

APPLICATIONS OF SRIVASTAVA–ATTIYA OPERATOR TO THE CLASSES OF STRONGLY STARLIKE AND STRONGLY CONVEX FUNCTIONS

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Abstract. Srivastava-Attiya operator is used to define some new subclasses of strongly starlike and strongly convex functions of order β and type α in the open unit disk \mathbb{U} . For each of these new function classes, several inclusion relationships are established. Some interesting corollaries and applications of the results presented here are also discussed.

1. Introduction

Let \mathcal{A} denote the class of functions $f(z)$ normalized by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad (1.1)$$

which are *analytic* in the *open unit disk* $\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$. A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{S}^*(\alpha)$ of starlike functions of order α in \mathbb{U} , if it satisfies the following inequality:

$$\Re \left(\frac{z f'(z)}{f(z)} \right) > \alpha \quad (z \in \mathbb{U}, 0 \leq \alpha < 1), \quad (1.2)$$

and is said to be in the class $\mathcal{K}(\alpha)$ of convex functions of order α in \mathbb{U} , if it satisfies the following inequality:

$$\Re \left(1 + \frac{z f''(z)}{f'(z)} \right) > \alpha \quad (z \in \mathbb{U}, 0 \leq \alpha < 1). \quad (1.3)$$

On the other hand, a function $f(z) \in \mathcal{A}$ is said to be in the class of strongly starlike functions of order β and type α , denoted by $\mathcal{S}_s^*(\alpha, \beta)$, if it satisfies the following inequality:

$$\left| \arg \left(\frac{z f'(z)}{f(z)} - \alpha \right) \right| < \frac{\pi}{2} \beta \quad (z \in \mathbb{U}; 0 \leq \alpha < 1; 0 < \beta \leq 1), \quad (1.4)$$

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and is said to be in a corresponding class $\mathcal{K}_c(\alpha, \beta)$ of strongly convex functions of order β and type α , if

$$\left| \arg \left(1 + \frac{zf''(z)}{f'(z)} - \alpha \right) \right| < \frac{\pi}{2} \beta \quad (z \in \mathbb{U}; 0 \leq \alpha < 1; 0 < \beta \leq 1). \tag{1.5}$$

It is obvious that $f(z) \in \mathcal{K}_c(\alpha, \beta)$, if and only if $zf'(z) \in \mathcal{S}_s^*(\alpha, \beta)$. We also observe that

$$\mathcal{S}_s^*(0, \beta) = \mathcal{S}_s^*(\beta) \quad \text{and} \quad \mathcal{K}_c(0, \beta) = \mathcal{K}_c(\beta),$$

where for $0 < \beta \leq 1$, $\mathcal{S}_s^*(\beta)$ and $\mathcal{K}_c(\beta)$ are, respectively, the classes of strongly star-like functions of order β and strongly convex functions of order β in \mathbb{U} . Furthermore, we have the following relationships:

$$\mathcal{S}_s^*(\alpha, 1) = \mathcal{S}^*(\alpha) \quad \text{and} \quad \mathcal{K}_c(\alpha, 1) = \mathcal{K}(\alpha)$$

The generalized Hurwitz-Lerch Zeta function $\phi(z, s, a)$ is defined by (cf, e.g., [9, p. 121 *et seq.*]):

$$\phi(z, s, a) = \sum_{n=0}^{\infty} \frac{z^n}{(a+n)^s} \tag{1.6}$$

$$(a \in \mathbb{C}/\mathbb{Z}_0^-; s \in \mathbb{C} \text{ when } |z| < 1; \Re(s) > 1 \text{ when } |z| = 1),$$

contains, as its special cases, well-known functions as the Riemann and Hurwitz (or generalized) Zeta function, Lerch Zeta function, the Polylogarithmic function and the Lipschitz-Lerch Zeta function. One may refer to the Srivastava and Choi [9] (see also, [8]) for further details and references to these functions.

