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# INITIAL COEFFICIENT BOUND FOR $m$–FOLD SYMMETRIC BI–$\lambda$–CONVEX FUNCTIONS

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*Abstract.* Let the functions $f(z) = z + a_2z^2 + \cdots$ and its inverse $f^{-1}$ be analytic and univalent in the unit disk. Such class of functions are called bi-univalent and denoted by $\sigma$ [9]. In an article, Pommerenke [10] remarked that, for an $m$-fold symmetric functions in the class $\mathscr{P}$, the well known lemma stated by Caratheodary for a one fold symmetric functions in $\mathscr{P}$ still holds good. Making use of this remark, we introduce two new subclasses of *bi-univalent* functions in which both $f$ and $f^{-1} = g$ are $m$-fold symmetric analytic functions with $(1-\lambda)\frac{zf'(z)}{f(z)} + \lambda\left(1+\frac{zf''(z)}{f'(z)}\right)$ and $(1-\lambda)\frac{wg'(w)}{g(w)} + \lambda\left(1+\frac{wg''(w)}{g'(w)}\right)$ in $\mathscr{P}$ and obtain coefficient bounds for functions in this new classes.

## 1. Introduction and definitions

Let $\mathscr{A}$ denote the class of functions of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in \mathbb{U}, \tag{1.1}$$

which are *analytic* in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$.

Further, by $\mathscr{S}$, we mean the class of all functions in $\mathscr{A}$ which are univalent in $\mathbb{U}$. For more details on univalent functions, see [3]. It is well known that every function $f \in \mathscr{S}$ has an inverse $f^{-1}$, defined by

$$f^{-1}(f(z)) = z \quad (z \in \mathbb{U}) \tag{1.2}$$

and

$$f(f^{-1}(w)) = w \quad \left(|w| < r_0(f); \quad r_0(f) \geqslant \frac{1}{4}\right). \tag{1.3}$$

Indeed, the inverse function may have an analytic continuation to $\mathbb{U}$, with

$$f^{-1}(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \cdots. \tag{1.4}$$

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A function $f \in \mathscr{A}$ is said to be *bi-univalent* in $\mathbb{U}$ if both $f(z)$ and $f^{-1}(z)$ are univalent in $\mathbb{U}$.

Let $\sigma$ denote the class of *bi-univalent* functions in $\mathbb{U}$, given by equation (1.1). Lewin [9] investigated the class of *bi-univalent* functions $\sigma$ and obtained a bound $|a_2| \leqslant 1.51$. Motivated by the work of Lewin [9], Brannan and Clunie [1] conjectured that $|a_2| \leqslant \sqrt{2}$. Some examples of bi-univalent functions are $\frac{z}{1-z}$, $\frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$ and $-\log(1-z)$ (see also the work of Srivastava et al. [12]). The coefficient estimate problem for each of the following Taylor-Maclaurin coefficients: $|a_n|$ $(n \in \mathbb{N},\ n \geqq 3)$ is still open ([12]).

In recent times, the study of *bi-univalent* functions gained momentum mainly due to the work of Srivastava et al. [12]. Motivated by this, many researchers (see [4, 7, 8, 12, 13, 14, 16, 17, 18, 19]) recently investigated several interesting subclasses of the class $\sigma$ and found non-sharp estimates on the first two Taylor-Maclaurin coefficients. In a very recent article, Sivasubramanian et al. [11] obtained covering theorem, distortion theorem, growth theorem and the radius of convexity for the functions of the class $\sigma$ which answered some of the questions raised by Goodman [6, pages 170–172, question number 2].

For each function $f$ in $\mathscr{S}$, the function

$$h(z) = \sqrt[m]{f(z^m)}$$

is univalent and maps the unit disk $\mathbb{U}$ into a region with $m$-fold symmetry.

A function is $m$-fold symmetric (see [10]) if it has the normalized form

$$f(z) = z + \sum_{k=1}^{\infty} a_{mk+1} z^{mk+1}, \quad z \in \mathbb{U}, \tag{1.5}$$

and we denote the class of $m$-fold symmetric univalent functions by $\mathscr{S}_m$, which are normalized by the above series expansion. In fact, the functions in the class $\mathscr{S}$ are one fold symmetric.

