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PARAMETERIZED INEQUALITIES ABOUT A POINT IN THE PLANE OF A TRIANGLE

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Abstract. Four conjectures with a parameter about a point in the plane of a triangle are proved. A theorem which allows assumptions to be added to help proving and reducing the complexity is also presented. The proving procedure could be applied to the similar problems.

1. Introduction

Point and triangle are the elementary objects in geometry. There are plenty of results concerning these two things such as the Erdös-Mordell inequality [1]. [3] presented some inequalities and conjectures about a point and a triangle on the same plane. [2] presented proofs to two conjectures based on the successive difference substitution method [4]. [6] proved another conjecture with a parameter based on the complete discrimination system for polynomials [5] and put forward four conjectures with a parameter, two of these conjectures are the parameterized versions of those two conjectures proved in [2].

As shown in Fig. 1, P and triangle ABC are on a same plane. Let D, E, F denote the feet of the perpendiculars from P to BC, CA, AB (may be produced) respectively. And let a, b, c, r_1, r_2, r_3, R_1, R_2, R_3 denote the lengths of the line segments BC, CA, AB, PD, PE, PF, PA, PB, PC respectively.

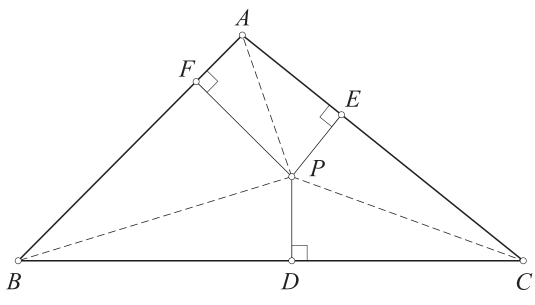

Figure 1: P *and* $\triangle ABC$

The four conjectures presented in [6] are rewritten as the following four propositions.

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PROPOSITION 1. Notations as above, when $\lambda \geqslant 0$, there exists

$$\frac{2R_1^2-r_2^2-r_3^2}{a^2+b^2+c^2+\lambda a^2}+\frac{2R_2^2-r_3^2-r_1^2}{a^2+b^2+c^2+\lambda b^2}+\frac{2R_3^2-r_1^2-r_2^2}{a^2+b^2+c^2+\lambda c^2}\geqslant\frac{3}{2(\lambda+3)}. \tag{1}$$

PROPOSITION 2. Notations as above, when $\lambda \geqslant 0$, there exists

$$\frac{R_2^2+R_3^2-r_2^2-r_3^2}{a^2+b^2+c^2+\lambda a^2}+\frac{R_3^2+R_1^2-r_3^2-r_1^2}{a^2+b^2+c^2+\lambda b^2}+\frac{R_1^2+R_2^2-r_1^2-r_2^2}{a^2+b^2+c^2+\lambda c^2}\geqslant\frac{3}{2(\lambda+3)}. \tag{2}$$

PROPOSITION 3. Notations as above, when $\lambda \geqslant 0$, there exists

$$\frac{2R_1^2-r_2^2-r_3^2}{(b+c)^2+\lambda bc}+\frac{2R_2^2-r_3^2-r_1^2}{(c+a)^2+\lambda ca}+\frac{2R_3^2-r_1^2-r_2^2}{(a+b)^2+\lambda ab}\geqslant\frac{3}{2(\lambda+4)}. \tag{3}$$

PROPOSITION 4. Notations as above, when $\lambda \geqslant 0$, there exists

$$\frac{R_2^2+R_3^2-r_2^2-r_3^2}{(b+c)^2+\lambda bc}+\frac{R_3^2+R_1^2-r_3^2-r_1^2}{(c+a)^2+\lambda ca}+\frac{R_1^2+R_2^2-r_1^2-r_2^2}{(a+b)^2+\lambda ab}\geqslant\frac{3}{2(\lambda+4)}. \tag{4}$$

Obviously, (3) and (4) are the parameterized versions of Conjecture 3 and 4 in [3] respectively which were proved in [2]. The rest parts are arranged as follows. In Section 2, some preliminary equations are recalled and a theorem to explain why some constrains could be made is presented. After that, the proofs to these four propositions are presented. The work is concluded in Section 3.

2. Main result

2.1. Preliminary

In this subsection, some equations are recalled. And a theorem which theoretically supports us to make some assumption to reduce the complexity is presented.

