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LYAPUNOV–TYPE INEQUALITY FOR QUASILINEAR SYSTEMS WITH ANTI–PERIODIC BOUNDARY CONDITIONS

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Abstract. In this paper, we establish a new Lyapunov-type inequality for quasilinear systems with the anti-periodic boundary conditions. It improves some result of Wang [17]. As an application, we also obtain lower bounds for the eigenvalues of corresponding systems.

1. Introduction

In this paper, we state and prove a new Lyapunov-type inequality for the following system

$$-\left(\phi_{p_k}\left(u_k'\right)\right)' = f_k(x)\phi_{\alpha_{kk}}\left(u_k\right)\prod_{\substack{i=1\\ i\neq k}}^{n}|u_i|^{\alpha_{ki}}, \tag{1.1}$$

where $n\in\mathbb{N},\ \phi_\gamma(u)=|u|^{\gamma-2}u,\ \gamma>1,\ f_k\in C([a,b],\mathbb{R})$ for $k=1,2,...,n$ and $x\in\mathbb{R}$, $(u_1(x),u_2(x),\ ...,u_n(x))$ is a real nontrivial solution of system (1.1) such that the anti-periodic boundary conditions

$$u_k^{(m)}(a)+u_k^{(m)}(b)=0 \tag{1.2}$$

for $m=0,1,\ k=1,2,...,n,\ a,b\in\mathbb{R}$ with $a<b$, u_k for $k=1,2,...,n$ are not identically zero on $[a,b],\ 1<p_k<\infty$ and α_{ki} for $k,i=1,2,...,n$ are nonnegative constants.

As an application, we have also investigated in the lower bounds on the eigenvalues of the following problem.

Let $(\lambda_1,\lambda_2,...,\lambda_n)$ be generalized eigenvalue of problem (1.1)-(1.2) and $r(x)$ be a function for $x\in\mathbb{R}$. Then, problem (1.1)–(1.2) with $f_k(x)=\lambda_k r(x)$ for $k=1,2,...,n$ and $x\in\mathbb{R}$ reduces to the following problem

$$-\left(\phi_{p_k}\left(u_k'\right)\right)' = \lambda_k r(x)\phi_{\alpha_{kk}}\left(u_k\right)\prod_{\substack{i=1\\ i\neq k}}^{n}|u_i|^{\alpha_{ki}}, \tag{1.3}$$

$$u_k^{(m)}(a)+u_k^{(m)}(b)=0 \tag{1.4}$$

for $m=0,1$ and $k=1,2,...,n$.

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As usual, it is easier to find upper bounds for eigenvalues than lower bounds. In fact, they can be obtained by using elementary inequalities. Finding the estimated lower bounds is based on giving a suitable Lyapunov inequality for the corresponding systems.

Before we proceed with the description of our main results, we discuss a few hints concerning the literature on the results obtained for system (1.1) and its special case with the Dirichlet boundary conditions [2, 5–8, 10–16]. Also, some Lyapunov-type inequalities may be obtained for partial differential equations [3, 4, 9].

First, we give the following result given by Lyapunov [11] for system (1.1) with $n=1$ and $\alpha_{11}=p_1=2$, that is,

$$-u_1''=f_1(x)\,u_1, \tag{1.5}$$

under the Dirichlet boundary condition

$$u_1(a)=0=u_1(b). \tag{1.6}$$

THEOREM A. *If $f_1\in C([a,b],[0,\infty))$ and u_1 is a nontrivial solution on $[a,b]$ for problem (1.5)–(1.6), then the so-called Lyapunov inequality*

$$\frac{4}{b-a}\leqslant\int_a^b f_1(s)\,ds \tag{1.7}$$

holds.

We know that the constant 4 in the left-hand side of inequality (1.7) cannot be replaced by a larger number (see [10, p. 345]).

Çakmak and Tiryaki [7] obtained the following inequality for system (1.1) with $\alpha_{ik}=\alpha_{kk}$ for $k,i=1,2,\dots,n$ under the Dirichlet boundary conditions.