Srivastava and Attiya in [8] (see also, [7]), introduced the following family of linear operator:

$$J_{\mu, b} : \mathcal{A} \longrightarrow \mathcal{A},$$

defined by

$$J_{\mu, b}(f)(z) = G_{\mu, b}(z) * f(z) \quad (z \in \mathbb{U}; b \in \mathbb{C}/\mathbb{Z}_0^-; \mu \in \mathbb{C}; f \in \mathcal{A}), \tag{1.7}$$

where $*$ denote the Hadamard product (or convolution) of analytic functions and function $G_{\mu, b}$ is given by

$$\begin{aligned} G_{\mu, b}(z) &:= (1+b)^\mu [\phi(z, \mu, b) - b^{-\mu}] \\ &= z + \sum_{n=2}^{\infty} \left(\frac{b+1}{b+n} \right)^\mu z^n \quad (z \in \mathbb{U}). \end{aligned} \tag{1.8}$$

Now using (1.8) in (1.7), we get

$$J_{\mu, b}(f)(z) = z + \sum_{n=2}^{\infty} \left(\frac{b+1}{b+n} \right)^\mu a_n z^n \quad (z \in \mathbb{U}; f \in \mathcal{A}). \tag{1.9}$$

For $f(z) \in \mathcal{A}$ and $z \in \mathbb{U}$, Srivastava and Attiya in [8] showed that

$$J_{0,b}(f)(z) := f(z), \tag{1.10}$$

$$J_{1,0}(f)(z) = \int_0^z \frac{f(t)}{t} dt := A(f)(z), \tag{1.11}$$

$$J_{1,\gamma}(f)(z) = \frac{1+\gamma}{z^\gamma} \int_0^z t^{\gamma-1} f(t) dt := \mathcal{S}_\gamma(f)(z) \quad (\gamma > -1), \tag{1.12}$$

$$J_{\sigma,1}(f)(z) = z + \sum_{n=2}^{\infty} \left(\frac{2}{n+1} \right)^\sigma a_n z^n := I^\sigma(f)(z) \quad (\sigma > 0), \tag{1.13}$$

where $A(f)$ and \mathcal{S}_γ are the integral operators introduced by Alexandor [1] and Bernardi [2], respectively, and $I^\sigma(f)$ is the Jung-Kim-Srivastava integral operator [4] closely related to some multiplier transformation studied by Fleet [3] (see also, [5]).

By applying the definition (1.9), Srivastava and Attiya obtained the following relation [8, p. 210, Eq. (24)]:

$$zJ'_{\mu+1,b}(f)(z) = (b+1)J_{\mu,b}(f)(z) - bJ_{\mu+1,b}(f)(z). \tag{1.14}$$

Using the linear operator $J_{\mu,b}$, we now introduce the following subclasses of \mathcal{A} :

$$\mathcal{S}_s^*(\mu, b, \alpha, \beta) := \left\{ f : f(z) \in \mathcal{A}, J_{\mu,b}(f)(z) \in \mathcal{S}_s^*(\alpha, \beta) \text{ and } \frac{zJ'_{\mu,b}(f)(z)}{J_{\mu,b}(f)(z)} \neq \alpha \ (z \in \mathbb{U}) \right\} \tag{1.15}$$

and

$$\mathcal{K}_c(\mu, b, \alpha, \beta) := \left\{ f : f(z) \in \mathcal{A}, J_{\mu,b}(f)(z) \in \mathcal{K}_c(\alpha, \beta) \text{ and } \frac{(zJ'_{\mu,b}(f)(z))'}{J'_{\mu,b}(f)(z)} \neq \alpha \ (z \in \mathbb{U}) \right\}. \tag{1.16}$$

It is obvious from the definitions (1.15) and (1.16) that

$$f(z) \in \mathcal{K}_c(\mu, b, \alpha, \beta) \iff z f'(z) \in \mathcal{S}_s^*(\mu, b, \alpha, \beta).$$

2. Main Results

In order to derive our main results, we shall need the following lemmas.