According to the triangle inequality, three positive variables, say u, v, w, are usually called to express a, b, c, the lengths of three sides of $\triangle ABC$ as follows,

$$a=u+v, \quad b=v+w, \quad c=u+w. \tag{5}$$

When the problems are homogeneous with respect to $\{a,b,c\}$, we could divide into six cases according to the size order of $\{a,b,c\}$. In each case, one of $\{a,b,c\}$ is the maximum and set to 1, and the other two are then less than 1.

Actually, in many problems, especially in geometric problems, some constrains could be assumed without loss of generality. However, these operations are usually based on experience. We present the following theorem to illustrate such things in an analytic way.

THEOREM 1. *Let m and n be two positive integers, $\Omega_1 \subset \mathscr{R}^m$ and $\Omega_2 \subset \mathscr{R}^n$. Let Φ denote a proposition defined in Ω_1 and Λ the set where Φ holds, i.e. $\Lambda \triangleq \{\boldsymbol{x} \in \Omega_1 \,|\Phi(\boldsymbol{x})$ is correct$\}$. For a function $f : \Omega_1 \times \Omega_2 \to \mathscr{R}$, if $\forall \boldsymbol{x}_0 \in \Omega_1$, $\forall \boldsymbol{y}_0 \in \Omega_2$, there exist $\boldsymbol{x}_1 \in \Lambda$, $\boldsymbol{y}_1 \in \Omega_2$, such that $f(\boldsymbol{x}_1,\boldsymbol{y}_1) = f(\boldsymbol{x}_0,\boldsymbol{y}_0)$, then the following three statements are equivalent,*

I. $\forall(\boldsymbol{x},\boldsymbol{y}) \in \Omega_1 \times \Omega_2$, $f(\boldsymbol{x},\boldsymbol{y}) \geqslant 0$,

II. $\forall(\boldsymbol{x},\boldsymbol{y}) \in \Lambda \times \Omega_2$, $f(\boldsymbol{x},\boldsymbol{y}) \geqslant 0$,

III. For every $\boldsymbol{x} \in \Omega_1$, if $\Phi(\boldsymbol{x})$ holds, then $f(\boldsymbol{x},\boldsymbol{y}) \geqslant 0$ holds for every $\boldsymbol{y} \in \Omega_2$.

Proof. The equivalency between II. and III. is obvious, since $\boldsymbol{x} \in \Lambda \Leftrightarrow \Phi(\boldsymbol{x})$ holds.

I. $\Rightarrow$ II. Since $\Lambda \subseteq \Omega_1$, for every $(\boldsymbol{x},\boldsymbol{y}) \in \Lambda \times \Omega_2$, it is belongs to $\Omega_1 \times \Omega_2$, therefore $f(\boldsymbol{x},\boldsymbol{y}) \geqslant 0$.

I. $\Leftarrow$ II. Because $\forall(\boldsymbol{x},\boldsymbol{y}) \in \Omega_1 \times \Omega_2$, there exists $(\boldsymbol{x}_1,\boldsymbol{y}_1) \in \Lambda \times \Omega_2$, such that $f(\boldsymbol{x},\boldsymbol{y}) = f(\boldsymbol{x}_1,\boldsymbol{y}_1)$. Based on the hypothesis, we have $f(\boldsymbol{x}_1,\boldsymbol{y}_1) \geqslant 0$. Therefore, $f(\boldsymbol{x},\boldsymbol{y}) \geqslant 0$.

From all above, the theorem has been proved. □

REMARK 1. To prove an inequality, say $f(x,y,z) \geqslant 0$ on $\mathscr{R}^3$. If f is symmetric and homogeneous with respect to $\{x,y,z\}$, based on our experience, we could add a constrain, $x \geqslant y \geqslant z$, to help proving the inequality. But if f is not symmetric with respect to $\{x,y,z\}$, our experience can not provide such constrain confidentially. The above theorem provides a theoretical support for adding helpful constrains. Furthermore, there could be some derivatives about this theorem. For example, the conclusion of f, to be non-negative, may be changed to be in an interval.

Let (x,y,z) be the normalized barycentric coordinates of P with respect to $\triangle ABC$. That is to say, x, y, $z \in \mathscr{R}$ and $x+y+z=1$. There are some other well-known equations, we just list them here without proof.

$$R_1 = \sqrt{b^2z^2 + c^2y^2 + yz(b^2+c^2-a^2)}, \tag{6}$$

$$R_2 = \sqrt{c^2x^2 + a^2z^2 + xz(a^2+c^2-b^2)}, \tag{7}$$

$$R_3 = \sqrt{a^2y^2 + b^2x^2 + xy(a^2+b^2-c^2)}, \tag{8}$$

$$S = \frac{\sqrt{(a+b+c)(b+c-a)(a+c-b)(a+b-c)}}{4}, \tag{9}$$

$$r_1 = \frac{2S|x|}{a}, \quad r_2 = \frac{2S|y|}{b}, \quad r_3 = \frac{2S|z|}{c}. \tag{10}$$

2.2. Main result

In this subsection, we prove the Proposition 1–4.

