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A GENERALIZATION OF OSTROWSKI TYPE INEQUALITY ON TIME SCALES WITH k POINTS

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Abstract. In this paper, we generalize the Ostrowski type inequality including a parameter on time scales with k points and unify corresponding continuous and discrete versions. We also discuss some particular Ostrowski type inequalities on time scales as special cases.

1. Introduction

In 1938, Ostrowski established the following interesting integral inequality which is a relationship between the value of a function f on some point in (a,b) and the integration on $[a,b]$:

Suppose that $f:[a,b]\to\mathbb{R}$ is continuous on $[a,b]$ and differentiable in (a,b), and its derivative $f':(a,b)\to\mathbb{R}$ is bounded in (a,b), i.e., $\|f'\|_\infty := \sup_{t\in(a,b)}|f'(t)|<\infty$. Then for any $t\in[a,b]$, we have the inequality

$$\left|(b-a)f(t)-\int_a^b f(s)ds\right| \leqslant \left[\frac{(b-a)^2}{4}+\left(t-\frac{a+b}{2}\right)^2\right]\|f'\|_\infty.$$

The inequality is sharp in the sense that the constant $\frac{1}{4}$ cannot be replaced by a smaller one.

Later, many authors have introduced some results on the extensions, generalizations, and applications of Ostrowski type inequality which give explicit error bounds for some known and new quadrature formulas, see [1].

The development of the theory of time scales was initiated by Hilger [2] in 1988 as a theory capable to contain both difference and differential calculus in a consistent way. Since then, many researchers have studied the theory of certain integral inequalities or dynamic equations on time scales (cf. [3–6]). Particularly, Bohner and Matthews [3, Theorem 3.5] established the following Ostrowski type inequality on time scales:

Suppose that $a,\ b,\ s,\ t\in\mathbb{T}$, $a<b$, and $f:[a,b]\to\mathbb{R}$ is differentiable. Then

$$\left|f(t)-\frac{1}{b-a}\int_a^b f(s)\triangle s\right| \leqslant \frac{M}{b-a}(h_2(t,a)+h_2(t,b)), \tag{1.1}$$

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where $\mathbb{T}$ *is a time scale,* h_2 *is a function defined in Section 2, and* $M = \sup_{a<t<b} |f^{\triangle}(t)| < \infty$. *This inequality is sharp in the sense that the right-hand side of (1.1) cannot be replaced by a smaller one.*

Recently, Liu and Ngô [7, Theorem 4] generalized the inequality (1.1) by adding k points to the interval $[a,b]$, and they obtained the following result:

Suppose that

1) $a,b \in \mathbb{T}$ *and* $\mathbb{I}_k := \{x_0, x_1, \dots, x_k\} \subset \mathbb{T}$, *where* $a = x_0 < x_1 < \cdots < x_{k-1} < x_k = b$, *is a partition of the interval* $[a,b]$*;*

2) $\{\alpha_0, \alpha_1, \dots, \alpha_{k+1}\} \subset \mathbb{T}$ *is a set of k+2 points such that* $\alpha_0 = a$, $\alpha_i \in [x_{i-1}, x_i]$ *for* $i = 0,1,\dots,k$ *and* $\alpha_{k+1} = b$*;*

3) $f : [a,b] \to \mathbb{R}$ *is a differentiable function.*

Then we have the inequality

$$\left| \int_a^b f^{\sigma}(t) \triangle t - \sum_{i=0}^{k} (\alpha_{i+1} - \alpha_i) f(x_i) \right| \leqslant M \sum_{i=0}^{k-1} \left(h_2(x_i, \alpha_{i+1}) + h_2(x_{i+1}, \alpha_{i+1}) \right), \quad (1.2)$$

where $\mathbb{T}$ *is a time scale,* h_2 *is a function defined in Section 2, and* $M = \sup_{a<t<b} |f^{\triangle}(t)| < \infty$. *This inequality is sharp in the sense that the right-hand side of (1.2) cannot be replaced by a smaller one.*

In the present paper, by introducing a parameter, we will first extend some generalization of Ostrowski type inequality on time scales with k points and unify corresponding continuous and discrete versions. In the end, we also discuss some particular integral inequalities on time scales as special cases.

