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ON UNICITY OF MEROMORPHIC FUNCTION AND ITS kTH ORDER DERIVATIVE

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Abstract. In this paper, we study the uniqueness problems on meromorphic function and its kth order derivative. The results in this paper improve the results given by K. W. Yu (On entire and meromorphic functions that share small functions with their derivatives, J. Inequal. Pure Appl. Math. 4(1)(2003), Art. 21), L. P. Liu and Y. X. Gu (Uniqueness of meromorphic functions that share one small function with their derivatives, Kodai Math. J. 27(2004), 272-279) and supplement a result of S. H. Lin and W. C. Lin (Uniqueness of meromorphic functions concerning weakly weighted sharing, Kodai Math. J. 29(2006), 269-280).

1. Introduction, definitions and results

By a meromorphic function we shall always mean a meromorphic function in the complex plane. Set $E(a,f)=\{z: f(z)-a=0\}$, where a zero point with multiplicity m is counted m times in the set. If these zeros points are only counted once, then we denote the set by $\overline{E}(a,f)$. Let f and g be two nonconstant meromorphic functions. If $E(a,f)=E(a,g)$, then we say that f and g share the value a CM; if $\overline{E}(a,f)=\overline{E}(a,g)$, then we say that f and g share the value a IM. Let m be a positive integer or infinity and $a\in C\cup\{\infty\}$. We denote by $E_{m)}(a,f)$ the set of all a-points of f with multiplicities not exceeding m, where an a-point is counted according to its multiplicity. Also we denote by $\overline{E}_{m)}(a,f)$ the set of distinct a-points of f with multiplicities not greater than m. We denote by $N_{k)}(r,1/(f-a))$ the counting function for zeros of $f-a$ with multiplicity $\leqslant k$, and by $\overline{N}_{k)}(r,1/(f-a))$ the corresponding one for which multiplicity is not counted. Let $N_{(k}(r,1/(f-a))$ be the counting function for zeros of $f-a$ with multiplicity at least k and $\overline{N}_{(k}(r,1/(f-a))$ the corresponding one for which multiplicity is not counted. Set

$$N_k\left(r,\frac{1}{f-a}\right)=\overline{N}\left(r,\frac{1}{f-a}\right)+\overline{N}_{(2}\left(r,\frac{1}{f-a}\right)+\ldots+\overline{N}_{(k}\left(r,\frac{1}{f-a}\right).$$

By the above definition, we have

$$\overline{N}\left(r,\frac{1}{h}\right)+\overline{N}_{(2}\left(r,\frac{1}{h}\right)=N_2\left(r,\frac{1}{h}\right)\leqslant N\left(r,\frac{1}{h}\right).$$

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Let $N_E(r,a;f,g)(\overline{N}_E(r,a;f,g))$ be the counting function(reduced counting function) of all common zeros of $f-a$ and $g-a$ with the same multiplicities and $N_0(r,a;f,g)$ $(\overline{N}_0(r,a;f,g))$ be the counting function(reduced counting function) of all common zeros of $f-a$ and $g-a$ ignoring multiplicities. If

$$\overline{N}\left(r,\frac{1}{f-a}\right)+\overline{N}\left(r,\frac{1}{g-a}\right)-2\overline{N}_E(r,a;f,g)=S(r,f)+S(r,g),$$

then we say that f and g share a "CM". On the other hand, if

$$\overline{N}\left(r,\frac{1}{f-a}\right)+\overline{N}\left(r,\frac{1}{g-a}\right)-2\overline{N}_0(r,a;f,g)=S(r,f)+S(r,g),$$

then we say that f and g share a "IM". It is assumed that the reader is familiar with the notations of Nevanlinna theory, that can be found, for instance, in [7] and [15]. Denote

$$\Theta(\infty,f)=1-\limsup_{r\to\infty}\frac{\overline{N}(r,f)}{T(r,f)},\qquad \delta_p(0,f)=1-\limsup_{r\to\infty}\frac{N_p(r,1/f)}{T(r,f)}.$$

Rubel and Yang [12], Gundersen [5], Yang [13] and many other authors have obtained elegant results on the uniqueness problems of entire functions that share values CM or IM with their first or k-th derivatives. In the aspect of only one CM value, R. Brück posed the following conjecture.