Analogous to the concept of $m$-fold symmetric univalent functions, we here introduce the concept of $m$-fold symmetric *bi-univalent* functions. For details of the the concept of $m$-fold symmetric *bi-univalent* functions one may refer to the work of Srivastava et al. [15]. Each function in the class $f$ in $\sigma$, generates an $m$-fold symmetric *bi-univalent* function for each integer $m$. The normalized form of $f$ is given as in (1.5) and $f^{-1}$ is given as follows. The series expansion for $f^{-1}$ is given by

$$\begin{aligned} g(w) &= w - a_{m+1}w^{m+1} + \left[(m+1)a_{m+1}^2 - a_{2m+1}\right] w^{2m+1} \\ &\quad - \left[\frac{1}{2}(m+1)(3m+2)a_{m+1}^3 - (3m+2)a_{m+1}a_{2m+1} + a_{3m+1}\right] w^{3m+1} + \cdots. \end{aligned} \tag{1.6}$$

where $f^{-1} = g$. We denote the class of $m$-fold symmetric *bi-univalent* functions by $\sigma_m$. For $m = 1$, the formula (1.6) coincides with the formula (1.4) of the class $\sigma$.

Some examples of $m$-fold symmetric bi-univalent functions are $\left(\frac{z^m}{1-z^m}\right)^{\frac{1}{m}}$, $\left(\frac{1}{2}\log\left(\frac{1+z^m}{1-z^m}\right)\right)^{\frac{1}{m}}$ and $(-\log(1-z^m))^{\frac{1}{m}}$ with the corresponding inverse functions $\left(\frac{w^m}{1+w^m}\right)^{\frac{1}{m}}$, $\left(\frac{e^{2w^m}-1}{e^{2w^m}+1}\right)^{\frac{1}{m}}$ and $\left(\frac{e^{w^m}-1}{e^{w^m}}\right)^{\frac{1}{m}}$ respectively.

Denote also, by $\mathscr{P}$, the class of analytic functions of the form $p(z) = 1 + p_1 z + p_2 z^2 + \cdots$ such that $\Re(p(z)) > 0$ in $\mathbb{U}$.

Pommerenke [10] remarked that, for an $m$-fold symmetric functions in the class $\mathscr{P}$, the well known lemma stated by Caratheodary for a one fold symmetric functions in $\mathscr{P}$ still holds good. As an application of this remark, we introduce some new subclasses of *bi-univalent* functions in which both $f$ and $f^{-1}$ are m-fold symmetric analytic functions with $(1-\lambda)\frac{zf'(z)}{f(z)} + \lambda\left(1+\frac{zf''(z)}{f'(z)}\right)$ and $(1-\lambda)\frac{wg'(w)}{g(w)} + \lambda\left(1+\frac{wg''(w)}{g'(w)}\right)$ in $\mathscr{P}$ and obtain coefficient bounds of $|a_{m+1}|$ and $|a_{2m+1}|$ for functions in this new classes.

We use the following lemma of Caratheodary class to derive our main result.

LEMMA 1.1. *If $h \in \mathscr{P}$, then $|c_k| \leqslant 2$ for each $k$, where $\mathscr{P}$ is the family of all functions $h$ analytic in $\mathbb{U}$ for which $\Re(h(z)) > 0$ and*

$$h(z) = 1 + c_1 z + c_2 z^2 + \ldots \qquad for\ \ z \in \mathbb{U}. \tag{1.7}$$

*and the extremal is obtained for the function* $h(z) = \frac{1+z}{1-z}$.