2. A new generalized Ostrowski type inequality on time scales with k points

Throughout this paper, we assume that $\mathbb{T}$ is a time scale, which is an arbitrary nonempty closed subset of real numbers. For more definitions and properties about the theory of time scales, one can refer to [2, 8, 9]. Before introducing the new generalized Ostrowski type inequalities, we firstly define the function $h_k(t,s)$ and give the generalized Montgomery identity for k points.

DEFINITION 1. Let $h_k : \mathbb{T} \times \mathbb{T} \to \mathbb{R}$, $k \in \mathbb{N}_0$ be defined by

$$h_0(t,s) = 1 \text{ for all } s,t \in \mathbb{T}$$

and then recursively by

$$h_{k+1}(t,s) = \int_s^t h_k(\tau, s) \triangle \tau \text{ for all } s,t \in \mathbb{T}.$$

LEMMA 1. (Generalized Montgomery identity for a parameter) *Suppose that*

1) $a,\ b\in\mathbb{T}$ *and* $\mathbb{I}_k:=\{x_0,x_1,\dots,x_k\}\subset\mathbb{T}$, *where* $a=x_0<x_1<\cdots<x_{k-1}<x_k=b$, *is a partition of the interval* $[a,b]$*;*

2) $\{\alpha_0,\alpha_1,\dots,\alpha_{k+1}\}\subset\mathbb{T}$ *is a set of* $k+2$ *points such that* $\alpha_0=a,\ \alpha_i\in[x_{i-1},x_i]$ *for* $i=0,1,\dots,k$ *and* $\alpha_{k+1}=b$*;*

3) $f:[a,b]\to\mathbb{R}$ *is a differentiable function.*

We then have the inequality

$$
\begin{aligned}
&(1-\lambda)\sum_{i=0}(\alpha_{i+1}-\alpha_i)f(x_i)+\lambda\sum_{i=0}^{k}(\alpha_{i+1}-\alpha_i)\frac{f(x_i)+f(x_{i+1})}{2}\\
&=\int_a^b f^{\sigma}(t)\triangle t+\int_a^b K(t,I_k)f^{\triangle}(t)\triangle t,
\end{aligned}
\tag{2.1}
$$

where

$$
K(t,I_k)=\begin{cases}
t-\left(\alpha_1-\lambda\frac{\alpha_1-a}{2}\right), & t\in[a,\alpha_1),\\
t-\left(\alpha_1+\lambda\frac{\alpha_2-\alpha_1}{2}\right), & t\in[\alpha_1,x_1),\\
t-\left(\alpha_2-\lambda\frac{\alpha_2-\alpha_1}{2}\right), & t\in[x_1,\alpha_2),\\
\quad\vdots & \quad\vdots\\
t-\left(\alpha_{k-1}+\lambda\frac{\alpha_k-\alpha_{k-1}}{2}\right), & t\in[\alpha_{k-1},x_{k-1}),\\
t-\left(\alpha_k-\lambda\frac{\alpha_k-\alpha_{k-1}}{2}\right), & t\in[x_{k-1},\alpha_k),\\
t-\left(\alpha_k+\lambda\frac{\alpha_{k+1}-\alpha_k}{2}\right), & t\in[\alpha_k,b].
\end{cases}
$$