LEMMA 1. (see [6]) Let a function $p(z)$ be analytic in \mathbb{U} with

$$p(0) = 1 \quad \text{and} \quad p(z) \neq 0 \quad (z \in \mathbb{U}).$$

If there exists a point $z_0 \in \mathbb{U}$ such that

$$|\arg(p(z))| < \frac{\pi}{2} \beta \quad (|z| < |z_0|) \quad \text{and} \quad |\arg(p(z_0))| = \frac{\pi}{2} \beta \quad (0 < \beta \leq 1), \tag{2.1}$$

then

$$\frac{z_0 p'(z_0)}{p(z_0)} = ik\beta, \tag{2.2}$$

where

$$k \geq \frac{1}{2} \left(a + \frac{1}{a} \right) \quad \text{when} \quad \arg(p(z_0)) = \frac{\pi}{2} \beta, \tag{2.3}$$

$$k \leq -\frac{1}{2} \left(a + \frac{1}{a} \right) \quad \text{when} \quad \arg(p(z_0)) = -\frac{\pi}{2} \beta, \tag{2.4}$$

and

$$(p(z_0))^{\frac{1}{\beta}} = \pm ia \quad (a > 0).$$

LEMMA 2. (see, [8, p. 210]) Let $f \in \mathcal{A}$, $z, t_j \in \mathbb{U}$ ($j = 1, \dots, n$), $n \in \mathbb{N}$ and $b \in \mathbb{C}/\mathbb{Z}_0^-$, we have

$$J_{2,0}(f)(z) =: \int_0^z \frac{1}{t_1} \int_0^{t_1} \frac{f(t_2)}{t_2} dt_2 dt_1, \tag{2.5}$$

$$J_{n,0}(f)(z) =: \int_0^z \frac{1}{t_1} \int_0^{t_1} \frac{1}{t_2} \int_0^{t_2} \dots \frac{1}{t_{n-1}} \int_0^{t_{n-1}} \frac{f(t_n)}{t_n} dt_n dt_{n-1} \dots dt_1, \tag{2.6}$$

$$J_{2,b}(f)(z) =: \frac{(1+b)^2}{z^b} \int_0^z \frac{1}{t_1} \int_0^{t_1} t_2^{b-1} f(t_2) dt_2 dt_1, \tag{2.7}$$

$$J_{n,b}(f)(z) =: \frac{(1+b)^n}{z^b} \int_0^z \frac{1}{t_1} \int_0^{t_1} \frac{1}{t_2} \int_0^{t_2} \dots \frac{1}{t_{n-1}} \int_0^{t_{n-1}} t_n^{b-1} f(t_n) dt_n dt_{n-1} \dots dt_1. \tag{2.8}$$

Theorem 1 below gives our first main inclusion relationship.

THEOREM 1. Let $f \in \mathcal{A}$. Suppose also that

$$\mu \in \mathbb{C}, \quad 0 \leq \alpha < 1, \quad 0 < \beta \leq 1 \quad \text{and} \quad b > -\alpha.$$

Then

$$\mathcal{S}_s^*(\mu, b, \alpha, \beta) \subset \mathcal{S}_s^*(\mu + 1, b, \alpha, \beta).$$

proof. Let $f \in \mathcal{S}_s^*(\mu, b, \alpha, \beta)$. Then, upon setting

$$p(z) = \frac{1}{1-\alpha} \left(\frac{zJ'_{\mu+1,b}(f)(z)}{J_{\mu+1,b}(f)(z)} - \alpha \right) \quad (z \in \mathbb{U}), \tag{2.9}$$

we see that the function $p(z)$ is analytic in \mathbb{U} , with $p(0) = 1$ and $p(z) \neq 0$ for $z \in \mathbb{U}$.