THEOREM B. [7, Theorem 1] *If $f_k\in C([a,b],\mathbb{R})$ for $k=1,2,\dots,n$ and $(u_1(x), u_2(x),\dots,u_n(x))$ is a nontrivial solution on $[a,b]$ for system (1.1) with $\alpha_{ik}=\alpha_{kk}$ for $k,i=1,2,\dots,n$, the Dirichlet boundary conditions*

$$u_k(a)=0=u_k(b) \tag{1.8}$$

for $k=1,2,\dots,n$ and

$$\sum_{k=1}^{n}\frac{\alpha_{kk}}{p_k}=1, \tag{1.9}$$

then the inequality

$$\prod_{k=1}^{n}\left[(c_k-a)^{1-p_k}+(b-c_k)^{1-p_k}\right]^{\frac{\alpha_k}{p_k}}\leqslant\prod_{k=1}^{n}\left(\int_a^b f_k^+(s)ds\right)^{\frac{\alpha_k}{p_k}} \tag{1.10}$$

holds, where $|u_k(c_k)|=\max\limits_{a<x<b}|u_k(x)|$ and $f_k^+(x)=\max\{0,f_k(x)\}$ is the nonnegative part of $f_k(x)$for $k=1,2,\dots,n$.

Recently, Yang et al. [15] obtained the following inequality under the Dirichlet boundary conditions.

THEOREM C. [15, Theorem 1] *Assume that there exist nontrivial solutions* $(e_1, e_2, \ldots, e_n)$ *of the following linear homogeneous system*

$$e_k\left(1-\frac{\alpha_{kk}}{p_k}\right)-\sum_{\substack{i=1\\i\neq k}}^{n}\frac{\alpha_{ik}}{p_k}e_i=0, \tag{1.11}$$

where $e_k \geqslant 0$ *for* $k=1,2,\ldots,n$ *and* $\sum_{k=1}^{n} e_k^2 > 0$. *If* r_k, $f_k \in C([a,b],\mathbb{R})$, $r_k(x)>0$, $k=1,2,\ldots,n$, $x\in\mathbb{R}$, *and* $(u_1(x), u_2(x), \ldots, u_n(x))$ *is a nontrivial solution on* $[a,b]$ *for the following system*

$$-\left(r_k(x)\phi_{p_k}\left(u_k'\right)\right)'=f_k(x)\phi_{\alpha_{kk}}(u_k)\prod_{\substack{i=1\\i\neq k}}^{n}|u_i|^{\alpha_{ki}} \tag{1.12}$$

with the Dirichlet boundary conditions (1.8), then the inequality

$$1<\prod_{k=1}^{n}\left[2^{-p_k}\left(\int_a^b r_k^{1/(1-p_k)}(s)ds\right)^{p_k-1}\int_a^b f_k^+(s)ds\right]^{e_k} \tag{1.13}$$

holds, where $f_k^+(x)=\max\{0,f_k(x)\}$ *for* $k=1,2,\ldots n$.

Yang et al. [16] also obtained Theorem C with $n=3$ under the Dirichlet boundary conditions.

More recently, by using the anti-periodic boundary conditions instead of the Dirichlet boundary conditions, Wang [17] obtained a new Lyapunov-type inequality for $(n+1)$-th order half-linear differential equation as follows.

THEOREM D. [17, Theorem 2.1] *If* $f_1 \in C([a,b],\mathbb{R})$ *and* $u_1(x)$ *is a nontrivial solution on* $[a,b]$ *for* $(n+1)$*-th order half-linear differential equation*

$$-(\phi_{p_1}(u_1^{(n)}))'=f_1(x)\phi_{p_1}(u_1) \tag{1.14}$$

with the anti-periodic boundary conditions

$$u_1^{(m)}(a)+u_1^{(m)}(b)=0 \tag{1.15}$$

for $m=0,1,2,\ldots,n$, *then the inequality*

$$2\left(\frac{2}{b-a}\right)^{n(p_1-1)}<\int_a^b|f_1(s)|ds \tag{1.16}$$

holds.

Note that if we take $\alpha_{kk}=p_k$ for $k=1,2,\ldots,n$, and for $i\neq k$, $\alpha_{ki}=0$ for $i=1, 2,\ldots,n$, then we obtain a single equation from system (1.1). For example, when $n=1$ in the problem (1.1)–(1.2) or (1.14)–(1.15), we have the following problem

$$\begin{cases}-(\phi_{p_1}(u_1'))'=f_1(x)\phi_{p_1}(u_1),\\ u_1(a)+u_1(b)=0,\\ u_1'(a)+u_1'(b)=0.\end{cases} \tag{1.17}$$

If we take $n=1$ in the problem (1.14)–(1.15), then we obtain the following result from Theorem D.