Proof. Integrating by parts, we have

$$
\begin{aligned}
&\int_a^b K(t,I_k)f^{\triangle}(t)\triangle t\\
&=\sum_{i=0}^{k-1}\Bigg[\int_{x_i}^{\alpha_{i+1}}\left(t-\left(\alpha_{i+1}-\lambda\frac{\alpha_{i+1}-\alpha_i}{2}\right)\right)f^{\triangle}(t)\triangle t\\
&\quad+\int_{\alpha_{i+1}}^{x_{i+1}}\left(t-\left(\alpha_{i+1}+\lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}\right)\right)f^{\triangle}(t)\triangle t\Bigg]\\
&=\sum_{i=0}^{k-1}\Bigg[\left(\alpha_{i+1}-\left(\alpha_{i+1}-\lambda\frac{\alpha_{i+1}-\alpha_i}{2}\right)\right)f(\alpha_{i+1})-\left(\alpha_{i+1}-\left(x_i-\lambda\frac{\alpha_{i+1}-\alpha_i}{2}\right)\right)f(x_i)\\
&\quad-\int_{x_i}^{\alpha_{i+1}}f^{\sigma}(t)\triangle t+\left(x_{i+1}-\left(\alpha_{i+1}+\lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}\right)\right)f(x_{i+1})\\
&\quad-\left(\alpha_{i+1}-\left(\alpha_{i+1}+\lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}\right)\right)f(\alpha_{i+1})-\int_{\alpha_{i+1}}^{x_{i+1}}f^{\sigma}(t)\triangle t\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{i=0}^{k-1} \frac{\lambda}{2}(\alpha_{i+2}-\alpha_i) f(\alpha_{i+1}) + \sum_{i=0}^{k-1}(1-\lambda)(\alpha_{i+2}-\alpha_i) f(x_i) \\
&\quad + \left(1-\frac{\lambda}{2}\right)[(b-\alpha_k) f(b) + (\alpha(1)-a) f(a)] - \int_a^b f^\sigma(t)\triangle t \\
&= (1-\lambda)\sum_{i=0}^{k}(\alpha_{i+1}-\alpha_i) f(x_i) + \lambda \sum_{i=0}^{k}(\alpha_{i+1}-\alpha_i)\frac{f(x_i)+f(x_{i+1})}{2} - \int_a^b f^\sigma(t)\triangle t,
\end{aligned}
$$

from which equation (2.1) follows. □

Now, we introduce our main result in the following theorem:

THEOREM 1. *Suppose that*

1) $a,\ b \in \mathbb{T}$ *and* $\mathbb{I}_k := \{x_0, x_1, \dots, x_k\} \subset \mathbb{T}$, *where* $a = x_0 < x_1 < \cdots < x_{k-1} < x_k = b$, *is a partition of the interval* $[a,b]$*;*

2) $\{\alpha_0, \alpha_1, \dots, \alpha_k\} \subset \mathbb{T}$ *is a set of* $k+2$ *points such that* $\alpha_0 = a$, $\alpha_i \in [x_{i-1}, x_i]$ *for* $i = 0, 1, \dots, k$ *and* $\alpha_{k+1} = b$*;*

3) $f : [a,b] \to \mathbb{R}$ *is a differentiable function.*

We then have the inequality

$$
\begin{aligned}
&\left| \int_a^b f^\sigma(t)\triangle t - \left[(1-\lambda)\sum_{i=0}^{k}(\alpha_{i+1}-\alpha_i) f(x_i) + \lambda\sum_{i=0}^{k}(\alpha_{i+1}-\alpha_i)\frac{f(x_i)+f(x_{i+1})}{2}\right]\right| \\
&\leqslant M\sum_{i=0}^{k-1}\left[h_2\left(x_i, \alpha_{i+1} - \lambda\frac{\alpha_{i+1}-\alpha_i}{2}\right) + h_2\left(\alpha_{i+1}, \alpha_{i+1} - \lambda\frac{\alpha_{i+1}-\alpha_i}{2}\right)\right. \\
&\quad \left. + h_2\left(\alpha_{i+1}, \alpha_{i+1} + \lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}\right) + h_2\left(x_{i+1}, \alpha_{i+1} + \lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}\right)\right], \qquad (2.2)
\end{aligned}
$$

where $M = \sup_{a<t<b} |f^\triangle(t)| < \infty$.

Proof. By applying Lemma 1, we get

$$
\begin{aligned}
&\left| \int_a^b f^\sigma(t)\triangle t - \left[(1-\lambda)\sum_{i=0}^{k}(\alpha_{i+1}-\alpha_i) f(x_i) + \lambda\sum_{i=0}^{k}(\alpha_{i+1}-\alpha_i)\frac{f(x_i)+f(x_{i+1})}{2}\right]\right| \\
&= \left| \int_a^b K(t, I_k) f^\triangle(t)\triangle t \right| \\
&\leqslant M\sum_{0}^{k-1}\left[\int_{x_i}^{\alpha_{i+1}}\left|t - \left(\alpha_{i+1} - \lambda\frac{\alpha_{i+1}-\alpha_i}{2}\right)\right|\triangle t + \int_{\alpha_{i+1}}^{x_{i+1}}\left|t - \left(\alpha_{i+1} + \lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}\right)\right|\triangle t\right]
\end{aligned}
$$

