Journal of
Mathematical
Inequalities
Volume 8, Number 3 (2014), 465–473 doi:10.7153/jmi-08-34

ON THE STRONG LAW OF LARGE NUMBERS FOR WEIGHTED SUMS OF φ–MIXING RANDOM VARIABLES

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(*Communicated by N. Elezović*)

Abstract. Let $\{X_n, n \geqslant 1\}$ be a sequence of φ-mixing random variables with non-identical distribution and $\{a_{ni}; 1 \leqslant i \leqslant n, n \geqslant 1\}$ be an array of real constants. In this paper, we study the strong law of large numbers for the maximal weighted sums of φ-mixing random variables. The results obtained generalize and improve the previous known result of Bai and Cheng (*Z.D. Bai and P.E. Cheng, 2000. Marcinkiewicz strong laws for linear statistics. Statist. Probab. Lett. vol. 46, no. 2, pp. 105–112.*) for independent and identically distributed random variables to φ-mixing case.

1. Introduction

Throughout this paper, let $\{X_n, n \geqslant 1\}$ be a sequence of random variables defined on a probability space $(\Omega, \mathcal{F}, P)$. It is desirable to know that the Kolmogorov strong law of large numbers and the Maricinkiewicz strong law of large numbers are defined as follows:

$$\frac{1}{n}\sum_{i=1}^{n} X_i \to 0 \text{ almost surely (a.s., in short) and } \frac{1}{n^{1/r}}\sum_{i=1}^{n} X_i \to 0 \text{ a.s. for } 1 < r < 2.$$

For more general case, Bai and Cheng [1] showed an extension of the Maricinkiewicz strong law of large numbers for weighted sums of independent and identically distributed (i.i.d., in short) random variables under certain moment conditions as follows.

Mathematics subject classification (2010): 60F15.
Keywords and phrases: φ-mixing random variables, strong law of large numbers, weighted sums.
Supported by the National Natural Science Foundation of China (11061012, 71271042, 11361019), the Project supported by Program to Sponsor Teams for Innovation in the Construction of Talent Highlands in Guangxi Institutions of Higher Learning ((2011)47), the Plan of Jiangsu Specially-appointed Professors, Plan of Introducing High-level Talents in Entrepreneurship and Innovation to Jiangsu Province, the Major Program of key research center in financial risk management of Jiangsu universities philosophy social sciences (No:2012JDXM009) and the Fundamental Research Funds for the Central Universities of China (ZYGX2012J119), the Guangxi China Science Foundation (2013GXNSFDA019001), the Guangxi Provincial Scientific Research Projects (201204LX157, 2013YB104).

THEOREM A. (Bai and Cheng, [1]) *Suppose that* $1 < \alpha,\ \beta < \infty,\ 1 \leqslant p < 2$ *and* $1/p = 1/\alpha + 1/\beta$. *Let* $\{X, X_n; n \geqslant 1\}$ *be a sequence of i.i.d. random variables with* $EX = 0$, *and let* $\{a_{ni}; 1 \leqslant i \leqslant n, n \geqslant 1\}$ *be an array of real constants such that*

$$\limsup_{n\to\infty} \left(\frac{1}{n} \sum_{i=1}^{n} |a_{ni}|^{\alpha} \right)^{1/\alpha} < \infty. \tag{1.1}$$

If $E|X|^{\beta} < \infty$, *then*

$$\lim_{n\to\infty} n^{-1/p} \sum_{i=1}^{n} a_{ni} X_i = 0 \quad a.s. \tag{1.2}$$

Inspired by Bai and Cheng [1], our main purpose of this paper is to generalize and improve the above result of Bai and Cheng [1] for i.i.d. random variables to the case of φ-mixing. We study the strong law of large numbers for φ-mixing random variables without assumptions of identical distribution. As an application, a strong law of large numbers for weighted sums of φ-mixing random variables is obtained. The results presented in this work are obtained by using the truncated method and the maximal type inequality of φ-mixing random variables by Wang et al [2].

