ADDITIVE $\rho$–FUNCTIONAL INEQUALITIES
IN NON–ARCHIMEDEAN NORMED SPACES

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Abstract. In this paper, we solve the additive $\rho$-functional inequalities

$$\|f(x+y) - f(x) - f(y)\| \leq \|\rho \left(2f\left(\frac{x+y}{2}\right) - f(x) - f(y)\right)\| \quad (0.1)$$

and

$$\|2f\left(\frac{x+y}{2}\right) - f(x) - f(y)\| \leq \|\rho (f(x+y) - f(x) - f(y))\|, \quad (0.2)$$

where $\rho$ is a fixed non-Archimedean number with $|\rho| < 1$.

Furthermore, we prove the Hyers-Ulam stability of the additive $\rho$-functional inequalities (0.1) and (0.2) in non-Archimedean Banach spaces and prove the Hyers-Ulam stability of additive $\rho$-functional equations associated with the additive $\rho$-functional inequalities (0.1) and (0.2) in non-Archimedean Banach spaces.

1. Introduction and preliminaries

A valuation is a function $|\cdot|$ from a field $K$ into $[0, \infty)$ such that 0 is the unique element having the 0 valuation, $|rs| = |r| \cdot |s|$ and the triangle inequality holds, i.e.,

$$|r+s| \leq |r| + |s|, \quad \forall r, s \in K.$$

A field $K$ is called a valued field if $K$ carries a valuation. The usual absolute values of $\mathbb{R}$ and $\mathbb{C}$ are examples of valuations.

Let us consider a valuation which satisfies a stronger condition than the triangle inequality. If the triangle inequality is replaced by

$$|r+s| \leq \max\{|r|, |s|\}, \quad \forall r, s \in K,$$

then the function $|\cdot|$ is called a non-Archimedean valuation, and the field is called a non-Archimedean field. Clearly $|1| = |1| = 1$ and $|n| \leq 1$ for all $n \in \mathbb{N}$. A trivial example of a non-Archimedean valuation is the function $|\cdot|$ taking everything except for 0 into 1 and $|0| = 0$.

Throughout this paper, we assume that the base field is a non-Archimedean field, hence call it simply a field.


Keywords and phrases: Hyers-Ulam stability, additive $\rho$-functional equation, additive $\rho$-functional inequality, non-Archimedean normed space.

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DEFINITION 1.1. ([7]) Let $X$ be a vector space over a field $K$ with a non-Archimedean valuation $| \cdot |$. A function $\| \cdot \| : X \to [0, \infty)$ is said to be a non-Archimedean norm if it satisfies the following conditions:

(i) $\|x\| = 0$ if and only if $x = 0$;
(ii) $\|rx\| = |r|\|x\|$ ($r \in K, x \in X$);
(iii) the strong triangle inequality $\|x + y\| \leq \max\{\|x\|, \|y\|\}, \quad \forall x, y \in X$ holds. Then $(X, \| \cdot \|)$ is called a non-Archimedean normed space.

DEFINITION 1.2. (i) Let $\{x_n\}$ be a sequence in a non-Archimedean normed space $X$. Then the sequence $\{x_n\}$ is called Cauchy if for a given $\varepsilon > 0$ there is a positive integer $N$ such that

$$\|x_n - x_m\| \leq \varepsilon$$

for all $n, m \geq N$.

(ii) Let $\{x_n\}$ be a sequence in a non-Archimedean normed space $X$. Then the sequence $\{x_n\}$ is called convergent if for a given $\varepsilon > 0$ there are a positive integer $N$ and an $x \in X$ such that

$$\|x_n - x\| \leq \varepsilon$$

for all $n \geq N$. Then we call $x \in X$ a limit of the sequence $\{x_n\}$, and denote by $\lim_{n \to \infty} x_n = x$.

(iii) If every Cauchy sequence in $X$ converges, then the non-Archimedean normed space $X$ is called a non-Archimedean Banach space.