$$
\begin{aligned}
&= M\sum_{0}^{k-1}\Bigg[\int_{x_i}^{\alpha_{i+1}-\lambda\frac{\alpha_{i+1}-\alpha_i}{2}}\left|t-\left(\alpha_{i+1}-\lambda\frac{\alpha_{i+1}-\alpha_i}{2}\right)\right|\triangle t\\
&\quad+\int_{\alpha_{i+1}-\lambda\frac{\alpha_{i+1}-\alpha_i}{2}}^{\alpha_{i+1}}\left|t-\left(\alpha_{i+1}-\lambda\frac{\alpha_{i+1}-\alpha_i}{2}\right)\right|\triangle t\\
&\quad+\int_{\alpha_{i+1}}^{\alpha_{i+1}+\lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}}\left|t-\left(\alpha_{i+1}+\lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}\right)\right|\triangle t\\
&\quad+\int_{\alpha_{i+1}+\lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}}^{x_{i+1}}\left|t-\left(\alpha_{i+1}+\lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}\right)\right|\triangle t\Bigg]\\
&= M\sum_{0}^{k-1}\Bigg[\int_{\alpha_{i+1}-\lambda\frac{\alpha_{i+1}-\alpha_i}{2}}^{x_i}\left(t-\left(\alpha_{i+1}-\lambda\frac{\alpha_{i+1}-\alpha_i}{2}\right)\right)\triangle t\\
&\quad+\int_{\alpha_{i+1}-\lambda\frac{\alpha_{i+1}-\alpha_i}{2}}^{\alpha_{i+1}}\left(t-\left(\alpha_{i+1}-\lambda\frac{\alpha_{i+1}-\alpha_i}{2}\right)\right)\triangle t\\
&\quad+\int_{\alpha_{i+1}+\lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}}^{\alpha_{i+1}}\left(t-\left(\alpha_{i+1}+\lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}\right)\right)\triangle t\\
&\quad+\int_{\alpha_{i+1}+\lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}}^{x_{i+1}}\left(t-\left(\alpha_{i+1}+\lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}\right)\right)\triangle t\Bigg]\\
&= M\sum_{i=0}^{k-1}\Bigg[h_2\left(x_i,\alpha_{i+1}-\lambda\frac{\alpha_{i+1}-\alpha_i}{2}\right)+h_2\left(\alpha_{i+1},\alpha_{i+1}-\lambda\frac{\alpha_{i+1}-\alpha_i}{2}\right)\\
&\quad+h_2\left(\alpha_{i+1},\alpha_{i+1}+\lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}\right)+h_2\left(x_{i+1},\alpha_{i+1}+\lambda\frac{\alpha_{i+2}-\alpha_{i+1}}{2}\right)\Bigg],
\end{aligned}
$$

which completes the proof. □

REMARK 1. If we take $\lambda = 0$ in Theorem 1, then inequality (1.2) can be directly obtained and this theorem also covers the result in [7].

Taking different time scales in Theorem 1, one can directly obtain some new results stated in the corollaries below. If $\mathbb{T} = \mathbb{R}$, then our delta integral is the usual Riemann integral, and hence,

$$h_2(t,s) = \frac{(t-s)^2}{2} \quad \text{for all } s,\ t \in \mathbb{R},$$

from which one can have the following corollary:

COROLLARY 1. (Continuous case) *If $\mathbb{T} = \mathbb{R}$, then we have the inequality*

$$
\begin{aligned}
&\left|\int_a^b f(t)dt-\left[(1-\lambda)\sum_{i=0}^{k}(\alpha_{i+1}-\alpha_i)f(x_i)+\lambda\sum_{i=0}^{k}(\alpha_{i+1}-\alpha_i)\frac{f(x_i)+f(x_{i+1})}{2}\right]\right|\\
&\leqslant M\left(\frac{1}{4}((1-\lambda)^2+\lambda^2)\sum_{i=0}^{k-1}(x_{i+1}-x_i)^2+\sum_{i=0}^{k-1}(\alpha_{i+1}-\frac{x_{i+1}+x_i}{2})^2\right),
\end{aligned}
$$

where $M = \sup_{a<t<b} |f^{\triangle}(t)| < \infty$.