Firstly, we will recall the definition of φ-mixing random variables.

Let $\{X_n; n \geqslant 1\}$ be a sequence of random variables defined on a fixed probability space (Ω, F, P). Let n and m be positive integers. Write $F_n^m = \sigma(X_i; n \leqslant i \leqslant m)$. For any given two σ-algebras $\mathfrak{I}$, $\mathfrak{R}$ in F, let

$$\varphi(\mathfrak{I}, \mathfrak{R}) = \sup_{A \in \mathfrak{I}, B \in \mathfrak{R}, P(A) > 0} |P(B|A) - P(B)|. \tag{1.3}$$

Define the φ-mixing coefficients by

$$\varphi(n) = \sup_{k \geqslant 1} \varphi\left(F_1^k, F_{k+n}^{\infty}\right), \ n \geqslant 0. \tag{1.4}$$

DEFINITION 1.1. A sequence $\{X_n; n \geqslant 1\}$ of random variables is said to be a φ-mixing sequence of random variables if $\varphi(n) \downarrow 0$ as $n \to \infty$.

The concept of φ-mixing random variables was introduced by Dobrushin [3]. Many applications have been found. For example, we can refer to: Dobrushin [3], Chen [4] and Utev [5] for central limit theorem, Herrndorf [6] and Peligrad [7] for weak invariance principle, Wang et al [2, 8, 9] for some moment inequalities and complete convergence, and so forth.

We will use the following concept in this paper.

DEFINITION 1.2. Let $\{X_n; n \geqslant 1\}$ be a sequence of random variables and let X be a random variable. If there exists a constant C $(0 < C < \infty)$ such that

$$P(|X_n| \geqslant t) \leqslant CP(|X| \geqslant t), \tag{1.5}$$

for all $t \geqslant 0$ and $n \geqslant 1$, then $\{X_n; n \geqslant 1\}$ is said to be stochastically dominated by X.

In order to prove our main results of this paper, we need the following lemmas.

LEMMA 1.1. (Wang et al [2]) *Let* $\{X_n; n \geqslant 1\}$ *be a sequence of* φ*-mixing random variables satisfying* $\sum_{n=1}^{\infty} \varphi^{1/2}(n) < \infty$. *Assume that* $EX_n = 0$ *and* $E|X_n|^q < \infty$ *for all* $n \geqslant 1$ *and* $q \geqslant 2$. *Then there exists a positive constant* $C = C(q)$ *depending only on* q *and* $\varphi(\cdot)$ *such that*

$$E\left(\max_{1\leqslant k\leqslant n}\left|\sum_{i=a+1}^{a+k} X_i\right|^q\right) \leqslant C\left[\sum_{i=a+1}^{a+n} E|X_i|^q + \left(\sum_{i=a+1}^{a+n}\left(EX_i^2\right)\right)^{q/2}\right], \tag{1.6}$$

for each $a \geqslant 0$ *and* $n \geqslant 1$. *In particular,*

$$E\left(\max_{1\leqslant k\leqslant n}\left|\sum_{i=1}^{k} X_i\right|^q\right) \leqslant C\left[\sum_{i=1}^{n} E|X_i|^q + \left(\sum_{i=1}^{n}\left(EX_i^2\right)\right)^{q/2}\right], \tag{1.7}$$

for each $n \geqslant 1$.

LEMMA 1.2. *Let* $\{X_n; n \geqslant 1\}$ *be a sequence of random variables which is stochastically dominated by a random variable* X. *For any* $u > 0$, $t > 0$ *and* $n \geqslant 1$, *the following two statements hold:*

$$E|X_n|^u I(|X_n| \leqslant t) \leqslant C_1\left[E|X|^u I(|X| \leqslant t) + t^u P(|X| > t)\right]; \tag{1.8}$$

$$E|X_n|^u I(|X_n| > t) \leqslant C_2 E|X|^u I(|X| > t). \tag{1.9}$$

where C_1 *and* C_2 *are positive constants.*

2. Main Results

Throughout this paper, the symbol C will stand for a positive constant whose value may change from one appearance to the next, and $a_n = O(b_n)$ will mean $a_n \leqslant C(b_n)$.