Using the identity (1.14) in (2.9), and differentiating with respect to z , we get

$$\frac{zJ'_{\mu,b}(f)(z)}{J_{\mu,b}(f)(z)} - \alpha = (1-\alpha)p(z) + \frac{(1-\alpha)zp'(z)}{b+\alpha+(1-\alpha)p(z)}. \tag{2.10}$$

Suppose now that there exists a point $z_0 \in \mathbb{U}$ such that the conditions (2.1) to (2.4) of Lemma 1 are satisfied. Thus, if $\arg(p(z_0)) = \frac{\pi}{2} \beta$ for $z_0 \in \mathbb{U}$, then

$$\begin{aligned} \frac{z_0 J'_{\mu,b}(f)(z_0)}{J_{\mu,b}(f)(z_0)} - \alpha &= (1 - \alpha)p(z_0) \left(1 + \frac{z_0 p'(z_0)/p(z_0)}{b + \alpha + (1 - \alpha)p(z_0)} \right) \\ &= (1 - \alpha) a^\beta e^{\frac{i\pi\beta}{2}} \left(1 + \frac{ik\beta}{b + \alpha + (1 - \alpha)a^\beta e^{\frac{i\pi\beta}{2}}} \right) \end{aligned}$$

This implies that

$$\begin{aligned} &\arg\left(\frac{z_0 J'_{\mu,b}(f)(z_0)}{J_{\mu,b}(f)(z_0)} - \alpha\right) \\ &= \frac{\pi\beta}{2} + \arg\left(1 + \frac{ik\beta}{b + \alpha + (1 - \alpha)a^\beta e^{\frac{i\pi\beta}{2}}}\right) \\ &= \frac{\pi\beta}{2} + \tan^{-1}\left(\frac{k\beta\left(b + \alpha + (1 - \alpha)a^\beta \cos \frac{\pi\beta}{2}\right)}{(b + \alpha)^2 + (1 - \alpha)^2 a^{2\beta} + 2(b + \alpha)(1 - \alpha)a^\beta \cos \frac{\pi\beta}{2} + k\beta(1 - \alpha)a^\beta \sin \frac{\pi\beta}{2}}\right) \\ &\geq \frac{\pi\beta}{2} \quad \left(\text{since } k \geq \frac{1}{2}\left(a + \frac{1}{a}\right) \geq 1 \text{ and } z_0 \in \mathbb{U}\right). \end{aligned} \tag{2.11}$$

Thus, (2.11) would contradict our assumption that $f(z) \in \mathcal{S}_s^*(\mu, b, \alpha, \beta)$.

On the other hand, if we set $\arg(p(z_0)) = -\frac{\pi}{2} \beta$, then it can *similarly* be shown that

$$\arg\left(\frac{z_0 J'_{\mu,b}(f)(z_0)}{J_{\mu,b}(f)(z_0)} - \alpha\right) \leq -\frac{\pi}{2} \beta \quad \left(\text{since } k \leq \frac{1}{2}\left(a + \frac{1}{a}\right) \leq 1 \text{ and } z_0 \in \mathbb{U}\right),$$

which again contradicts the assumption that $f(z) \in \mathcal{S}_s^*(\mu, b, \alpha, \beta)$.

Hence the function $p(z)$ defined by (2.9) satisfies the following inequality:

$$|\arg(p(z))| < \frac{\pi}{2} \beta \quad (z \in \mathbb{U}),$$

which implies that $f(z) \in \mathcal{S}_s^*(\mu, b, \alpha, \beta)$. This completes the proof of Theorem 1. \square

We next prove the following inclusion relationships.