CONJECTURE. [3] *Let f be a nonconstant entire function. Suppose that $\rho_1(f)$ is not a positive integer or infinite, if f and f' share one finite value a CM, then*

$$\frac{f'-a}{f-a}=c,$$

for some non-zero constant c, where $\rho_1(f)$ is the first iterated order of f which is defined by

$$\rho_1(f)=\limsup_{r\to\infty}\frac{\log\log T(r,f)}{\log r}.$$

In 1998, Gundersen and Yang [6] *proved that the conjecture is true if f is of finite order, and in 1999, Yang* [14] *generalized their result to the k-th derivatives. In 2004, Chen and Shon* [4] *proved that the conjecture is true for entire functions of first iterated order $\rho_1(f)<1/2$. In 2003, Yu considered the case that a is a small function and obtained the following result.*

THEOREM A. [16] *Let f be a nonconstant entire function, let k be a positive integer, and let a be a small meromorphic function of f such that $a(z)\not\equiv 0,\infty$. If $f-a$ and $f^{(k)}-a$ share the value 0 CM and $\delta(0,f)>3/4$, then $f\equiv f^{(k)}$.*

THEOREM B. [16] *Let f be a nonconstant meromorphic function, let k be a positive integer, and let a be a small function of f such that $a(z)\not\equiv 0,\infty$, f and a do not have any common pole. If $f-a$ and $f^{(k)}-a$ share the value 0 CM and $4\delta(0,f)+2(8+k)\Theta(\infty,f)>19+2k$, then $f\equiv f^{(k)}$.*

In 2004, Liu and Gu obtained the following result.

THEOREM C. [11] *Let $k \geqslant 1$ and let f be a nonconstant meromorphic function, and let a be a small function of f such that $a(z) \not\equiv 0,\infty$. If $f-a$ and $f^{(k)}-a$ share the value 0 CM and a do not have any common poles of same multiplicity and $2\delta(0,f)+4\Theta(\infty,f)>5$, then $f\equiv f^{(k)}$.*

In 2006, Lin and Lin [10] introduced the following notion of weakly weighted sharing which is the generalization of the idea of weighted sharing introduced in [8].

DEFINITION 1. [10] Let f and g share a "IM" and k be a positive integer or ∞. $\overline{N}^{E}_{k)}(r,a;f,g)$ denotes the reduced counting function of those a-points of f whose multiplicities are equal to the corresponding a-points of g, both of their multiplicities are not greater than k. $\overline{N}^{O}_{(k}(r,a;f,g)$ denotes the reduced counting function of those a-points of f which are a-points of g, both of their multiplicities are not less than k.

DEFINITION 2. [10] For $a\in C\cup\{\infty\}$, if k is a positive integer or ∞ and

$$\overline{N}_{k)}\left(r,\frac{1}{f-a}\right)-\overline{N}^{E}_{k)}(r,a;f,g)=S(r,f),$$
$$\overline{N}_{k)}\left(r,\frac{1}{g-a}\right)-\overline{N}^{E}_{k)}(r,a;f,g)=S(r,g),$$
$$\overline{N}_{(k+1}\left(r,\frac{1}{f-a}\right)-\overline{N}^{O}_{(k+1}(r,a;f,g)=S(r,f),$$
$$\overline{N}_{(k+1}\left(r,\frac{1}{g-a}\right)-\overline{N}^{O}_{(k+1}(r,a;f,g)=S(r,g),$$

or if $k=0$ and

$$\overline{N}\left(r,\frac{1}{f-a}\right)-\overline{N}_0(r,a;f,g)=S(r,f),\overline{N}\left(r,\frac{1}{g-a}\right)-\overline{N}_0(r,a;f,g)=S(r,g),$$

then we say f and g weakly share a with weight k. Here we write f, g share "(a,k)" to mean that f, g weakly share a with weight k.

Lin and Lin [10] improved Theorem C with the notion of weakly weighted sharing.

THEOREM D. [10] *Let $k\geqslant 1$ and $2\leqslant m\leqslant\infty$. Let f be a nonconstant meromorphic function, and let a be a small function of f such that $a(z)\not\equiv 0,\infty$. If f and $f^{(k)}$ share "(a,m)" and $2\delta_{2+k}(0,f)+4\Theta(\infty,f)>5$, then $f\equiv f^{(k)}$.*

Recently, A. Banerjee [2] introduced another sharing notion which is also a scaling between "IM" and "CM" but weaker than weakly weighted sharing.