In view of Pommerenke [10], the $m$-fold symmetric functions in the class $\mathscr{P}$ is of the form

$$p(z) = 1 + c_m z^m + c_{2m} z^{2m} + \cdots.$$

## 2. Coefficient bounds for the class $\mathscr{M}_{\sigma,m}[\alpha,\lambda]$

DEFINITION 2.1. A function $f(z)$, given by (1.5), is said to be in the class $\mathscr{M}_{\sigma,m}[\alpha,\lambda]$ if the following conditions are satisfied;

$$f \in \sigma_m, \quad \left|\arg\left((1-\lambda)\frac{zf'(z)}{f(z)} + \lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\right)\right| < \frac{\alpha\pi}{2}, \ (z \in \mathbb{U};\ 0 < \alpha \leqslant 1) \tag{2.1}$$

and

$$\left|\arg\left((1-\lambda)\frac{wg'(w)}{g(w)} + \lambda\left(1+\frac{wg''(w)}{g'(w)}\right)\right)\right| < \frac{\alpha\pi}{2}, \ (w \in \mathbb{U};\ 0 < \alpha \leqslant 1) \tag{2.2}$$

where the function $g(w)$ is given by (1.6).

For the case of one fold symmetric functions with $\lambda = 0$, the class $\mathscr{M}_{\sigma,m}[\alpha,\lambda]$ reduces to the class $\mathscr{S}^*_\sigma[\alpha]$ introduced and studied by Brannan and Taha [2].

We first state and prove the following theorem.

THEOREM 2.1. *Let $f(z)$, given by (1.5), be in the class $\mathscr{M}_{\sigma,m}[\alpha,\lambda]$, $0<\alpha\leqslant 1$ and $0\leqslant\lambda\leqslant 1$.*
*Then*

$$|a_{m+1}| \leqslant \frac{2\alpha}{m\sqrt{(1+\lambda m)(\alpha-\lambda\alpha m+\lambda m+1)}} \tag{2.3}$$

*and*

$$|a_{2m+1}| \leqslant \frac{2(m+1)\alpha^2}{m^2(1+\lambda m)^2}+\frac{\alpha}{m(2\lambda m+1)}. \tag{2.4}$$

*Proof.* From (2.1) and (2.2) we get

$$(1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)=[p(z)]^\alpha \tag{2.5}$$

and

$$(1-\lambda)\frac{wg'(w)}{g(w)}+\lambda\left(1+\frac{wg''(w)}{g'(w)}\right)=[q(w)]^\alpha \tag{2.6}$$

where $p(z)$ and $q(w)$ in $\mathscr{P}$ with the series representation

$$p(z)=1+p_m z^m+p_{2m}z^{2m}+p_{3m}z^{3m}+\dots \tag{2.7}$$

and

$$q(w)=1+q_m w^m+q_{2m}w^{2m}+q_{3m}w^{3m}+\dots. \tag{2.8}$$

Now, equating the coefficients in (2.5) and (2.6), we get

$$(1+\lambda m)ma_{m+1}=\alpha p_m, \tag{2.9}$$

$$2m(2\lambda m+1)a_{2m+1}-(1+2\lambda m+\lambda m^2)ma_{m+1}^2=\alpha p_{2m}+\frac{\alpha(\alpha-1)}{2}p_m^2, \tag{2.10}$$

$$-(1+\lambda m)ma_{m+1}=\alpha q_m \tag{2.11}$$

and

$$(1+2m+2\lambda m+3\lambda m^2)ma_{m+1}^2-2m(2m\lambda+1)a_{2m+1}=\alpha q_{2m}+\frac{\alpha(\alpha-1)}{2}q_m^2. \tag{2.12}$$

From (2.9) and (2.11), we get

$$p_m=-q_m \tag{2.13}$$

and

$$2(1+\lambda m)^2m^2a_{m+1}^2=\alpha^2(p_m^2+q_m^2). \tag{2.14}$$

Also from (2.10), (2.12) and (2.14), a simple computation shows that

$$\begin{aligned} 2m^2(1+\lambda m)a_{m+1}^2 &= \alpha(p_{2m}+q_{2m}) + \frac{\alpha(\alpha-1)}{2}(p_m^2+q_m^2) \\ &= \alpha(p_{2m}+q_{2m}) + \frac{\alpha(\alpha-1)}{2}\frac{2(1+\lambda m)^2m^2}{\alpha^2}a_{m+1}^2. \end{aligned}$$

Therefore, we have

$$a_{m+1}^2 = \frac{\alpha^2(p_{2m}+q_{2m})}{(1+\lambda m)m^2(\alpha-\lambda\alpha m+\lambda m+1)}.$$

Applying Lemma 1.1 for the coefficients $p_{2m}$ and $q_{2m}$, we get

$$|a_{m+1}| \leqslant \frac{2\alpha}{m\sqrt{(1+\lambda m)(\alpha-\lambda\alpha m+\lambda m+1)}}.$$

This gives the desired estimate on $|a_2|$ as asserted in (2.3).