THEOREM 2. *Let $f \in \mathcal{A}$. Suppose also that*

$$\mu \in \mathbb{C}, \quad 0 \leq \alpha < 1, \quad 0 < \beta \leq 1 \quad \text{and} \quad b > -\alpha.$$

Then

$$\mathcal{K}_c(\mu, b, \alpha, \beta) \subset \mathcal{K}_c(\mu + 1, b, \alpha, \beta).$$

Proof. We observe from Theorem 1 that

$$\begin{aligned} f(z) \in \mathcal{H}_c(\mu, b, \alpha, \beta) &\iff z f'(z) \in \mathcal{S}_s^*(\mu, b, \alpha, \beta) \\ &\implies z f'(z) \in \mathcal{S}_s^*(\mu + 1, b, \alpha, \beta) \\ &\iff f(z) \in \mathcal{H}_c(\mu + 1, b, \alpha, \beta), \end{aligned}$$

which establishes Theorem 2. \square

THEOREM 3. *Let $f \in \mathcal{A}$. Suppose also that*

$$b \in \mathbb{C}/\mathbb{Z}_0^-, \quad \mu \in \mathbb{C}, \quad 0 \leq \alpha < 1, \quad 0 < \beta \leq 1 \quad \text{and} \quad c > -\alpha,$$

and that

$$\frac{z(J_{\mu,b} \mathcal{I}_c(f))'(z)}{J_{\mu,b} \mathcal{I}_c(f)(z)} \neq \alpha \quad (z \in \mathbb{U}).$$

Then

$$f(z) \in \mathcal{S}_s^*(\mu, b, \alpha, \beta) \implies \mathcal{I}_c f(z) \in \mathcal{S}_s^*(\mu, b, \alpha, \beta).$$

Proof. We begin by assuming that $f(z) \in \mathcal{S}_s^*(\mu, b, \alpha, \beta)$ and defining a function $q(z)$ by

$$q(z) = \frac{1}{1-\alpha} \left(\frac{z(J_{\mu,b} \mathcal{I}_c(f))'(z)}{J_{\mu,b} \mathcal{I}_c(f)(z)} - \alpha \right) \quad (z \in \mathbb{U}), \tag{2.12}$$

where $q(z)$ is analytic in \mathbb{U} , with $q(0) = 1$ and $q(z) \neq 0$ for $z \in \mathbb{U}$.

It can easily be verified from (1.9) and (1.12) that

$$z(J_{\mu,b} \mathcal{I}_c(f))'(z) = (c+1)J_{\mu,b}(f)(z) - c J_{\mu,b} \mathcal{I}_c(f)(z). \tag{2.13}$$

Thus, by using (2.13) and (2.12), we find that

$$\frac{zJ'_{\mu,b}(f)(z)}{J_{\mu,b}(f)(z)} - \alpha = (1-\alpha)q(z) + \frac{(1-\alpha)zq'(z)}{c+\alpha+(1-\alpha)q(z)}.$$

The remaining part of the proof of the Theorem 3 is similar to that of Theorem 1 and so we omit it. \square

From Theorem 3, we easily see the following result.

THEOREM 4. *Under the parametric constraints stated with Theorem 3, let*

$$f(z) \in \mathcal{A} \quad \text{and} \quad \frac{(z(J_{\mu,b} \mathcal{I}_c(f))')'(z)}{(J_{\mu,b} \mathcal{I}_c(f))'(z)} \neq \alpha \quad (z \in \mathbb{U}).$$

Then

$$f(z) \in \mathcal{H}_c(\mu, b, \alpha, \beta) \implies \mathcal{I}_c(f)(z) \in \mathcal{H}_c(\mu, b, \alpha, \beta).$$

REMARK. Upon setting $b = 1$, Theorems 1 to 4 would yield the corresponding known results due to Liu [5] for $\mu > 0$.

3. Corollaries and Applications

In this concluding section, we deduce the following consequences of our main results (Theorems 1 to 4) established in Section 2.

First of all, on setting $\mu = 1$ and $b = 0$, Theorem 1 would yield the following result.