THEOREM E. [17, Corollary 2.6] *If* $f_1 \in C([a,b],\mathbb{R})$ *and* $u_1(x)$ *is a nontrivial solution on* $[a,b]$ *for problem (1.17), then the inequality*

$$\frac{2^{p_1}}{(b-a)^{p_1-1}} < \int_a^b |f_1(s)|\,ds \tag{1.18}$$

holds.

In this paper, our motivation comes from the recent papers of Çakmak and Tiryaki [7], Yang et al. [15] and Wang [17]. We state and prove a new Lyapunov-type inequality for problem (1.1)–(1.2).

Since our attention is restricted to the Lyapunov-type inequality for the quasilinear systems of differential equations, we shall assume the existence of the nontrivial solution of system (1.1). For readers who contributed to the existence of the solution of this type system, we refer to the paper by Afrouzi and Heidarkhani [1].

2. Main Results

We state a lemma which we will use in the proof of our main result. The proof of the following lemma proceeds along the lines of that of Lemma 3.1 in Wang [17] and hence is omitted.

LEMMA 2.1. *If* $(u_1(x), u_2(x), \ldots, u_n(x))$ *is a nontrivial solution of system (1.1) satisfying the anti-periodic boundary conditions (1.2) with* $m = 0,1$ *and* $k = 1,2,\ldots,n$, *then we have*

$$|u_i(x)|^{\alpha_{ki}} \leqslant 2^{-\alpha_{ki}} (b-a)^{\frac{(p_i-1)\alpha_{ki}}{p_i}} \left(\int_a^b |u_i'(s)|^{p_i} ds \right)^{\frac{\alpha_{ki}}{p_i}} \tag{2.1}$$

for $k,i = 1,2,\ldots,n$.

Now, we give the main result of this paper.

THEOREM 2.1. *Assume that there exist nontrivial solutions* $(e_1, e_2, \ldots, e_n)$ *of the following linear homogeneous system*

$$e_k \left(1 - \frac{\alpha_{kk}}{p_k}\right) - \sum_{\substack{i=1 \\ i \neq k}}^{n} \frac{\alpha_{ik}}{p_k} e_i = 0, \tag{2.2}$$

where $e_k \geqslant 0$ *for* $k = 1,2,\ldots,n$. *If* $f_k \in C([a,b],\mathbb{R})$ *for* $k = 1,2,\ldots,n$ *and* $(u_1(x), u_2(x), \ldots, u_n(x))$ *is a nontrivial solution on* $[a,b]$ *for problem (1.1)–(1.2), then the inequality*

$$1 < \prod_{k=1}^{n} \left[2^{-\sum_{i=1}^{n} \alpha_{ki}} (b-a)^{\sum_{i=1}^{n} \frac{(p_i-1)\alpha_{ki}}{p_i}} \int_a^b f_k^+(s)\,ds \right]^{e_k} \tag{2.3}$$

holds, where $f_k^+(x) = \max\{0, f_k(x)\}$ *for* $k = 1,2,\ldots,n$.

Proof. Let $u_k^{(m)}(a)+u_k^{(m)}(b)=0$ for $m=0,1$, $k=1,2,\ldots,n$ where $n\in\mathbb{N}$, $a,b\in\mathbb{R}$ with $a<b$ and u_k for $k=1,2,\ldots,n$ are not identically zero on $[a,b]$. Multiplying the k-th equation of system (1.1) by u_k, integrating from a to b and by using the anti-periodic boundary conditions (1.2), we get

$$\int_a^b \left|u_k'(s)\right|^{p_k} ds = \int_a^b f_k(s)\prod_{i=1}^n \left|u_i(s)\right|^{\alpha_{ki}} ds \tag{2.4}$$

for $k=1,2,\ldots,n$. By using the inequalities (2.1) in Lemma 2.1, we obtain

$$\begin{aligned}\int_a^b \left|u_k'(s)\right|^{p_k} ds &= \int_a^b f_k(s)\prod_{i=1}^n \left|u_i(s)\right|^{\alpha_{ki}} ds \leqslant \int_a^b f_k^+(s)\prod_{i=1}^n \left|u_i(s)\right|^{\alpha_{ki}} ds\\ &< \max_{a\leqslant x\leqslant b}\prod_{i=1}^n \left|u_i(x)\right|^{\alpha_{ki}}\int_a^b f_k^+(s)ds\\ &\leqslant 2^{-\sum\limits_{i=1}^n \alpha_{ki}}(b-a)^{\sum\limits_{i=1}^n \frac{(p_i-1)\alpha_{ki}}{p_i}}\prod_{i=1}^n\left(\int_a^b \left|u_i'(s)\right|^{p_i} ds\right)^{\frac{\alpha_{ki}}{p_i}}\int_a^b f_k^+(s)ds\end{aligned} \tag{2.5}$$