Next, in order to find the bound on $|a_{2m+1}|$, by a simple computation using (2.12), (2.10) and (2.13), we get

$$\begin{aligned} &4m^2(2\lambda m+1)(\lambda m+1)a_{2m+1} \\ &= \alpha\left[(1+2m+2\lambda m+3\lambda m^2)p_{2m}+(1+2\lambda m+\lambda m^2)q_{2m}\right] \\ &\quad + \frac{\alpha(\alpha-1)}{2}\left[(1+2m+2\lambda m+3\lambda m^2)p_m^2+(1+2\lambda m+\lambda m^2)q_m^2\right] \end{aligned} \tag{2.15}$$

Applying Lemma 1.1 again for the coefficients $p_m$, $p_{2m}$, $q_m$ and $q_{2m}$, we get

$$|a_{2m+1}| \leqslant \frac{(m+1)\alpha^2}{m^2(1+\lambda m)}.$$

This completes the proof of Theorem 2.1. □

For the case of one fold symmetric functions with $\lambda = 0$, Theorem 2.1 reduces to the results of Brannan and Taha [2].

COROLLARY 2.1. ([2]) *Let $f(z)$, given by (1.1), be in the class $\mathscr{S}_\sigma^*[\alpha]$, $(0 < \alpha \leqslant 1)$. Then*

$$|a_2| \leqslant \frac{2\alpha}{\sqrt{1+\alpha}} \tag{2.16}$$

*and*

$$|a_3| \leqslant 2\alpha^2. \tag{2.17}$$

For the case of one fold symmetric functions with $\lambda = 1$, Theorem 2.1 reduces to the results of Brannan and Taha [2].

COROLLARY 2.2. ([2]) *Let $f(z)$, given by (1.1), be in the class $\mathscr{C}_\sigma[\alpha]$, $(0 < \alpha \leqslant 1)$. Then*

$$|a_2| \leqslant \alpha \tag{2.18}$$

*and*

$$|a_3| \leqslant \alpha^2. \tag{2.19}$$

### 3. Coefficient bound for the function class $\mathscr{M}^*_{\sigma,m}(\beta,\lambda)$

DEFINITION 3.1. A function $f(z)$, given by (1.5), is said to be in the class $\mathscr{M}^*_{\sigma,m}(\beta,\lambda)$ if the following conditions are satisfied;

$$f \in \sigma_m \text{ and } \Re\left((1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\right) > \beta,\ (z \in \mathbb{U};\ 0 \leqslant \beta < 1) \tag{3.1}$$

and

$$\Re\left((1-\lambda)\frac{wg'(w)}{g(w)}+\lambda\left(1+\frac{wg''(w)}{g'(w)}\right)\right) > \beta,\ (w \in \mathbb{U};\ 0 \leqslant \beta < 1) \tag{3.2}$$

where the function $g$ is given by (1.6).

It is easy that for the choice of one fold symmetric functions with $\lambda = 0$ and $\lambda = 1$, the class $\mathscr{M}^*_{\sigma,m}(\beta,\lambda)$ reduces to the classes $\mathscr{S}^*_{\sigma}(\beta)$ and $\mathscr{C}_{\sigma}(\beta)$ respectively. The later classes were introduced and studied by Brannan and Taha. ([2].