First, substitute (6)–(10) into the inequality (1) and obtain an equivalent[1],

$$F_1 = \frac{f_1(a,b,c,x,y,z)}{4a^2b^2c^2(\lambda+3)(\lambda a^2+\sigma)(\lambda b^2+\sigma)(\lambda c^2+\sigma)} \geqslant 0, \tag{11}$$

in which

$$\sigma = a^2+b^2+c^2, \tag{12}$$

$$\begin{aligned} f_1(a,b,c,x,y,z) = {} & (\lambda+3)\cdot\big(f_{11}(a,b,c)\cdot x^2 + f_{12}(c,a,b)\cdot xy + f_{11}(b,c,a)\cdot y^2 \\ & + f_{12}(a,b,c)\cdot yz + f_{11}(c,a,b)\cdot z^2 + f_{12}(b,c,a)\cdot xz\big) \\ & -6a^2b^2c^2\left(a^2\lambda+\sigma\right)\left(b^2\lambda+\sigma\right)\left(c^2\lambda+\sigma\right), \end{aligned} \tag{13}$$

$$\begin{aligned} f_{11}(a,b,c) = {} & b^2c^2(\lambda a^2+\sigma)\Big(\Big(a^4b^2+a^4c^2+6a^2b^4+6a^2c^4+b^6 \\ & +c^6-b^4c^2-b^2c^4-4a^2b^2c^2\Big)\lambda+2\sigma(\sigma+2bc)(\sigma-2bc)\Big), \end{aligned} \tag{14}$$

$$f_{12}(a,b,c) = 8a^2b^2c^2(\sigma-2a^2)(\sigma+\lambda b^2)(\sigma+\lambda c^2). \tag{15}$$

Let Ω denote the region of (a,b,c), the lengths of three sides of $\triangle ABC$. And Ψ denotes the region of (x,y,z), the normalized barycentric coordinates of P with respect to $\triangle ABC$. That is to say,

$$\Omega \triangleq \{(x_1,x_2,x_3)\in\mathscr{R}_+^3 \,|\, x_1+x_2>x_3, x_2+x_3>x_1, x_3+x_1>x_2\}, \tag{16}$$

$$\Psi \triangleq \{(x_1,x_2,x_3)\in\mathscr{R}^3 \,|\, x_1+x_2+x_3=1\}. \tag{17}$$

Obviously, for any $(x_1,x_2,x_3)\in\Omega$, all its permutations are in Ω. And $\forall(x_1,x_2,x_3)\in\Psi$, all its permutations are in Ψ, too. We could divide Ω into six subsets according to the size order, $a\geqslant b\geqslant c$, $b\geqslant c\geqslant a$ etc. And let Ω_1 denote $\{(a,b,c)\in\Omega\,|\,a\geqslant b\geqslant c\}$. Since

$$\begin{aligned} & f_1(a,b,c,x,y,z) = f_1(b,c,a,y,z,x) = f_1(c,a,b,z,x,y) \\ = {} & f_1(a,c,b,x,z,y) = f_1(c,b,a,z,y,x) = f_1(b,a,c,y,x,z), \end{aligned} \tag{18}$$

according to Theorem1, if $f_1(a,b,c,x,y,z)\geqslant 0$ holds on $\Omega_1\times\Psi$, then it will hold on $\Omega\times\Psi$. Therefore, we just need prove $f_1(a,b,c,x,y,z)\geqslant 0$ when $a\geqslant b\geqslant c$.

Additionally, because $f_1(a,b,c,x,y,z)$ is homogeneous with respect to $\{a,b,c\}$, based on (5), we could set

$$a=u+v=1, \quad b=u+w=1-v+w, \quad c=v+w. \tag{19}$$

Notice that u, v, w are all positive real numbers. Since $a\geqslant b\geqslant c$, we also have

$$1-v=u\geqslant v\geqslant w>0, \;\; v\in\left(0,\frac{1}{2}\right], \;\; w\in(0,v]. \tag{20}$$

[1]All the calculations are implemented in the Maple 2017.