The functional equation

$$f(x + y) = f(x) + f(y)$$

is called the Cauchy equation. In particular, every solution of the Cauchy equation is said to be an additive mapping. Hyers [6] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers’ Theorem was generalized by Aoki [1] for additive mappings and by Rassias [10] for linear mappings by considering an unbounded Cauchy difference. A generalization of the Rassias theorem was obtained by Găvruta [3] by replacing the unbounded Cauchy difference by a general control function in the spirit of Rassias’ approach.

The functional equation

$$f \left( \frac{x + y}{2} \right) = \frac{1}{2} f(x) + \frac{1}{2} f(y)$$

is called the Jensen equation.

In [4], Gilányi showed that if $f$ satisfies the functional inequality

$$\|2f(x) + 2f(y) - f(xy^{-1})\| \leq \|f(xy)\|$$

(1.1)
then $f$ satisfies the Jordan-von Neumann functional equation

$$2f(x) + 2f(y) = f(xy) + f(xy^{-1}).$$


In [8], Park defined additive $\rho$-functional inequalities and additive $\rho$-functional equations and proved the Hyers-Ulam stability of the additive $\rho$-functional inequalities and the additive $\rho$-functional equations in (Archimedean) Banach spaces.

In Section 2, we solve the additive functional inequality (0.1) and prove the Hyers-Ulam stability of the additive functional inequality (0.1) in non-Archimedean Banach spaces. We moreover prove the Hyers-Ulam stability of an additive functional equation associated with the functional inequality (0.1) in non-Archimedean Banach spaces.

In Section 3, we solve the additive functional inequality (0.2) and prove the Hyers-Ulam stability of the additive functional inequality (0.2) in non-Archimedean Banach spaces. We moreover prove the Hyers-Ulam stability of an additive functional equation associated with the functional inequality (0.2) in non-Archimedean Banach spaces.

Throughout this paper, assume that $X$ is a non-Archimedean normed space and that $Y$ is a non-Archimedean Banach space. Let $|\rho| \neq 1$ and let $\rho$ be a non-Archimedean number with $|\rho| < 1$.

2. Additive $\rho$-functional inequality (0.1)

We solve the additive $\rho$-functional inequality (0.1) in non-Archimedean normed spaces.

**Lemma 2.1.** Let $G$ be an Abelian semigroup with division by 2. A mapping $f : G \rightarrow Y$ satisfies

$$\|f(x + y) - f(x) - f(y)\| \leq \rho \left( 2f\left( \frac{x+y}{2} \right) - f(x) - f(y) \right) \quad (2.1)$$

for all $x, y \in G$ if and only if $f : G \rightarrow Y$ is additive.

**Proof.** Assume that $f : G \rightarrow Y$ satisfies (2.1).

Letting $x = y = 0$ in (2.1), we get

$$\|f(0)\| \leq 0.$$

So $f(0) = 0$.

Letting $y = x$ in (2.1), we get

$$\|f(2x) - 2f(x)\| \leq 0$$

and so $f(2x) = 2f(x)$ for all $x \in G$. Thus

$$f\left( \frac{x}{2} \right) = \frac{1}{2}f(x) \quad (2.2)$$
for all \( x \in G \).

It follows from (2.1) and (2.2) that
\[
\|f(x+y) - f(x) - f(y)\| \leq \|\rho \left(2f\left(\frac{x+y}{2}\right) - f(x) - f(y)\right)\|
\]
\[
= |\rho|\|f(x+y) - f(x) - f(y)\|
\]
and so
\[
f(x+y) = f(x) + f(y)
\]
for all \( x, y \in G \).

The converse is obviously true. \( \square \)

**Corollary 2.2.** Let \( G \) be an Abelian semigroup with division by 2. A mapping \( f : G \to Y \) satisfies
\[
f(x+y) - f(x) - f(y) = \rho \left(2f\left(\frac{x+y}{2}\right) - f(x) - f(y)\right)
\]
(2.3)
for all \( x, y \in G \) if and only if \( f : G \to Y \) is additive.