Now, we state and prove the main results of this paper.

THEOREM 2.1. *Let* $\{X_n; n \geqslant 1\}$ *be a sequence of* φ*-mixing random variables which is stochastically dominated by a random variable* X *with* $E|X|^\beta < \infty$. *Suppose that* $0 < \alpha$, $\beta < \infty$, $0 < p < 2$ *and* $1/p = 1/\alpha + 1/\beta$. *Further assume that* $\sum_{n=1}^{\infty} \varphi^{1/2}(n) < \infty$ *and* $EX_n = 0$ *for* $\beta > 1$. *Let* $\{a_{ni}; 1 \leqslant i \leqslant n, n \geqslant 1\}$ *be an array of real constants such that*

$$\sum_{i=1}^{n} |a_{ni}|^\alpha = O(n). \tag{2.1}$$

Then,

$$\lim_{n\to\infty} n^{-1/p} \max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{j} a_{ni}X_i\right| = 0 \quad a.s. \tag{2.2}$$

Proof of Theorem 2.1. For fixed $n \geqslant 1$, define

$$X_i^{(n)} = X_i I\left(|X_i| \leqslant n^{1/\beta}\right);$$

$$Z_i = X_i - X_i^{(n)} = X_i I\left(|X_i| > n^{1/\beta}\right),\ i \geqslant 1.$$

It is easy to check that

$$\begin{aligned} n^{-1/p}\max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{j} a_{ni}X_i\right| &\leqslant n^{-1/p}\max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{j} a_{ni}Z_i\right| + n^{-1/p}\max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{j} a_{ni}X_i^{(n)}\right| \\ &\leqslant n^{-1/p}\max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{j} a_{ni}Z_i\right| + n^{-1/p}\max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{j} a_{ni}EX_i^{(n)}\right| \\ &\quad + n^{-1/p}\max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{j} a_{ni}\left(X_i^{(n)} - EX_i^{(n)}\right)\right| \\ &\triangleq I_1 + I_2 + I_3. \end{aligned} \tag{2.3}$$

Firstly, we will show that

$$I_1 \triangleq n^{-1/p}\max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{j} a_{ni}Z_i\right| \to 0 \quad \text{a.s.} \tag{2.4}$$

For $0 < \gamma < \alpha$, it follows from (2.1) and c_r inequality that

$$\sum_{i=1}^{n} |a_{ni}|^{\gamma} \leqslant \left(\sum_{i=1}^{n} |a_{ni}|^{\alpha}\right)^{\gamma/\alpha}\left(\sum_{i=1}^{n} 1\right)^{1-\gamma/\alpha} \leqslant Cn;$$

For $\gamma \geqslant \alpha$, it also follows from (2.1) and Hölder inequality that

$$\sum_{i=1}^{n} |a_{ni}|^{\gamma} \leqslant \left(\sum_{i=1}^{n} |a_{ni}|^{\alpha}\right)^{\gamma/\alpha} \leqslant Cn^{\gamma/\alpha}.$$

So, we can see that

$$\sum_{i=1}^{n} |a_{ni}|^{\gamma} \leqslant Cn^{\max(1,\gamma/\alpha)}. \tag{2.5}$$

By $E|X|^{\beta} < \infty$, we can obtain that

$$\sum_{i=1}^{\infty} P(Z_i \neq 0) = \sum_{i=1}^{\infty} P\left(|X_i| > i^{1/\beta}\right) \leqslant C\sum_{i=1}^{\infty} P\left(|X| > i^{1/\beta}\right) \leqslant CE|X|^{\beta} < \infty. \tag{2.6}$$