THEOREM 3.1. *Let $f(z)$, given by (1.5), be in the class $\mathscr{M}^*_{\sigma,m}(\beta,\lambda)$, $0 \leqslant \beta < 1$. Then*

$$|a_{m+1}| \leqslant \frac{1}{m}\sqrt{\frac{2(1-\beta)}{1+\lambda m}} \tag{3.3}$$

*and*

$$|a_{2m+1}| \leqslant \frac{(1-\beta)(m+1)}{m^2(1+\lambda m)}. \tag{3.4}$$

*Proof.* The argument inequalities (3.1) and (3.2) can be written in the following forms,

$$(1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right) = \beta+(1-\beta)p(z) \tag{3.5}$$

and

$$(1-\lambda)\frac{wg'(w)}{g(w)}+\lambda\left(1+\frac{wg''(w)}{g'(w)}\right) = \beta+(1-\beta)q(w) \tag{3.6}$$

where $p(z)$ and $q(w)$ in $\mathscr{P}$ and have the forms (2.7) and (2.8) respectively. As in the proof of theorem 2.1, by equating the coefficients in (3.5) and (3.6),we get

$$(1+\lambda m)ma_{m+1} = (1-\beta)p_m, \tag{3.7}$$

$$2m(2\lambda m+1)a_{2m+1}-(1+2\lambda m+\lambda m^2)ma^2_{m+1} = (1-\beta)p_{2m}, \tag{3.8}$$

$$-(1+\lambda m)ma_{m+1} = (1-\beta)q_m \tag{3.9}$$

and

$$(1+2m+2\lambda m+3\lambda m^2)ma^2_{m+1}-2m(2m\lambda+1)a_{2m+1} = (1-\beta)q_{2m}. \tag{3.10}$$

From (3.7) and (3.9), we get

$$p_m = -q_m \tag{3.11}$$

and

$$2(1+\lambda m)^2 m^2 a_{m+1}^2 = (1-\beta)^2 (p_m^2 + q_m^2). \tag{3.12}$$

Also from (3.8) and (3.10), we get

$$2m^2(1+\lambda m)a_{m+1}^2 = (1-\beta)(p_{2m}+q_{2m}).$$

Thus, we have

$$|a_{m+1}|^2 \leqslant \frac{(1-\beta)(|p_{2m}|+|q_{2m}|)}{2m^2(1+\lambda m)} \leqslant \frac{4(1-\beta)}{2m^2(1+\lambda m)}.$$

This gives the bound on $|a_{m+1}|$ as asserted in (3.3).

Next, in order to find the bound on $|a_{2m+1}|$, by a simple computation using (3.10) and (3.8), we get

$$\begin{aligned} &4m^2(2\lambda m+1)(\lambda m+1)a_{2m+1} \\ &= (1-\beta)\left((1+2m+2\lambda m+3\lambda m^2)p_{2m} + (1+2\lambda m+\lambda m^2)q_{2m}\right) \end{aligned}$$

or equivalently

$$a_{2m+1} = \frac{(1-\beta)\left((1+2m+2\lambda m+3\lambda m^2)p_{2m} + (1+2\lambda m+\lambda m^2)q_{2m}\right)}{4m^2(2\lambda m+1)(\lambda m+1)}. \tag{3.13}$$

Applying Lemma 1.1 for the coefficients $p_{2m}$ and $q_{2m}$, we get

$$|a_{2m+1}| \leqslant \frac{(1-\beta)(m+1)}{m^2(1+\lambda m)} \tag{3.14}$$

which is the bound on $|a_{2m+1}|$ as asserted in (3.4). □

For the case of one fold symmetric functions with $\lambda = 0$, Theorem 3.1 reduces to the following results of Brannan and Taha ([2]).

COROLLARY 3.1. ([2]) *Let $f(z)$, given by (1.1), be in the class $\mathscr{S}_\sigma^*(\beta)$, $(0 \leqslant \beta < 1)$. Then*

$$|a_2| \leqslant \sqrt{2(1-\beta)} \tag{3.15}$$

*and*

$$|a_3| \leqslant 2(1-\beta). \tag{3.16}$$

For the case of one fold symmetric functions with $\lambda = 1$ in Theorem 3.1, we get the following results of Brannan and Taha [2].

COROLLARY 3.2. ([2]) *Let* $f(z)$*, given by* (1.1)*, be in the class* $\mathscr{C}_\sigma(\beta)$, $(0 \leqslant \beta < 1)$. *Then*

$$|a_2| \leqslant \sqrt{(1-\beta)} \tag{3.17}$$

*and*

$$|a_3| \leqslant (1-\beta). \tag{3.18}$$

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