We prove the Hyers-Ulam stability of the additive \( \rho \)-functional inequality (2.1) in Banach spaces.

**Theorem 2.3.** Let \( r < 1 \) and \( \theta \) be nonnegative real numbers, and let \( f : X \to Y \) be a mapping such that
\[
\|f(x+y) - f(x) - f(y)\| \leq \|\rho \left(2f\left(\frac{x+y}{2}\right) - f(x) - f(y)\right)\| + \theta(\|x\|^r + \|y\|^r)
\]
(2.4)
for all \( x, y \in X \). Then there exists a unique additive mapping \( A : X \to Y \) such that
\[
\|f(x) - A(x)\| \leq \frac{2\theta}{2r} \|x\|^r
\]
(2.5)
for all \( x \in X \).

**Proof.** Letting \( y = x \) in (2.4), we get
\[
\|f(2x) - 2f(x)\| \leq 2\theta \|x\|^r
\]
(2.6)
for all \( x \in X \). So
\[
\|f(x) - 2f\left(\frac{x}{2}\right)\| \leq \frac{2}{2r} \theta \|x\|^r
\]
for all $x \in X$. Hence
\begin{equation}
\left\| 2^l f \left( \frac{x}{2^l} \right) - 2^m f \left( \frac{x}{2^m} \right) \right\| 
\leq \max \left\{ \left\| 2^l f \left( \frac{x}{2^l} \right) - 2^{l+1} f \left( \frac{x}{2^{l+1}} \right) \right\|, \ldots, \left\| 2^{m-1} f \left( \frac{x}{2^{m-1}} \right) - 2^m f \left( \frac{x}{2^m} \right) \right\| \right\}
\end{equation}
\begin{equation}
= \max \left\{ \left\| 2^l f \left( \frac{x}{2^l} \right) - 2 f \left( \frac{x}{2^{l+1}} \right) \right\|, \ldots, \left\| 2^{m-1} f \left( \frac{x}{2^{m-1}} \right) - 2 f \left( \frac{x}{2^m} \right) \right\| \right\}
\end{equation}
\begin{equation}
\leq \max \left\{ \left\| 2^l f \left( \frac{x}{2^l} \right) - 2 f \left( \frac{x}{2^{l+1}} \right) \right\|, \ldots, \left\| 2^{m-1} f \left( \frac{x}{2^{m-1}} \right) - 2 f \left( \frac{x}{2^m} \right) \right\| \right\}
\end{equation}
\begin{equation}
= \frac{2 \theta}{\left\| x \right\|} \left\| x \right\|^{r}
\end{equation}
for all nonnegative integers $m$ and $l$ with $m > l$ and all $x \in X$. It follows from (2.7) that the sequence $\{2^k f \left( \frac{x}{2^k} \right) \}$ is Cauchy for all $x \in X$. Since $Y$ is a non-Archimedean Banach space, the sequence $\{2^k f \left( \frac{x}{2^k} \right) \}$ converges. So one can define the mapping $A : X \to Y$ by
\begin{equation}
A(x) := \lim_{k \to \infty} 2^k f \left( \frac{x}{2^k} \right)
\end{equation}
for all $x \in X$. Moreover, letting $l = 0$ and passing the limit $m \to \infty$ in (2.7), we get (2.5).