Hence, by *Borel-Cantelli lemma*, we easily get that $P(Z_i \neq 0, \text{i.o.}) = 0$. It follows that

$$\begin{aligned} I_1 &\triangleq n^{-1/p}\max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{j} a_{ni}Z_i\right| \\ &\leqslant Cn^{-1/p}\sum_{i=1}^{n} |a_{ni}Z_i| \end{aligned}$$

$$\leqslant Cn^{-1/p}\left(\max_{1\leqslant i\leqslant n}|a_{ni}|^{\alpha}\right)^{1/\alpha}\left|\sum_{i=1}^{n}Z_i\right|$$
$$\leqslant Cn^{-1/p}\left(\sum_{i=1}^{n}|a_{ni}|^{\alpha}\right)^{1/\alpha}\left|\sum_{i=1}^{n}Z_i\right|$$
$$\leqslant Cn^{-1/\beta}\left|\sum_{i=1}^{n}Z_i\right|\to 0 \quad \text{a.s.}$$

Secondly, we will show that

$$I_2 \triangleq n^{-1/p}\max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{j}a_{ni}EX_i^{(n)}\right|\to 0 \tag{2.7}$$

When $0<\beta\leqslant 1$, it follows from $E|X|^{\beta}<\infty$, Markov inequality, (1.8) of Lemma 1.2 and (2.5) that

$$\begin{aligned}
I_2 &\triangleq n^{-1/p}\max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{j}a_{ni}EX_i^{(n)}\right| \\
&\leqslant Cn^{-1/p}\sum_{i=1}^{n}\left|a_{ni}EX_i^{(n)}\right| \\
&= Cn^{-1/p}\sum_{i=1}^{n}|a_{ni}|E|X_i|I\left(|X_i|\leqslant n^{1/\beta}\right) \\
&\leqslant Cn^{-1/p}\sum_{i=1}^{n}|a_{ni}|\left(E|X|^{\beta}n^{(1-\beta)/\beta}I(|X|\leqslant n^{1/\beta})+n^{1/\beta}P(|X|>n^{1/\beta})\right) \\
&\leqslant Cn^{-1/\alpha-1+\max(1,1/\alpha)}\to 0, n\to\infty.
\end{aligned} \tag{2.8}$$

When $\beta>1$, it also follows from $E|X|^{\beta}<\infty$, $EX_n=0$, Markov inequality, (1.9) of Lemma 1.2 and (2.5) that

$$\begin{aligned}
I_2 &\triangleq n^{-1/p}\max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{j}a_{ni}EX_i^{(n)}\right| \\
&\leqslant Cn^{-1/p}\sum_{i=1}^{n}\left|a_{ni}EX_i^{(n)}\right| \\
&= Cn^{-1/p}\sum_{i=1}^{n}|a_{ni}|E|X_i|I\left(|X_i|>n^{1/\beta}\right) \\
&\leqslant Cn^{-1/p}\sum_{i=1}^{n}|a_{ni}|E|X|I\left(|X|>n^{1/\beta}\right) \\
&\leqslant Cn^{-1/p}\sum_{i=1}^{n}|a_{ni}|E|X|\left(\frac{|X|}{n^{1/\beta}}\right)^{\beta-1}I\left(|X|>n^{1/\beta}\right)
\end{aligned}$$

$$\leqslant Cn^{-1/\alpha-1+\max(1,1/\alpha)} \to 0,\ n \to \infty. \tag{2.9}$$

(2.8) and (2.9) yield (2.7).