COROLLARY 1. *Let*

$$f(z) \in \mathcal{A} \quad \text{and} \quad f(z) \neq \alpha \int_0^z \frac{f(t)}{t} dt \quad (z \in \mathbb{U}).$$

If $f(z)$ satisfies the following inequality:

$$\left| \arg \left(\frac{f(z)}{\int_0^z \frac{f(t)}{t} dt} - \alpha \right) \right| < \frac{\pi}{2} \beta \quad (0 \leq \alpha < 1; 0 < \beta \leq 1),$$

then

$$\int_0^z \frac{1}{t_1} \int_0^{t_1} \frac{f(t_2)}{t_2} dt_2 dt_1 \in \mathcal{S}_s^*(\alpha, \beta) \quad (t_1, t_2 \in \mathbb{U}).$$

Next, if we set $\mu = 1$ and $b = 1$ in Theorem 1, we get Corollary 2 below.

COROLLARY 2. *Let*

$$f(z) \in \mathcal{A} \quad \text{and} \quad zf(z) \neq (\alpha + 1) \int_0^z f(t) dt \quad (z \in \mathbb{U}).$$

If $f(z)$ satisfies the following inequality:

$$\left| \arg \left(\frac{zf(z)}{\int_0^z f(t) dt} - (\alpha + 1) \right) \right| < \frac{\pi}{2} \beta \quad (0 \leq \alpha < 1; 0 < \beta \leq 1),$$

then

$$\frac{4}{z} \int_0^z \frac{1}{t_1} \int_0^{t_1} f(t_2) dt_2 dt_1 \in \mathcal{S}_s^*(\alpha, \beta) \quad (t_1, t_2 \in \mathbb{U}).$$

By putting $\mu = 1$ and $b = 0$, in Theorem 2, we arrive at Corollary 3 below.

COROLLARY 3. *Let*

$$f(z) \in \mathcal{A} \quad \text{and} \quad zf'(z) \neq \alpha f(z) \quad (z \in \mathbb{U}).$$

If $f(z) \in \mathcal{S}_s^*(\alpha, \beta)$, then

$$\int_0^z \frac{1}{t_1} \int_0^{t_1} \frac{f(t_2)}{t_2} dt_2 dt_1 \in \mathcal{K}_c(\alpha, \beta) \quad (t_1, t_2 \in \mathbb{U}).$$

Upon setting $\mu = 1$ and $b = 1$, Theorem 2 would yield the following result.

COROLLARY 4. *Let*

$$f(z) \in \mathcal{A} \quad \text{and} \quad z^2 f'(z) \neq (\alpha + 1) \left(z f(z) - \int_0^z f(t) dt \right) \quad (z \in \mathbb{U}).$$

If $f(z)$ satisfies the following inequality:

$$\left| \arg \left(\frac{z^2 f'(z)}{z f(z) - \int_0^z f(t) dt} - (\alpha + 1) \right) \right| < \frac{\pi}{2} \beta \quad (0 \leq \alpha < 1; 0 < \beta \leq 1),$$

then

$$\frac{4}{z} \int_0^z \frac{1}{t_1} \int_0^{t_1} f(t_2) dt_2 dt_1 \in \mathcal{K}_c(\alpha, \beta) \quad (t_1, t_2 \in \mathbb{U}).$$

Putting $\mu = 0$ in Theorem 1 (or in Theorem 3), we get the following Corollary for the integral operator $\mathcal{I}_c(f)$ given by (1.7).

COROLLARY 5. *If $f(z) \in \mathcal{A}$. Suppose also that*

$$0 \leq \alpha < 1, \quad 0 < \beta \leq 1 \quad \text{and} \quad c > -\alpha,$$

and that

$$\frac{z \mathcal{I}'_c(f)(z)}{\mathcal{I}_c(f)(z)} \neq \alpha \quad (z \in \mathbb{U}).$$

Then

$$f(z) \in \mathcal{S}_s^*(\alpha, \beta) \implies \mathcal{I}_c(f)(z) \in \mathcal{S}_s^*(\alpha, \beta).$$

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