Furthermore, we could use other two non-negative variables s, t to replace v, w by

$$v=\frac{1}{2+s},\quad w=\frac{1}{2+s+t}. \tag{21}$$

Substitute (19), (21) and $z=1-x-y$ sequentially into $f_1(a,b,c,x,y,z)$, then we obtain an equivalent of $f_1(a,b,c,x,y,z)\geqslant 0$ as follows,

$$G_1=\frac{g_{12}\cdot x^2+g_{11}\cdot x+g_{10}}{(2+s)^{10}(2+s+t)^{10}}\geqslant 0, \tag{22}$$

in which g_{11} is a polynomial of y, s, t, λ with 1210 terms, g_{10} is a polynomial of y, λ, s, t with 1804 terms and

$$g_{12}=8\left(s^2+st+4s+t+4\right)^2(\lambda+3)\widetilde{g_{12}}. \tag{23}$$

Here $\widetilde{g_{12}}$ is a polynomial of λ, s, t with 297 positive-coefficient terms including a positive constant. Because λ, s, t are all non-negative, g_{12} is positive. Since $g_{12}x^2+g_{11}x+g_{10}$ is a quadratic polynomial with respect to x, in order to obtain its positive semidefinition, we only need prove $\delta_{1x}\triangleq 4g_{12}g_{10}-g_{11}^2\geqslant 0$. Calculation shows that

$$\delta_{1x}=64\left(s^2+st+4s+t+4\right)^2(\lambda+3)(4+2s+t)^2\cdot\left(h_{12}y^2+h_{11}y+h_{10}\right), \tag{24}$$

in which h_{11} is a polynomial of λ, s, t with 2118 terms, h_{10} is also a polynomial of λ, s, t with 2309 terms and

$$h_{12}=16\,(2+s)^2(s+1)^2(s+t+3)^2(2+s+t)^2(\lambda+3)\cdot\widetilde{h_{12}}. \tag{25}$$

Here $\widetilde{h_{12}}$ is a polynomial of λ, s, t with 858 positive-coefficient terms including a positive constant. Since λ, s, t are all non-negative, h_{12} is positive. To prove the positive semidefinition of $h_{12}y^2+h_{11}y+h_{10}$, we need prove $\delta_{1y}\triangleq 4h_{12}h_{10}-h_{11}^2\geqslant 0$. Calculation shows that δ_{1y} is a polynomial of λ, s, t with 13424 positive-coefficient terms.

Therefore, when λ, s, t are all non-negative, δ_{1y} is non-negative and so are $h_{12}y^2+h_{11}y+h_{10}$, δ_{1x} and $g_{12}x^2+g_{11}x+g_{10}$. That is to say (22) holds and so does $f_1\geqslant 0$ when $a\geqslant b\geqslant c$. According to Theorem1, $f_1\geqslant 0$ holds in $\Omega\times\Psi$. Consequentially, the inequality (1) is correct for any $\lambda\geqslant 0$. From all above, we just present a proof to the Proposition 1.

REMARK 2. Compared with the proofs in [2] and [6], the above proof is more straightforward. Although the assumption, $a\geqslant b\geqslant c$, may be made based on the experience manually, the theoretical support provided by Theorem 1 makes us more confidential and makes it possible for an automated reasoning algorithm to do the same thing.

For the other three propostions, the proving procedure is quite similar. We just summarize some key data below. Notice that the first subscript denotes which proposition it belongs to.

Substituting (6)–(10) into the inequalities (2), (3) and (4), the equivalent inequalities respectively are

$$F_2 = \frac{f_2(a,b,c,x,y,z)}{4a^2b^2c^2(\lambda+3)(\lambda a^2+\sigma)(\lambda b^2+\sigma)(\lambda c^2+\sigma)} \geqslant 0, \tag{26}$$

$$F_3 = \frac{f_3(a,b,c,x,y,z)}{4a^2b^2c^2(\lambda+4)(\lambda ab+(a+b)^2)(\lambda ac+(a+c)^2)(\lambda bc+(b+c)^2)} \geqslant 0, \tag{27}$$

$$F_4 = \frac{f_4(a,b,c,x,y,z)}{4a^2b^2c^2(\lambda+4)(\lambda ab+(a+b)^2)(\lambda ac+(a+c)^2)(\lambda bc+(b+c)^2)} \geqslant 0, \tag{28}$$