Finally, we will show that

$$I_3 \triangleq n^{-1/p} \max_{1\leqslant j\leqslant n} \left| \sum_{i=1}^{j} a_{ni} \left(X_i^{(n)} - EX_i^{(n)} \right) \right| \to 0 \quad \text{a.s.} \tag{2.10}$$

Let $q > \frac{1}{\min\{\frac{1}{2},\frac{1}{\alpha},\frac{1}{\beta},\frac{1}{p}-\frac{1}{2}\}}$, it follows from Markov inequality and Lemma 1.1 that

$$\begin{aligned}
&\sum_{n=1}^{\infty} P\left(n^{-1/p} \max_{1\leqslant j\leqslant n} \left| \sum_{i=1}^{j} a_{ni} \left(X_i^{(n)} - EX_i^{(n)} \right) \right| > \varepsilon \right) \\
&\leqslant C \sum_{n=1}^{\infty} n^{-q/p} E \left(\max_{1\leqslant j\leqslant n} \left| \sum_{i=1}^{j} a_{ni} \left(X_i^{(n)} - EX_i^{(n)} \right) \right| \right)^q \\
&\leqslant C \sum_{n=1}^{\infty} n^{-q/p} \sum_{i=1}^{n} E \left| a_{ni} \left(X_i^{(n)} - EX_i^{(n)} \right) \right|^q \\
&\quad + C \sum_{n=1}^{\infty} n^{-q/p} \left(\sum_{i=1}^{n} a_{ni}^2 E \left| \left(X_i^{(n)} - EX_i^{(n)} \right) \right|^2 \right)^{q/2} \\
&\triangleq I_{31} + I_{32}.
\end{aligned}$$

Hence, to prove (2.10), we need only to prove that $I_{31} < \infty$ and $I_{32} < \infty$.

It follows from $E|X|^\beta < \infty$, c_r inequality, (1.8) of Lemma 1.2 and (2.5) that

$$\begin{aligned}
I_{31} &\leqslant C \sum_{n=1}^{\infty} n^{-q/p} \sum_{i=1}^{n} |a_{ni}|^q E |X_i|^q I \left(|X_i| \leqslant n^{1/\beta} \right) \\
&\leqslant C \sum_{n=1}^{\infty} n^{-q/p+q/\alpha} \left(E|X|^q I \left(|X| \leqslant n^{1/\beta} \right) + n^{q/\beta} P \left(|X| > n^{1/\beta} \right) \right) \\
&\leqslant C \sum_{n=1}^{\infty} n^{-q/\beta} \sum_{i=1}^{n} E|X|^q I \left((i-1)^{1/\beta} < |X| \leqslant i^{1/\beta} \right) + C \sum_{n=1}^{\infty} P \left(|X| > n^{1/\beta} \right) \\
&\leqslant C \sum_{i=1}^{\infty} E|X|^q I \left((i-1)^{1/\beta} < |X| \leqslant i^{1/\beta} \right) \sum_{n=i}^{\infty} n^{-q/\beta} + CE|X|^\beta \\
&\leqslant C \sum_{i=1}^{\infty} i^{1-q/\beta} E|X|^q I \left((i-1)^{1/\beta} < |X| \leqslant i^{1/\beta} \right) + CE|X|^\beta \\
&\leqslant C \sum_{i=1}^{\infty} E|X|^\beta I \left((i-1)^{1/\beta} < |X| \leqslant i^{1/\beta} \right) + CE|X|^\beta \\
&\leqslant CE|X|^\beta < \infty.
\end{aligned} \tag{2.11}$$

By (2.5), we can have that

$$\sum_{i=1}^{n} |a_{ni}|^2 \leqslant Cn \text{ for } \alpha \geqslant 2 \text{ and } \sum_{i=1}^{n} |a_{ni}|^2 \leqslant Cn^{2/\alpha} \text{ for } \alpha < 2. \tag{2.12}$$

Then,

$$EX^2 I\left(|X| \leqslant n^{1/\beta}\right) + n^{2/\beta} P\left(|X| > n^{1/\beta}\right) \leqslant C\left(E|X|^\beta n^{(1/\beta)(2-\beta)} + n^{-1+2/\beta} E|X|^\beta\right)$$
$$\leqslant C n^{-1+2/\beta} \text{ for } \beta < 2;$$

$$EX^2 I\left(|X| \leqslant n^{1/\beta}\right) + n^{2/\beta} P\left(|X| > n^{1/\beta}\right) \leqslant CE|X|^2 < \infty \text{ for } \beta \geqslant 2.$$