and hence

$$\begin{aligned}\left(\int_a^b \left|u_k'(s)\right|^{p_k} ds\right)^{1-\frac{\alpha_{kk}}{p_k}} &< 2^{-\sum\limits_{i=1}^n \alpha_{ki}}(b-a)^{\sum\limits_{i=1}^n \frac{(p_i-1)\alpha_{ki}}{p_i}}\prod_{\substack{i=1\\ i\neq k}}^n\left(\int_a^b \left|u_i'(s)\right|^{p_i} ds\right)^{\frac{\alpha_{ki}}{p_i}}\\ &\quad\times\int_a^b f_k^+(s)ds\end{aligned} \tag{2.6}$$

for $k=1,2,\ldots,n$. Raising the both sides of the inequality (2.6) to the power e_k for each $k=1,2,\ldots,n$, respectively, and multiplying the resulting inequalities side by side, we obtain

$$\begin{aligned}&\prod_{k=1}^n\left(\int_a^b \left|u_k'(s)\right|^{p_k} ds\right)^{\left(1-\frac{\alpha_{kk}}{p_k}\right)e_k}\\ &< \prod_{k=1}^n\left[2^{-\sum\limits_{i=1}^n \alpha_{ki}}(b-a)^{\sum\limits_{i=1}^n \frac{(p_i-1)\alpha_{ki}}{p_i}}\prod_{\substack{i=1\\ i\neq k}}^n\left(\int_a^b \left|u_i'(s)\right|^{p_i} ds\right)^{\frac{\alpha_{ki}}{p_i}}\int_a^b f_k^+(s)ds\right]^{e_k}\end{aligned} \tag{2.7}$$

and hence

$$\begin{aligned}&\prod_{k=1}^n\left(\int_a^b \left|u_k'(s)\right|^{p_k} ds\right)^{\left(1-\frac{\alpha_{kk}}{p_k}\right)e_k}\\ &< \prod_{k=1}^n\left(\int_a^b \left|u_k'(s)\right|^{p_k} ds\right)^{\sum\limits_{\substack{i=1\\ i\neq k}}^n \frac{\alpha_{ik}}{p_k}e_i}\prod_{k=1}^n\left[2^{-\sum\limits_{i=1}^n \alpha_{ki}}(b-a)^{\sum\limits_{i=1}^n \frac{(p_i-1)\alpha_{ki}}{p_i}}\int_a^b f_k^+(s)\,ds\right]^{e_k}.\end{aligned} \tag{2.8}$$

It is easy to see that by using similar technique to the proof of Theorem 2.1 given by Wang [17], we obtain the following inequalities

$$\int_a^b \left|u_k'(s)\right|^{p_k} ds > 0 \tag{2.9}$$

for $k = 1,2,...,n$. Thus, we have

$$\prod_{k=1}^{n}\left(\int_a^b \left|u_k'(s)\right|^{p_k} ds\right)^{\theta_k} < \prod_{k=1}^{n}\left[2^{-\sum\limits_{i=1}^{n}\alpha_{ki}}(b-a)^{\sum\limits_{i=1}^{n}\frac{(p_i-1)\alpha_{ki}}{p_i}}\int_a^b f_k^+(s)\,ds\right]^{e_k}, \tag{2.10}$$

where $\theta_k = \left(1-\frac{\alpha_{kk}}{p_k}\right)e_k - \sum\limits_{\substack{i=1\\ i\neq k}}^{n}\frac{\alpha_{ik}}{p_k}e_i$ for $k = 1,2,...,n$. By assumption, system (2.2) has nontrivial solutions $(e_1,e_2,...,e_n)$ such that $\theta_k = 0$ for $k = 1,2,...,n$, where $e_k \geqslant 0$ for $k = 1,2,...,n$ and at least one $e_j > 0$ for $j = \{1,2,...,n\}$. Choosing one of the solutions $(e_1,e_2,...,e_n)$, we obtain the inequality (2.3) from (2.10). This completes the proof of Theorem 2.1. □

The proof of the following result proceeds along the lines of that of Corollary 1 in Yang et al. [15] and hence is omitted.