Now, let $T : X \to Y$ be another additive mapping satisfying (2.5). Then we have
\begin{equation}
\left\| A(x) - T(x) \right\| = \left\| 2^q A \left( \frac{x}{2^q} \right) - 2^q T \left( \frac{x}{2^q} \right) \right\|
\leq \max \left\{ \left\| 2^q A \left( \frac{x}{2^q} \right) - 2^q f \left( \frac{x}{2^q} \right) \right\|, \left\| 2^q T \left( \frac{x}{2^q} \right) - 2^q f \left( \frac{x}{2^q} \right) \right\| \right\}
\leq \frac{2 \theta}{\left\| x \right\|} \left\| x \right\|^{r},
\end{equation}
which tends to zero as $q \to \infty$ for all $x \in X$. So we can conclude that $A(x) = T(x)$ for all $x \in X$. This proves the uniqueness of $A$.

It follows from (2.4) that
\begin{equation}
\left\| A(x+y) - A(x) - A(y) \right\| = \lim_{n \to \infty} \left\| 2^n \left( f \left( \frac{x+y}{2^n} \right) - f \left( \frac{x}{2^n} \right) - f \left( \frac{y}{2^n} \right) \right) \right\|
\leq \lim_{n \to \infty} \left\| 2^n \left( f \left( \frac{x+y}{2^n} \right) - f \left( \frac{x}{2^n} \right) - f \left( \frac{y}{2^n} \right) \right) \right\|
\leq \lim_{n \to \infty} \frac{2^n \theta}{\left\| x \right\|^{r} + \left\| y \right\|^{r}}
\end{equation}
\begin{equation}
+ \lim_{n \to \infty} \frac{\left\| x \right\|^{r} + \left\| y \right\|^{r}}{\left\| x \right\|^{r} + \left\| y \right\|^{r}}
\end{equation}
\begin{equation}
= \left\| \rho \left( 2 A \left( \frac{x+y}{2} \right) - A(x) - A(y) \right) \right\|
\end{equation}
for all $x, y \in X$. So
\begin{equation}
\left\| A(x+y) - A(x) - A(y) \right\| \leq \left\| \rho \left( 2 A \left( \frac{x+y}{2} \right) - A(x) - A(y) \right) \right\|
\end{equation}
for all $x, y \in X$. By Lemma 2.1, the mapping $A : X \to Y$ is additive. □
THEOREM 2.4. Let $r > 1$ and $\theta$ be positive real numbers, and let $f : X \to Y$ be a mapping satisfying (2.4). Then there exists a unique additive mapping $A : X \to Y$ such that

$$\|f(x) - A(x)\| \leq \frac{2\theta}{|2|^{r}}\|x\|^{r}$$

(2.8)

for all $x \in X$.

Proof. It follows from (2.6) that

$$\left\| f(x) - \frac{1}{2} f(2x) \right\| \leq \frac{2\theta}{|2|^{r}}\|x\|^{r}$$

for all $x \in X$. Hence

$$\left\| \frac{1}{2^{r}} f(2^{l} x) - \frac{1}{2^{m}} f(2^{m} x) \right\| \leq \max \left\{ \left\| \frac{1}{2^{r}} f(2^{l} x) - \frac{1}{2^{l+1}} f(2^{l+1} x) \right\|, \ldots, \left\| \frac{1}{2^{m-1}} f(2^{m-1} x) - \frac{1}{2^{m}} f(2^{m} x) \right\| \right\}$$

(2.9)

$$= \max \left\{ \frac{1}{|2|^{r}} \left\| f(2^{l} x) - f(2^{l+1} x) \right\|, \ldots, \frac{1}{|2|^{m-1}} \left\| f(2^{m-1} x) - f(2^{m} x) \right\| \right\}$$

$$\leq \max \left\{ \frac{|2|^{r}}{|2|^{l+1}}, \ldots, \frac{|2|^{r(m-1)}}{|2|^{m+1}} \right\} 2\theta \|x\|^{r}$$

$$= \frac{2\theta}{|2|^{(1-r)l+1}}\|x\|^{r}$$

for all nonnegative integers $m$ and $l$ with $m > l$ and all $x \in X$. It follows from (2.9) that the sequence $\left\{ \frac{1}{2^{m}} f(2^{m} x) \right\}$ is a Cauchy sequence for all $x \in X$. Since $Y$ is complete, the sequence $\left\{ \frac{1}{2^{m}} f(2^{m} x) \right\}$ converges. So one can define the mapping $A : X \to Y$ by

$$A(x) := \lim_{n \to \infty} \frac{1}{2^{n}} f(2^{n} x)$$

for all $x \in X$. Moreover, letting $l = 0$ and passing the limit $m \to \infty$ in (2.9), we get (2.8).