It follows from c_r inequality, Markov inequality, (2.12) and Lemma 1.2 that

$$\begin{aligned}\sum_{i=1}^n a_{ni}^2 E\left|\left(X_i^{(n)} - EX_i^{(n)}\right)\right|^2 &\leqslant C\sum_{i=1}^n a_{ni}^2\left(EX_i^2 I\left(|X_i| \leqslant n^{1/\beta}\right)\right)\\ &\leqslant C\sum_{i=1}^n a_{ni}^2\left(EX^2 I\left(|X| \leqslant n^{1/\beta}\right) + n^{2/\beta} P\left(|X| > n^{1/\beta}\right)\right)\\ &\triangleq \Delta,\end{aligned} \tag{2.13}$$

(i) for $\alpha < 2,\ \beta < 2,\ \Delta = Cn^{-1+2/p}$;

(ii) for $\alpha < 2,\ \beta \geqslant 2,\ \Delta = Cn^{2/\alpha}$;

(iii) for $\alpha \geqslant 2,\ \beta < 2,\ \Delta = Cn^{2/\beta}$;

(iv) for $\alpha \geqslant 2,\ \beta \geqslant 2,\ \Delta = Cn$.

Then, denote that

$$\sum_{i=1}^n a_{ni}^2 E\left|\left(X_i^{(n)} - EX_i^{(n)}\right)\right|^2 \leqslant Cn^\lambda,$$

where $\lambda = \max\{-1 + \frac{2}{p}, \frac{2}{\alpha}, \frac{2}{\beta}, 1\}$. We can see that

$$\begin{aligned}\left(-\frac{1}{p} + \frac{\lambda}{2}\right) \times q &= q \times \max\left(-\frac{1}{2}, -\frac{1}{\beta}, -\frac{1}{\alpha}, -\frac{1}{p} + \frac{1}{2}\right)\\ &= -q \times \min\left(\frac{1}{2}, \frac{1}{\beta}, \frac{1}{\alpha}, \frac{1}{p} - \frac{1}{2}\right) < -1,\end{aligned}$$

Hence, we can obtain that

$$I_{32} \leqslant C\sum_{n=1}^\infty n^{(-1/p+\lambda/2)q} < \infty, \tag{2.14}$$

By *Borel-Cantelli lemma*,

$$I_3 \triangleq n^{-1/p} \max_{1 \leqslant j \leqslant n}\left|\sum_{i=1}^j a_{ni}\left(X_i^{(n)} - EX_i^{(n)}\right)\right| \to 0 \quad \text{a.s.}$$

The proof of Theorem 2.1 is completed. □

By taking $a_{ni} = 1$ in Theorem 2.1, then (2.1) is always valid for any $\alpha > 0$. Hence, for any $0 < p < \min(\beta, 2)$, let $\alpha = p\beta/(\beta - p) > 0$, we can obtain the following result.

COROLLARY 2.1. *Let* $\{X_n; n \geqslant 1\}$ *be a sequence of* φ*-mixing random variables which is stochastically dominated by a random variable* X *with* $E|X|^\beta < \infty$. *Assume further that* $\sum_{n=1}^{\infty} \varphi^{1/2}(n) < \infty$ *and* $EX_n = 0$ *for* $\beta > 1$. *Then for any* $0 < p < \min(\beta, 2)$,

$$\lim_{n\to\infty} n^{-1/p} \max_{1\leqslant j\leqslant n} \left|\sum_{i=1}^{j} X_i\right| = 0 \quad a.s. \tag{2.15}$$

REMARK 2.1. Theorem 2.1 generalizes and improves the above result of Bai and Cheng [1] for i.i.d. random variables to the case of φ-mixing without assumption of identical distribution and extends the rang of α, β, p, respectively.

Acknowledgements. The authors are exceptionally grateful to the referee and the editor for offering their valuable comments and some helpful suggestions that improved the clarity and readability of the paper.

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(Received October 19, 2012)

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