DEFINITION 3. [2] We denote by $\overline{N}(r,a;f|=p;g|=q)$ the reduced counting function of common a-points of f and g with multiplicities p and q respectively.

DEFINITION 4. [2] Let f, g share a "IM". Also let k be a positive integer or ∞ and $a\in C\cup\{\infty\}$. If

$$\sum_{p,q\leqslant k(p\neq q)}\overline{N}(r,a;f|=p;g|=q)=S(r),$$

then we say f and g share a with weight k in a relaxed manner. Here we write f and g share $(a,k)^*$ to mean that f and g share a with weight k in a relaxed manner.

Now we state the main results of this paper.

THEOREM 1. *Let f be a nonconstant meromorphic function, and let a be a small function of f such that $a(z)\not\equiv 0,\infty$. If $\overline{E}_{4)}(a,f)=\overline{E}_{4)}(a,f^{(k)})$, $E_{2)}(a,f)=E_{2)}(a,f^{(k)})$ and $2\delta_{2+k}(0,f)+(\frac{k}{2}+4)\Theta(\infty,f)>5+\frac{k}{2}$, then $f\equiv f^{(k)}$.*

THEOREM 2. *Let f be a nonconstant meromorphic function, and let a be a small function of f such that $a(z)\not\equiv 0,\infty$. If f and $f^{(k)}$ share $(a,2)^*$ and $3\delta_{2+k}(0,f)+(k+5)\Theta(\infty,f)>k+7$, then $f\equiv f^{(k)}$.*

If f is a nonconstant entire function, then $\Theta(\infty,f)=1$. So we have the following results.

COROLLARY 1. *Let f be a nonconstant entire function, and let a be a small function of f such that $a(z)\not\equiv 0,\infty$. If $\overline{E}_{4)}(a,f)=\overline{E}_{4)}(a,f^{(k)})$ and $E_{2)}(a,f)=E_{2)}(a,f^{(k)})$ and $\delta_{2+k}(0,f)>\frac{1}{2}$, then $f\equiv f^{(k)}$.*

COROLLARY 2. *Let f be a nonconstant entire function, and let a be a small function of f such that $a(z)\not\equiv 0,\infty$. If f and $f^{(k)}$ share $(a,2)^*$ and $\delta_{2+k}(0,f)>\frac{2}{3}$, then $f\equiv f^{(k)}$.*

REMARK. Corollary 1 and Corollary 2 improve Theorem A. Theorem 1 and Theorem 2 improve Theorem A-C and supplement Theorem D.

2. Some Lemmas

In this section, we present some lemmas which will be needed in the sequel. We will denote by H the following function:

$$H=\left(\frac{F''}{F'}-\frac{2F'}{F-1}\right)-\left(\frac{G''}{G'}-\frac{2G'}{G-1}\right).$$

LEMMA 1. [1] *Let H be defined as above. If $\overline{E}_{4)}(1,F)=\overline{E}_{4)}(1,G)$ and $E_{2)}(1,F)=E_{2)}(1,G)$, and $H\not\equiv 0$, then*

$$T(r,F)+T(r,G)\leqslant 2\left\{N_2\left(r,\frac{1}{F}\right)+N_2(r,F)+N_2\left(r,\frac{1}{G}\right)+N_2(r,G)\right\}+S(r,F)+S(r,G).$$

LEMMA 2. [2] *Let H be defined as above. If F and G share $(1,2)^*$ and $H\not\equiv 0$, then*

$$T(r,F)\leqslant N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)+N_2(r,F)+N_2(r,G)+\overline{N}\left(r,\frac{1}{F}\right)$$
$$+\overline{N}(r,F)-m\left(r,\frac{1}{G-1}\right)+S(r,F)+S(r,G),$$

the same inequality holds for $T(r,G)$.