COROLLARY 2.1. *Assume that*

$$\sum_{i=1}^{n}\alpha_{ik} = p_k \tag{2.11}$$

for $k = 1,2,...,n$. *If* $f_k \in C([a,b],\mathbb{R})$ *for* $k = 1,2,...,n$ *and* $(u_1(x),u_2(x),...,u_n(x))$ *is a nontrivial solution on* $[a,b]$ *for problem (1.1)–(1.2), then the inequality*

$$1 < \prod_{k=1}^{n}\left[2^{-\sum\limits_{i=1}^{n}\alpha_{ki}}(b-a)^{\sum\limits_{i=1}^{n}\frac{(p_i-1)\alpha_{ki}}{p_i}}\int_a^b f_k^+(s)\,ds\right] \tag{2.12}$$

holds, where $f_k^+(x) = \max\{0,f_k(x)\}$ *for* $k = 1,2,...,n$.

REMARK 2.1. We don't know if the right-hand side of inequality (2.12) can be replaced by a smaller one arbitrarily close to 1. This is an open problem for the readers.

REMARK 2.2. If we compare Theorems A-C with Theorem 2.1 (or Corollary 2.1), it is easy to see that the main difference between these results are the boundary conditions on the solution $(u_1(x),u_2(x),...,u_n(x))$. Therefore, they are different from each other.

REMARK 2.3. Since

$$f^+(x) \leqslant |f(x)|, \tag{2.13}$$

the integrals of $\int_a^b f_k^+(s)ds$ for $k=1,2,\dots,n$ in the above results can also be replaced by $\int_a^b |f_k(s)|\,ds$ for $k=1,2,\dots,n$, respectively.

REMARK 2.4. If we take $n=1$ in the problem (1.1)–(1.2), then we have the following inequality

$$\frac{2^{p_1}}{(b-a)^{p_1-1}} < \int_a^b f_1^+(s)\,ds \tag{2.14}$$

from the inequality (2.12) in Corollary 2.1. It is easy to see from the inequality (2.13) that the inequality (2.14) is better than (1.18) in the sense that (1.18) follows from (2.14), but not conversely. Therefore, Corollary 2.1 with $n=1$ improves Theorem E given by Wang [17].

Now, we present an application of the Lyapunov-type inequality obtained for system (1.1).

We obtain the following result which gives lower bounds for the n-th component of any generalized eigenvalue $(\lambda_1,\lambda_2,\dots,\lambda_n)$ of system (1.3). The proof of the following theorem is based on above generalization of the Lyapunov-type inequality, as in that of Theorem 9 of Çakmak and Tiryaki [7] and hence is omitted.

THEOREM 2.2. *Assume that there exist nontrivial solutions $(e_1,e_2,\dots,e_n)$ of system (2.2) and a function $h_1(\lambda_1,\lambda_2,\dots,\lambda_{n-1})$ such that*

$$h_1(\lambda_1,\lambda_2,\dots,\lambda_{n-1}) < |\lambda_n| \tag{2.15}$$

for any generalized eigenvalue $(\lambda_1,\lambda_2,\dots,\lambda_n)$ of problem (1.3)–(1.4), where

$$h_1(\lambda_1,\lambda_2,\dots,\lambda_{n-1}) = \left\{\left[\prod_{k=1}^{n-1}|\lambda_k|^{e_k}\right]\left[\prod_{k=1}^{n}\left(2^{-\sum_{i=1}^{n}\alpha_{ki}}(b-a)^{\sum_{i=1}^{n}\frac{(p_i-1)\alpha_{ki}}{p_i}}\int_a^b |r(s)|\,ds\right)^{e_k}\right]\right\}^{-\frac{1}{e_n}}. \tag{2.16}$$

REMARK 2.5. Since h_1 is a continuous function, then $h_1(\lambda_1,\lambda_2,\dots,\lambda_{n-1}) \to +\infty$ as any eigenvalue of $\lambda_k \to 0$ for $k=1,2,\dots,n-1$. Therefore, there exists a ball centered in the origin such that the generalized spectrum is contained in its exterior. Also, by rearranging terms in (2.15) we obtain

$$\prod_{k=1}^{n}\left[2^{-\sum_{i=1}^{n}\alpha_{ki}}(b-a)^{\sum_{i=1}^{n}\frac{(p_i-1)\alpha_{ki}}{p_i}}\int_a^b |r(s)|\,ds\right]^{-e_k} < \prod_{k=1}^{n}|\lambda_k|^{e_k}. \tag{2.17}$$

It is clear that when the interval collapses, left-hand side of (2.17) goes to infinity.

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