The rest of the proof is similar to the proof of Theorem 2.3. □

Let $A(x, y) := f(x + y) - f(x) - f(y)$ and $B(x, y) := \rho \left( 2 f \left( \frac{x+y}{2} \right) - f(x) - f(y) \right)$ for all $x, y \in X$.

For $x, y \in X$ with $\|A(x, y)\| \leq \|B(x, y)\|$, $\|A(x, y)\| - \|B(x, y)\| \leq \|A(x, y) - B(x, y)\|$.

For $x, y \in X$ with $\|A(x, y)\| > \|B(x, y)\|$, $\|A(x, y)\| = \|A(x, y) - B(x, y) + B(x, y)\|$$\leq \max \{ \|A(x, y) - B(x, y)\|, \|B(x, y)\| \}$$= \|A(x, y) - B(x, y)\|$$\leq \|A(x, y) - B(x, y)\| + \|B(x, y)\|$,
since \( \|A(x,y)\| > \|B(x,y)\| \). So we have
\[
\|f(x+y) - f(x) - f(y)\| - \|\rho \left( 2f \left( \frac{x+y}{2} \right) - f(x) - f(y) \right)\| \\
\leq \left| \|f(x+y) - f(x) - f(y)\| - \rho \left( 2f \left( \frac{x+y}{2} \right) - f(x) - f(y) \right)\| \right|.
\]

As corollaries of Theorems 2.3 and 2.4, we obtain the Hyers-Ulam stability results for the additive \( \rho \)-functional equation (2.3) in non-Archimedean Banach spaces.

**Corollary 2.5.** Let \( r < 1 \) and \( \theta \) be nonnegative real numbers, and let \( f : X \rightarrow Y \) be a mapping such that
\[
\|f(x+y) - f(x) - f(y)\| \leq \theta (\|x\|^r + \|y\|^r) \tag{2.10}
\]
for all \( x,y \in X \). Then there exists a unique additive mapping \( A : X \rightarrow Y \) satisfying (2.5).

**Corollary 2.6.** Let \( r > 1 \) and \( \theta \) be nonnegative real numbers, and let \( f : X \rightarrow Y \) be a mapping satisfying (2.10). Then there exists a unique additive mapping \( A : X \rightarrow Y \) satisfying (2.8).

### 3. Additive \( \rho \)-functional inequality (0.2)

We solve the additive \( \rho \)-functional inequality (0.2) in non-Archimedean normed spaces.

**Lemma 3.1.** Let \( G \) be an Abelian semigroup with division by 2. A mapping \( f : G \rightarrow Y \) satisfies \( f(0) = 0 \) and
\[
\|2f \left( \frac{x+y}{2} \right) - f(x) - f(y)\| \leq \|\rho (f(x+y) - f(x) - f(y))\| \tag{3.1}
\]
for all \( x,y \in G \) if and if \( f : G \rightarrow Y \) is additive.

**Proof.** Assume that \( f : X \rightarrow Y \) satisfies (3.1).

Letting \( y = 0 \) in (3.1), we get
\[
\|2f \left( \frac{x}{2} \right) - f(x)\| \leq 0 \tag{3.2}
\]
and so \( f \left( \frac{x}{2} \right) = \frac{1}{2} f(x) \) for all \( x \in G \).

It follows from (3.1) and (3.2) that
\[
\|f(x+y) - f(x) - f(y)\| = \|2f \left( \frac{x+y}{2} \right) - f(x) - f(y)\| \\
\leq |\rho| \|f(x+y) - f(x) - f(y)\|
\]
and so
\[ f(x + y) = f(x) + f(y) \]
for all \( x, y \in G \).