LEMMA 3. [10] *Let* f *be a nonconstant meromorphic function and let* k *be a positive integer. Then*

$$N_2\left(r,\frac{1}{f^{(k)}}\right)\leqslant N_{2+k}\left(r,\frac{1}{f}\right)+k\overline{N}(r,f)+S(r,f),$$
$$N_2\left(r,\frac{1}{f^{(k)}}\right)\leqslant T(r,f^{(k)})-T(r,f)+N_{2+k}\left(r,\frac{1}{f}\right)+S(r,f).$$

LEMMA 4. [9] *Let* f *be a transcendental meromorphic function and* $\alpha(\not\equiv 0,\infty)$ *be a meromorphic function such that* $T(r,\alpha)=S(r,f)$. *Let* b *and* c *are any two finite nonzero distinct complex numbers. If* $\psi=\alpha(f)^n(f^{(k)})^p$, *where* $n(\geqslant 0)$, $p(\geqslant 1)$ *and* $k(\geqslant 1)$ *are integers, then*

$$(p+n)T(r,f)\leqslant(p+n)N\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{\psi-b}\right)+N\left(r,\frac{1}{\psi-c}\right)$$
$$-N(r,f)-N\left(r,\frac{1}{\psi'}\right)+S(r,f).$$

LEMMA 5. [17] *Let* f *be a nonconstant meromorphic function,* k *be a positive integer, then*

$$N_p\left(r,\frac{1}{f^{(k)}}\right)\leqslant N_{p+k}\left(r,\frac{1}{f}\right)+k\overline{N}(r,f)+S(r,f),$$

where $N_p\left(r,\frac{1}{f^{(k)}}\right)$ *denotes the counting function of the zeros of* $f^{(k)}$ *where a zero of multiplicity* m *is counted* m *times if* $m\leqslant p$ *and* p *times if* $m>p$. *Clearly* $\overline{N}\left(r,\frac{1}{f^{(k)}}\right)=N_1\left(r,\frac{1}{f^{(k)}}\right)$.

3. Proof of Theorem 1

Let

$$F=\frac{f}{a},\qquad G=\frac{f^{(k)}}{a}. \tag{1}$$

Then it is easy to verify $\overline{E}_{4)}(1,F)=\overline{E}_{4)}(1,G)$ and $E_{2)}(1,F)=E_{2)}(1,G)$. Let H be defined as above. Suppose that $H\not\equiv 0$. It follows from Lemma 1 that

$$T(r,F)+T(r,G)\leqslant 2\left\{N_2\left(r,\frac{1}{F}\right)+N_2(r,F)+N_2\left(r,\frac{1}{G}\right)+N_2(r,G)\right\}+S(r,F)+S(r,G).$$

Using Lemma 3, we have

$$\begin{aligned} T(r,f)+T(r,f^{(k)}) &\leqslant 2\left\{N_2\left(r,\frac{1}{f}\right)+N_2(r,f)+N_2\left(r,\frac{1}{f^{(k)}}\right)+N_2(r,f^{(k)})\right\}+S(r,f) \\ &\leqslant N_2\left(r,\frac{1}{f}\right)+N_2(r,f)+N_2\left(r,\frac{1}{f^{(k)}}\right)+N_2(r,f^{(k)})+N_2\left(r,\frac{1}{f}\right) \\ &\quad +T(r,f^{(k)})-T(r,f)+N_{k+2}\left(r,\frac{1}{f}\right)+4\overline{N}(r,f)+S(r,f) \\ &\leqslant T(r,f^{(k)})-T(r,f)+4N_{k+2}\left(r,\frac{1}{f}\right)+(k+8)\overline{N}(r,f)+S(r,f), \end{aligned} \tag{2}$$

that is,

$$T(r,f) \leqslant 2N_{k+2}\left(r,\frac{1}{f}\right)+(\frac{k}{2}+4)\overline{N}(r,f)+S(r,f). \tag{3}$$

It follows that $2\delta_{2+k}(0,f)+(\frac{k}{2}+4)\Theta(\infty,f) \leqslant 5+\frac{k}{2}$, which contradicts $2\delta_{2+k}(0,f)+(\frac{k}{2}+4)\Theta(\infty,f) > 5+\frac{k}{2}$. Therefore $H \equiv 0$. That is

$$\frac{F''}{F'}-2\frac{F'}{F-1} \equiv \frac{G''}{G'}-2\frac{G'}{G-1}. \tag{4}$$

It follows that

$$\frac{1}{F-1}=\frac{A}{G-1}+B, \tag{5}$$

where $A(\neq 0)$ and B are constants. Therefore,

$$F=\frac{(B+1)G+(A-B-1)}{BG+(A-B)}. \tag{6}$$

Now we distinguish the following two cases.

Case 1. Suppose that $B \neq -1,0$.