COROLLARY 2. (Discrete case) *Suppose that* $\mathbb{T} = \mathbb{Z},\ a = 0,\ b = n$, *and*

1) $\mathbb{I}_k := \{j_0, j_1, \ldots, j_k\} \subset \mathbb{Z}$, *where* $a = j_0 < j_1 < \cdots < j_k = b$, *is a partition of the set* $[a,b] \bigcap \mathbb{Z}$;

2) $\{\alpha_0, \alpha_1, \ldots, \alpha_{k+1}\} \subset \mathbb{Z}$ *is a set of* $k+2$ *points such that* $\alpha_0 = a, \alpha_i \in [j_{i-1}, j_i]$ *for* $i = 1, 2, \ldots, k$ *and* $\alpha_{k+1} = b$;

3) $f(k) = x_k$.

We then have the inequality

$$\left| \sum_{i=1}^{n} x_i - \sum_{i=0}^{k} (\alpha_{i+1} - \alpha_i) \left((1-\lambda) x_j + \lambda \frac{x_{\alpha_{i+1}} + x_{\alpha_i}}{2} \right) \right|$$

$$\leqslant \frac{M}{2} \sum_{i=0}^{k-1} \left[j_i \left(j_i + (\lambda - 2)\alpha_{i+1} - \lambda \alpha_i - 1 \right) + j_{i+1} \left(j_{i+1} + (\lambda - 2)\alpha_{i+1} - \lambda \alpha_{i+2} - 1 \right) \right.$$

$$+ 2\alpha_{i+1} \left((1-\lambda)\alpha_{i+1} + \lambda \frac{\alpha_{i+1} + \alpha_i}{2} + 1 \right) + 2 \left(\lambda \frac{\alpha_{i+1} - \alpha_i}{2} - \frac{1}{2} \right)^2$$

$$\left. + 2 \left(\lambda \frac{\alpha_{i+2} - \alpha_{i+1}}{2} + \frac{1}{2} \right)^2 - 1 \right].$$

Proof. Under the assumptions above, it is known that

$$h_k(t,s) = \binom{t-s}{k} \quad \text{for all } s,\ t \in \mathbb{Z}.$$

Taking $k = 2$, the result follows. □

3. Some particular integral inequalities on time scales

In this section, we discuss some particular integral inequalities on time scales as special cases. Throughout this section, we assume that $a, b \in \mathbb{T}$ with $a < b$ and $f : [a,b] \to \mathbb{R}$ is differentiable, and g is a function of $[0,1]$ into $[0,1]$. We set $M = \sup_{a<t<b} |f^{\triangle}(t)| < \infty$.

PROPOSITION 1. *Suppose* $\alpha \in [a,b] \bigcap \mathbb{T}$. *Under the assumptions of Theorem 1 with* $x_0 = a,\ x_1 = b,\ \alpha_0 = a,\ \alpha_1 = \alpha$, *and* $\alpha_2 = b$, *we have the integral inequality on the time scale*

$$\left| \int_a^b f^{\sigma}(t) \triangle t - \left[\left(1 - \frac{\lambda}{2}\right)(\alpha - a) f(a) + \frac{\lambda}{2}(b-a) f(\alpha) + \left(1 - \frac{\lambda}{2}\right)(b - \alpha) f(b) \right] \right|$$

$$\leqslant M \left[h_2\left(a, \alpha - \lambda \frac{\alpha - a}{2}\right) + h_2\left(\alpha, \alpha - \lambda \frac{\alpha - a}{2}\right) + h_2\left(\alpha, \alpha + \lambda \frac{b - \alpha}{2}\right) \right.$$

$$\left. + h_2\left(b, \alpha + \lambda \frac{b-\alpha}{2}\right) \right],$$

for all $\lambda \in [0,1]$ *such that* $\alpha - \lambda \frac{\alpha - a}{2}$ *and* $\alpha + \lambda \frac{\alpha - a}{2}$ *are in* $\mathbb{T}$.