where

$$\begin{aligned}
f_2(a,b,c,x,y,z) =& (\lambda+3)\left(f_{21}(a,b,c)x^2+f_{22}(c,a,b)xy+f_{21}(b,c,a)y^2\right.\\
&\left.+f_{22}(a,b,c)yz+f_{21}(c,a,b)z^2+f_{22}(b,c,a)xz\right)\\
&-6a^2b^2c^2(a^2\lambda+\sigma)(b^2\lambda+\sigma)(c^2\lambda+\sigma),\\
f_3(a,b,c,x,y,z) =& (\lambda+4)\left(f_{31}(a,b,c)x^2+f_{32}(c,a,b)xy+f_{31}(b,c,a)y^2\right.\\
&\left.+f_{32}(a,b,c)yz+f_{31}(c,a,b)z^2+f_{32}(b,c,a)xz\right)\\
&-6a^2b^2c^2(bc\lambda+(b+c)^2)(ac\lambda+(a+c)^2)(ab\lambda+(a+b)^2),\\
f_4(a,b,c,x,y,z) =& (\lambda+4)\left(f_{41}(a,b,c)x^2+f_{42}(c,a,b)xy+f_{41}(b,c,a)y^2\right.\\
&\left.+f_{42}(a,b,c)yz+f_{41}(c,a,b)z^2+f_{42}(b,c,a)xz\right)\\
&-6a^2b^2c^2(bc\lambda+(b+c)^2)(ac\lambda+(a+c)^2)(ab\lambda+(a+b)^2),\\
f_{21}(a,b,c) =& b^2c^2\left(a^2\left(a^4b^2+a^4c^2-2a^2b^4+4a^2b^2c^2-2a^2c^4+b^6+3b^4c^2\right.\right.\\
&\left.+3b^2c^4+c^6\right)\lambda^2+\sigma\left(2a^6+a^4b^2+a^4c^2+4a^2b^4+8a^2b^2c^2\right.\\
&\left.\left.+4a^2c^4+b^6-b^4c^2-b^2c^4+c^6\right)\lambda+2(\sigma-2bc)(\sigma+2bc)\sigma^2\right),\\
f_{31}(a,b,c) =& b^2c^2\left(bc\lambda+(b+c)^2\right)\left(a(b+c)(a^4-2a^2b^2+8a^2bc-2a^2c^2+b^4\right.\\
&-2b^2c^2+c^4)\lambda+2a^6+2a^5b+2a^5c+5a^4b^2+5a^4c^2-4a^3b^3\\
&+12a^3b^2c+12a^3bc^2-4a^3c^3+8a^2b^2c^2+2ab^5+2ab^4c-4ab^3c^2\\
&\left.-4ab^2c^3+2abc^4+2ac^5+b^6-b^4c^2-b^2c^4+c^6\right),\\
f_{41}(a,b,c) =& b^2c^2\Big(abc\left((b+c)a^4+4(b^2+c^2)a^3+2(b+c)(b-c)^2a^2\right.\\
&\left.+(b-c)^2(b+c)^3\right)\lambda^2+(2a^6bc+(b+c)(5b^2+4cb+5c^2)a^5\\
&+17bc(b^2+c^2)a^4+2(b+c)(b^4+4b^3c-6b^2c^2+4bc^3+c^4)a^3\\
&+(b-c)^2(b+c)^2(4bca^2+ab^3+5ab^2c+5abc^2+ac^3+b^3c\\
&+bc^3))\lambda+(6b^2+4cb+6c^2)a^6+2(b+c)(5b^2+2cb+5c^2)a^5\\
&+(b^2+c^2)(5b^2+18cb+5c^2)a^4+4(b+c)(b^4+2b^3c-2b^2c^2
\end{aligned}$$

$$+2bc^3+c^4)a^3+(4b^6+8b^5c-16b^3c^3+8bc^5+4c^6)a^2$$
$$+2(b-c)^2(b+c)^5a+(b^2+c^2)(b-c)^2(b+c)^4\Big),$$
$$f_{22}(a,b,c)=4a^2b^2c^2(\sigma-2a^2)(a^2\lambda+\sigma)(b^2\lambda+c^2\lambda+2\sigma),$$
$$f_{32}(a,b,c)=8a^2b^2c^2(\sigma-2a^2)\left((c+a)^2+\lambda ca\right)\left((a+b)^2+\lambda ab\right),$$
$$f_{42}(a,b,c)=4a^2b^2c^2(\sigma-2a^2)((b+c)^2+cb\lambda)\left(a\lambda(b+c)+(a+c)^2+(a+b)^2\right).$$