If $A-B-1 \neq 0$, then from (6), we have

$$\overline{N}\left(r,\frac{1}{G+\frac{A-B-1}{B+1}}\right)=\overline{N}\left(r,\frac{1}{F}\right). \tag{7}$$

By the Second Fundamental Theorem, we have

$$T(r,G) < \overline{N}(r,G)+\overline{N}\left(r,\frac{1}{G}\right)+\overline{N}\left(r,\frac{1}{G+\frac{A-B-1}{B+1}}\right)+S(r,G), \tag{8}$$

that is,

$$\begin{aligned} T(r,f^{(k)}) &< \overline{N}(r,f)+\overline{N}\left(r,\frac{1}{f^{(k)}}\right)+\overline{N}\left(r,\frac{1}{f}\right)+S(r,f) \\ &< \overline{N}(r,f)+T(r,f^{(k)})-T(r,f)+N_{k+2}\left(r,\frac{1}{f}\right)+\overline{N}\left(r,\frac{1}{f}\right)+S(r,f), \end{aligned} \tag{9}$$

and so

$$T(r,f) < \overline{N}(r,f)+2N_{k+2}\left(r,\frac{1}{f}\right)+S(r,f). \tag{10}$$

It follows that $2\delta_{2+k}(0,f)+\Theta(\infty,f)\leqslant 2$, which contradicts $2\delta_{2+k}(0,f)+(\frac{k}{2}+4)\Theta(\infty,f) > 5+\frac{k}{2}$. Therefore $A-B-1=0$. From (6), we obtain

$$\overline{N}\left(r,\frac{1}{G+\frac{1}{B}}\right)=\overline{N}(r,F). \tag{11}$$

Similar to the arguments in the above, we also have a contradiction.

Case 2. Suppose that $B=-1$.

If $A+1\neq 0$. Then from (6), we have

$$\overline{N}\left(r,\frac{1}{G-(A+1)}\right)=\overline{N}(r,F). \tag{12}$$

Similar to the arguments in Case 1, we can get a contradiction. Therefore, $A+1=0$, then from (6), we have $FG\equiv 1$. From (1), we have

$$ff^{(k)}\equiv a^2. \tag{13}$$

In the following, we distinguish two subcases.

(a) If f is a rational function, then a becomes a nonzero constant since a is a small function of f and $a(z)\not\equiv 0,\infty$. If z_0 is a zero of f, z_0 must be a pole of $f^{(k)}$ by (13). This is a contradiction. So f have no zero. Similarly f have no pole. Since f is nonconstant, this is a contradiction.

(b) If f is transcendental then by Lemma 4 and (13), we get

$$\begin{aligned} 2T(r,f) &\leqslant 2N\left(r,\frac{1}{f}\right)+2T(r,ff^{(k)})+S(r,f)\leqslant 2N\left(r,\frac{1}{f}\right)+S(r,f) \\ &\leqslant 2N\left(r,\frac{1}{ff^{(k)}}\right)+S(r,f)=2N\left(r,\frac{1}{a^2}\right)+S(r,f)=S(r,f). \end{aligned} \tag{14}$$

This is a contradiction.

Case 3. Suppose that $B=0$.

If $A-1\neq 0$, then from (6), we have

$$\overline{N}\left(r,\frac{1}{G+(A-1)}\right)=\overline{N}\left(r,\frac{1}{F}\right). \tag{15}$$

Similar to the arguments in Case 1, we also have a contradiction. Therefore, $A-1=0$. From (6), we have $F\equiv G$, this implies $f\equiv f^{(k)}$. This completes the proof of Theorem 1.

4. Proof of Theorem 2

Let

$$F=\frac{f}{a},\qquad G=\frac{f^{(k)}}{a}. \tag{16}$$

Then it is easy to verify F and G share $(1,2)^*$. Let H be defined as above. Suppose that $H\not\equiv 0$. It follows from Lemma 2 that

$$\begin{aligned}T(r,G)\leqslant{}& N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)+N_2(r,F)+N_2(r,G)\\&+\overline{N}\left(r,\frac{1}{G}\right)+\overline{N}(r,G)+S(r,F)+S(r,G).\end{aligned} \tag{17}$$