REMARK 2. If we choose $\lambda = 0$ in Proposition 1, then we get the rectangle inequality on the time scale

$$\left| \int_a^b f^\sigma(t) \triangle t - [(\alpha - a)f(a) + (b - \alpha)f(b)] \right| \leqslant M(h_2(a,\alpha) + h_2(b,\alpha)).$$

PROPOSITION 2. *Suppose* $x \in [a,b] \bigcap \mathbb{T}$. *Under the assumptions of Theorem 1 with* $x_0 = a,\ x_1 = x,\ x_2 = b,\ \alpha_0 = a,\ \alpha_1 \in [a,x] \bigcap \mathbb{T},\ \alpha_2 \in [x,b] \bigcap \mathbb{T}$, *and* $\alpha_3 = b$, *we have the following integral inequality on the time scale:*

$$\begin{aligned}
&\left| \int_a^b f^\sigma(t) \triangle t - \left[\left(1 - \frac{\lambda}{2}\right)(\alpha_1 - a)f(a) + (1 - \lambda)(\alpha_2 - \alpha_1)f(x) \right.\right. \\
&\left.\left. + \left(1 - \frac{\lambda}{2}\right)(b - \alpha_2)f(b) + \frac{\lambda}{2}\left((\alpha_2 - a)f(\alpha_1) + (b - a)f(\alpha_1)\right)\right]\right| \\
&\leqslant M\left[h_2\left(a, \alpha_1 - \lambda \frac{\alpha_1 - a}{2}\right) + h_2\left(\alpha_1, \alpha_1 - \lambda \frac{\alpha_1 - a}{2}\right) + h_2\left(\alpha_1, \alpha_1 + \lambda \frac{\alpha_2 - \alpha_1}{2}\right) \right. \\
&+ h_2\left(x, \alpha_1 + \lambda \frac{\alpha_2 - \alpha_1}{2}\right) + h_2\left(x, \alpha_2 - \lambda \frac{\alpha_2 - \alpha_1}{2}\right) + h_2\left(\alpha_2, \alpha_2 - \lambda \frac{\alpha_2 - \alpha_1}{2}\right) \\
&\left. + h_2\left(\alpha_2, \alpha_2 + \lambda \frac{b - \alpha_2}{2}\right) + h_2\left(b, \alpha_2 + \lambda \frac{b - \alpha_2}{2}\right)\right],
\end{aligned}$$

for all $\lambda \in [0,1]$ *such that* $\alpha_1 - \lambda \frac{\alpha_1 - a}{2},\ \alpha_1 + \lambda \frac{\alpha_2 - \alpha_1}{2},\ \alpha_2 - \lambda \frac{\alpha_2 - \alpha_1}{2}$, *and* $\alpha_2 + \lambda \frac{b - \alpha_2}{2}$ *are in* $\mathbb{T}$.

PROPOSITION 3. *Suppose* $x \in [\frac{5a+b}{6}, \frac{a+5b}{6}] \bigcap \mathbb{T}$. *Under the assumptions of Theorem 1 with* $x_0 = a,\ x_1 = x,\ x_2 = b,\ \alpha_0 = a,\ \alpha_1 = \frac{5a+b}{6},\ \alpha_2 = \frac{a+5b}{6},\ \alpha_3 = b$, *and* $\lambda = 0$, *we have the Simpson inequality on the time scale*

$$\begin{aligned}
&\left| \int_a^b f^\sigma(t) \triangle t - \frac{b-a}{6}\left[f(a) + 4f\left(\frac{b+a}{2}\right) + f(b)\right]\right| \\
&\leqslant M\left[h_2\left(a, \frac{5a+b}{6}\right) + h_2\left(\frac{a+b}{2}, \frac{5a+b}{6}\right) + h_2\left(\frac{a+b}{2}, \frac{a+5b}{6}\right) + h_2\left(b, \frac{a+5b}{6}\right)\right].
\end{aligned}$$

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