The converse is obviously true. \( \Box \)

**Corollary 3.2.** Let \( G \) be an Abelian semigroup with division by 2. A mapping \( f : G \to Y \) satisfies \( f(0) = 0 \) and
\[
2f\left(\frac{x + y}{2}\right) - f(x) - f(y) = \rho(f(x + y) - f(x) - f(y))
\]
(3.3)
for all \( x, y \in G \) if and only if \( f : G \to Y \) is additive.

Now, we prove the Hyers-Ulam stability of the additive \( \rho \)-functional inequality (3.1) in non-Archimedean Banach spaces.

**Theorem 3.3.** Let \( r < 1 \) and \( \theta \) be nonnegative real numbers, and let \( f : X \to Y \) be a mapping with \( f(0) = 0 \) such that
\[
\left\| 2f\left(\frac{x + y}{2}\right) - f(x) - f(y) \right\| \leq \| \rho(f(x + y) - f(x) - f(y)) \| + \theta(\|x\|^r + \|y\|^r)
\]
(3.4)
for all \( x, y \in X \). Then there exists a unique additive mapping \( A : X \to Y \) such that
\[
\left\| f(x) - A(x) \right\| \leq \theta \|x\|^r
\]
(3.5)
for all \( x \in X \).

**Proof.** Letting \( y = 0 \) in (3.4), we get
\[
\left\| f(x) - 2f\left(\frac{x}{2}\right) \right\| = \left\| 2f\left(\frac{x}{2}\right) - f(x) \right\| \leq \theta \|x\|^r
\]
(3.6)
for all \( x \in X \). So
\[
\left\| 2^lf\left(\frac{x}{2^l}\right) - 2^mf\left(\frac{x}{2^m}\right) \right\|
\]
(3.7)
\[
\leq \max\left\{ \left\| 2^lf\left(\frac{x}{2^l}\right) - 2^{l+1}f\left(\frac{x}{2^{l+1}}\right) \right\|, \ldots, \left\| 2^{m-1}f\left(\frac{x}{2^{m-1}}\right) - 2^m f\left(\frac{x}{2^m}\right) \right\| \right\}
\]
\[
= \max\left\{ |2|^l \left\| f\left(\frac{x}{2^l}\right) - 2f\left(\frac{x}{2^{l+1}}\right) \right\|, \ldots, |2|^{m-1} \left\| f\left(\frac{x}{2^{m-1}}\right) - 2f\left(\frac{x}{2^m}\right) \right\| \right\}
\]
\[
\leq \max\left\{ \frac{|2|^l}{2^{l(r-1)}}, \ldots, \frac{|2|^{m-1}}{2^{r(m-1)}} \right\} \theta \|x\|^r
\]
\[
= \frac{\theta}{|2|^{(r-1)l}} \|x\|^r
\]
for all nonnegative integers \( m \) and \( l \) with \( m > l \) and all \( x \in X \). It follows from (3.7) that the sequence \( \{2^kf\left(\frac{x}{2^k}\right)\} \) is Cauchy for all \( x \in X \). Since \( Y \) is a non-Archimedean
Banach space, the sequence \( \{2^k f(x)\} \) converges. So one can define the mapping \( A : X \to Y \) by
\[
A(x) := \lim_{k \to \infty} 2^k f \left( \frac{x}{2^k} \right)
\]
for all \( x \in X \). Moreover, letting \( l = 0 \) and passing the limit \( m \to \infty \) in (3.7), we get (3.5).

The rest of the proof is similar to the proof of Theorem 2.3. \( \square \)

**Theorem 3.4.** Let \( r > 1 \) and \( \theta \) be positive real numbers, and let \( f : X \to Y \) be a mapping satisfying \( f(0) = 0 \) and (3.4). Then there exists a unique additive mapping \( A : X \to Y \) such that
\[
\|f(x) - A(x)\| \leq \frac{|2^r \theta| x}{|2|}|x|^r
\]
for all \( x \in X \).