Using Lemma 3 and Lemma 5, we have

$$\begin{aligned}T(r,f^{(k)})\leqslant{}& N_2(r,f)+N_2\left(r,\frac{1}{f}\right)+N_2(r,f^{(k)})+N_2\left(r,\frac{1}{f^{(k)}}\right)\\&+\overline{N}\left(r,\frac{1}{f^{(k)}}\right)+\overline{N}(r,f^{(k)})+S(r,f)\\ \leqslant{}& N_{k+2}\left(r,\frac{1}{f}\right)+T(r,f^{(k)})-T(r,f)+N_{k+2}\left(r,\frac{1}{f}\right)\\&+N_{k+1}\left(r,\frac{1}{f}\right)+(k+5)\overline{N}(r,f)+S(r,f)\\ \leqslant{}& 3N_{k+2}\left(r,\frac{1}{f}\right)+T(r,f^{(k)})-T(r,f)+(k+5)\overline{N}(r,f)+S(r,f),\end{aligned} \tag{18}$$

that is,

$$T(r,f)\leqslant 3N_{k+2}\left(r,\frac{1}{f}\right)+(k+5)\overline{N}(r,f)+S(r,f). \tag{19}$$

It follows that $3\delta_{2+k}(0,f)+(k+5)\Theta(\infty,f)\leqslant k+7$, which contradicts $3\delta_{2+k}(0,f)+(k+5)\Theta(\infty,f)>k+7$. Therefore $H\equiv 0$. Similar to the arguments in Theorem 1, we see that Theorem 2 holds.

REFERENCES

[1] A. BANERJEE, *On uniqueness of meromorphic functions when two differential monomials share one value*, Bull. Korean Math. Soc., **44**, 4 (2007), 607–622.

[2] A. BANERJEE AND S. MUKHERJEE, *Uniqueness of meromorphic functions concerning differential monomial sharing the same value*, Bull. Math. Soc. Sci. Math. Roumanie, **50** (2007), 191–206.

[3] R. BRÜCK, *On entire functions which share one value CM with their first derivative*, Results Math., **30** (1996), 21–24.

[4] Z.X. CHEN AND K.H. SHON, *On conjecture of R.Bruck concering the entire function sharing one value CM with its derivatives*, Taiwanese J. Math., **8** (2004), 235–244.

[5] G.G. GUNDERSEN, *Meromorphic functions that share finite values with their derivatives*, J. Math. Anal. Appl., **75** (1980), 441–446. Correction, **86** (1982), 307.

[6] G.G. GUNDERSEN AND L.Z. YANG, *Entire functions that share one vlaue with one or two of their derivatives*, J. Math. Anal. Appl., **223** (1998), 88–95.

[7] W.K. HAYMAN, *Meromorphic Functions*, Clarendon, Oxford, 1964.

[8] I. LAHIRI, *Weighted value sharing and uniqueness of meromorphic functions*, Complex Variables Theory Appl., **46** (2001), 241–253.

[9] I. LAHIRI AND S. DEWAN, *Inequealities arising out of the value distribution of a differential monomial*, J. Inequal.Pure Appl. Math., **4** (2003), Art. 27.

[10] S.H. LIN AND W.C. LIN, *Uniqueness of meromorphic functions concerning weakly weighted sharing*, Kodai Math. J., **29** (2006), 269–280.

[11] L.P. LIU AND Y.X. GU, *Uniqueness of meromorphic functions that share one small function with their derivatives*, Kodai Math. J., **27** (2004), 272–279.

[12] L.A. RUBEL AND C.C. YANG, *Values shared by an entire function and its derivatives*, In: Complex Analysis, Kentucky 1976 (Proc. Conf), Lecture Notes in Mathematics, Vol. 599, Springer-Verlag, Berlin, 1977, 101–103.

[13] L.Z. YANG, *Entire functions that share finite values with their derivatives*, Bull. Austral. Math. Soc., **41** (1990), 337–342.

[14] L.Z. YANG, *Solution of a differential equation and its applications*, Kodai Math. J., **22** (1999), 458–464.

[15] L. YANG, *Value distribution theory*, Springer-Verlag, Berlin, 1993.

[16] K.W. YU, *On entire and meromorphic functions that share small functions with their derivatives*, J. Inequal. Pure Appl. Math., **4**, 1 (2003), Art. 21.

[17] Q.C. ZHANG, *Meromorphic function that share one small function with its derivative*, J. Inequal. Pure Appl. Math., **6**, 4 (2005), Art.116.

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