To add the assumption $a\geqslant b\geqslant c$, we need verify some equations like (18). Calculation shows that for $i=2,3,4$, there exist

$$f_i(a,b,c,x,y,z)=f_i(b,c,a,y,z,x)=f_i(c,a,b,z,x,y)$$
$$=f_i(a,c,b,x,z,y)=f_i(c,b,a,z,y,x)=f_i(b,a,c,y,x,z). \tag{29}$$

Then substitute (19), (21) and $z=1-x-y$ sequentially into $f_i(a,b,c,x,y,z)$, $i=2,3,4$, the equivalents of $f_i(a,b,c,x,y,z)\geqslant 0$ are as follows,

$$G_i=\frac{g_{i2}\cdot x^2+g_{i1}\cdot x+g_{i0}}{(2+s)^{10}(2+s+t)^{10}}\geqslant 0. \tag{30}$$

Here g_{i1} and g_{i0} are all polynomials of λ, y, s, t and g_{i2} are polynomials of λ, s, t with positive-coefficient terms including positive constants. Therefore, g_{i2} are all positive. Let δix denote $4g_{i2}g_{i0}-g_{i1}^2$ for $i=2,3,4$. Then we have

$$\delta_{2x}=64(4+2s+t)^2(s^2+st+4s+t+4)^2(\lambda+3)\cdot(h_{22}y^2+h_{21}y+h_{20}), \tag{31}$$
$$\delta_{3x}=64(4+2s+t)^2(s^2+st+4s+t+4)^2(\lambda+4)\cdot(h_{32}y^2+h_{31}y+h_{30}), \tag{32}$$
$$\delta_{4x}=64(4+2s+t)^2(s^2+st+4s+t+4)^2(\lambda+4)\cdot(h_{42}y^2+h_{41}y+h_{40}). \tag{33}$$

For $i=2,3,4$, $h_{i2}y^2+h_{i1}y+h_{i0}$ could be treated as quadratic polynomials with respect to y and h_{i2}, h_{i1}, h_{i0} are polynomials of λ, s, t. Since h_{i2} are polynomials with positive-coefficient terms including positive constant terms, they are all positive. Therefore, if we prove $\delta_{iy}=4h_{i2}h_{i0}-h_{i1}^2\geqslant 0$, we could obtain the positive semidefinition of δ_{ix}. Calculation shows that δ_{iy} $(i=2,3,4)$ are polynomials of λ, s, t with positive coefficients. They are all non-negative. Therefore, δ_{2x}, δ_{3x}, δ_{4x} are all non-negative and so are G_2, G_3, G_4. Consequentially, F_2, F_3, F_4 are all non-negative when $a\geqslant b\geqslant c$. According to Theorem 1 and (29), $F_i\geqslant 0$, $i=2,3,4$ hold for every $(a,b,c,x,y,z)\in\Omega\times\Psi$. From all above, we finish the proofs to Proposition 2–4.

3. Conclusion

We prove four parameterized conjectures presented in [6]. As we know, there are kinds of methods to transform a geometric problem to an algebraic one. The proving procedure in Section 2 shows that an appropriate method could make things much easier. Meanwhile, Theorem 1 provides theoretic support to make useful assumption which reduces the range of variables. Actually, a similar procedure has been applied to some others problems.

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REFERENCES

[1] O. BOTTEMA, *Geometric inequalities*, Groningen: Wolters-Noordhoff, The Netherlands, 1969.
[2] FANGJIAN HUANG, *Proofs to two inequality conjectures for a point on the plane of a triangle*, Journal of Inequalities and Applications **2016**, 1 (2016), 76–87.
[3] JIAN LIU, *A new inequality for a point in the plane of a triangle*, Journal of Mathematical Inequalities **8**, 3 (2014), 597–611.
[4] LU YANG, *Solving harder problems with lesser mathematics*, Proc. ATCM'2005 (Sung-Chi Chu and Wei-Chi Yang and Hee-Chan Lew, eds.), ATCM Inc., Blacksburg, 2005, pp. 37–46.
[5] LU YANG, XIAORONG HOU AND ZHENBING ZENG, *A complete discrimination system for polynomials*, Sci. China **E 39**, 6 (1996), 628–646.
[6] YONG YANG, SHEN LI CHEN, DONG HUANG, XIANG WANG, AND XIAOGUANG LIN, *Proof of an inequality conjecture for a point in the plane of a triangle*, Journal of Mathematical Inequalities **11**, 2 (2017), 399–411.

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