**Proof.** It follows from (3.6) that
\[
\left\| f(x) - \frac{1}{2} f(2x) \right\| \leq \frac{|2^r \theta| x}{|2|}|x|^r
\]
for all \( x \in X \). Hence
\[
\left\| \frac{1}{2^l} f(2^l x) - \frac{1}{2^m} f(2^m x) \right\| \leq \frac{2^{|r l| \theta |x|}}{2^{|l+1|}} \frac{2^{r (m-1)} \theta |x|}{2^{(m-1)+1}} |x|^r
\]
for all nonnegative integers \( m \) and \( l \) with \( m > l \) and all \( x \in X \). It follows from (3.9) that the sequence \( \{ \frac{1}{2^l} f(2^n x) \} \) is a Cauchy sequence for all \( x \in X \). Since \( Y \) is complete, the sequence \( \{ \frac{1}{2^l} f(2^n x) \} \) converges. So one can define the mapping \( A : X \to Y \) by
\[
A(x) := \lim_{n \to \infty} \frac{1}{2^n} f(2^n x)
\]
for all \( x \in X \). Moreover, letting \( l = 0 \) and passing the limit \( m \to \infty \) in (3.9), we get (3.8).

The rest of the proof is similar to the proof of Theorem 2.3. \( \square \)
Let $A(x, y) := 2f \left( \frac{x+y}{2} \right) - f(x) - f(y)$ and $B(x, y) := \rho (f(x+y) - f(x) - f(y))$ for all $x, y \in X$.

For $x, y \in X$ with $\|A(x, y)\| \leq \|B(x, y)\|$, 
\[\|A(x, y)\| - \|B(x, y)\| \leq \|A(x, y) - B(x, y)\| .\]

For $x, y \in X$ with $\|A(x, y)\| > \|B(x, y)\|$, 
\[\|A(x, y)\| = \|A(x, y) - B(x, y) + B(x, y)\|\]
\[\leq \max \{\|A(x, y) - B(x, y)\|, \|B(x, y)\|\}\]
\[= \|A(x, y) - B(x, y)\|\]
\[\leq \|A(x, y) - B(x, y)\| + \|B(x, y)\|,\]

since $\|A(x, y)\| > \|B(x, y)\|$. So we have
\[\|2f \left( \frac{x+y}{2} \right) - f(x) - f(y)\| - \|\rho (f(x+y) - f(x) - f(y))\|\]
\[\leq \left\|2f \left( \frac{x+y}{2} \right) - f(x) - f(y) - \rho (f(x+y) - f(x) - f(y))\right\| .\]

As corollaries of Theorems 3.3 and 3.4, we obtain the Hyers-Ulam stability results for the additive $\rho$-functional equation (3.3) in non-Archimedean Banach spaces.

**COROLLARY 3.5.** Let $r < 1$ and $\theta$ be nonnegative real numbers, and let $f : X \to Y$ be a mapping satisfying $f(0) = 0$ and
\[\left\|2f \left( \frac{x+y}{2} \right) - f(x) - f(y) - \rho (f(x+y) - f(x) - f(y))\right\| \leq \theta (\|x\|^r + \|y\|^r)(3.10)\]
for all $x, y \in X$. Then there exists a unique additive mapping $A : X \to Y$ satisfying (3.5).

**COROLLARY 3.6.** Let $r > 1$ and $\theta$ be nonnegative real numbers, and let $f : X \to Y$ be a mapping satisfying $f(0) = 0$ and (3.10). Then there exists a unique additive mapping $A : X \to Y$ satisfying (3.8).

**REFERENCES**

ADDITIVE $\rho$-FUNCTIONAL INEQUALITIES


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