{K}_c(\mu, b, \alpha, \beta).$$

REMARK. Upon setting $b = 1$, Theorems 1 to 4 would yield the corresponding known results due to Liu [5] for $\mu > 0$.

3. Corollaries and Applications

In this concluding section, we deduce the following consequences of our main results (Theorems 1 to 4) established in Section 2.

First of all, on setting $\mu = 1$ and $b = 0$, Theorem 1 would yield the following result.

COROLLARY 1. *Let*

$$f(z) \in \mathscr{A} \qquad \textit{and} \qquad f(z) \neq \alpha \int_0^z \frac{f(t)}{t}\,dt \qquad (z \in \mathbb{U}).$$

If $f(z)$ *satisfies the following inequality*:

$$\left|\arg\left(\frac{f(z)}{\int_0^z \frac{f(t)}{t}\,dt} - \alpha\right)\right| < \frac{\pi}{2}\beta \qquad (0 \leqq \alpha < 1;\ 0 < \beta \leqq 1),$$

then

$$\int_0^z \frac{1}{t_1}\int_0^{t_1} \frac{f(t_2)}{t_2}\,dt_2\,dt_1 \in \mathscr{S}_s^*(\alpha,\beta) \qquad (t_1, t_2 \in \mathbb{U}).$$

Next, if we set $\mu = 1$ and $b = 1$ in Theorem 1, we get Corollary 2 below.

COROLLARY 2. *Let*

$$f(z) \in \mathscr{A} \qquad \textit{and} \qquad zf(z) \neq (\alpha+1)\int_0^z f(t)\,dt \qquad (z \in \mathbb{U}).$$

If $f(z)$ *satisfies the following inequality*:

$$\left|\arg\left(\frac{zf(z)}{\int_0^z f(t)\,dt} - (\alpha+1)\right)\right| < \frac{\pi}{2}\beta \qquad (0 \leqq \alpha < 1;\ 0 < \beta \leqq 1),$$

then

$$\frac{4}{z}\int_0^z \frac{1}{t_1}\int_0^{t_1} f(t_2)\,dt_2\,dt_1 \in \mathscr{S}_s^*(\alpha,\beta) \qquad (t_1, t_2 \in \mathbb{U}).$$

By putting $\mu = 1$ and $b = 0$, in Theorem 2, we arrive at Corollary 3 below.

COROLLARY 3. *Let*

$$f(z) \in \mathscr{A} \qquad \textit{and} \qquad zf'(z) \neq \alpha f(z) \qquad (z \in \mathbb{U}).$$

If $f(z) \in \mathscr{S}_s^*(\alpha,\beta)$, *then*

$$\int_0^z \frac{1}{t_1}\int_0^{t_1} \frac{f(t_2)}{t_2}\,dt_2\,dt_1 \in \mathscr{K}_c(\alpha,\beta) \qquad (t_1, t_2 \in \mathbb{U}).$$

Upon setting $\mu = 1$ and $b = 1$, Theorem 2 would yield the following result.

COROLLARY 4. *Let*

$$f(z) \in \mathscr{A} \qquad \text{and} \qquad z^2 f'(z) \neq (\alpha+1)\left(zf(z) - \int_0^z f(t)\,dt\right) \qquad (z \in \mathbb{U}).$$

If $f(z)$ *satisfies the following inequality*:

$$\left|\arg\left(\frac{z^2 f'(z)}{zf(z) - \int_0^z f(t)\,dt} - (\alpha+1)\right)\right| < \frac{\pi}{2}\,\beta \qquad (0 \leqq \alpha < 1;\ 0 < \beta \leqq 1),$$

then

$$\frac{4}{z}\int_0^z \frac{1}{t_1}\int_0^{t_1} f(t_2)\,dt_2\,dt_1 \in \mathscr{K}_c(\alpha,\beta) \qquad (t_1, t_2 \in \mathbb{U}).$$

Putting $\mu = 0$ in Theorem 1 (or in Theorem 3), we get the following Corollary for the integral operator $\mathscr{I}_c(f)$ given by (1.7).

COROLLARY 5. *If* $f(z) \in \mathscr{A}$. *Suppose also that*

$$0 \leqq \alpha < 1, \quad 0 < \beta \leqq 1 \qquad \text{and} \qquad c > -\alpha,$$

and that

$$\frac{z\,\mathscr{I}_c'(f)(z)}{\mathscr{I}_c(f)(z)} \neq \alpha \qquad (z \in \mathbb{U}).$$

Then

$$f(z) \in \mathscr{S}_s^*(\alpha,\beta) \implies \mathscr{I}_c(f)(z) \in \mathscr{S}_s^*(\alpha,\beta).$$

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