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ASYMPTOTIC BOUNDS FOR PRECISE LARGE DEVIATIONS IN A COMPOUND RISK MODEL UNDER DEPENDENCE STRUCTURES

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Abstract. In the paper, we consider a compound risk model, where all the claim sizes satisfy a dependence structure, and the accident inter-arrival time and the claim-number of the subsequent accident satisfy another dependence structure described by a conditional tail probability of the inter-arrival time given the subsequent claim-number. We obtain the asymptotic lower and upper bounds for the precise large deviations of the aggregate claims, with a feature that the asymptotic bounds hold uniformly for all x in an infinite t-interval.

1. Introduction

1.1. Risk model

Consider the compound renewal risk model, proposed by Tang et al. (2001), in which all the modelling components satisfy the following assumptions.

ASSUMPTION H_1. The inter-arrival times of the accidents, $\{\theta_i, i \geqslant 1\}$ are positive and independent, identically distributed (i.i.d.) random variables (r.v.s) with finite mean λ^{-1}, which generate a renewal counting process $N(t) = \sup\{n \geqslant 1, \tau_n = \sum_{i=1}^{n} \theta_i \leqslant t\}$ with mean function $\lambda(t) = EN(t)$ such that $\lambda(t)/\lambda t \to 1$ as $t \to \infty$.

ASSUMPTION H_2. The claim numbers caused by the successive accidents, $\{Z_i, i \geqslant 1\}$ are a sequence of i.i.d. and nonnegative integer-valued r.v.s with finite mean ν.

ASSUMPTION H_3. The claim sizes caused by the i-th accident, $\{X_{ij}, j \geqslant 1\}$, $i \geqslant 1$, are independent copies of $X_i, i \geqslant 1$, which are a sequence of nonnegative and i.i.d. r.v.s with common distribution F and finite mean μ.

ASSUMPTION H_4. The sequences $\{\theta_i, i \geqslant 1\}$, $\{Z_i, i \geqslant 1\}$ and $\{X_{ij}, i \geqslant 1, j \geqslant 1\}$ are mutually independent.

Then, the aggregate amount of claims accumulated up to time $t \geqslant 0$ is expressed as

$$S(t) = \sum_{i=1}^{N(t)} \sum_{j=1}^{Z_i} X_{ij}. \tag{1.1}$$

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1.2. Heavy-tailed distribution classes

In the subsection, we present some classes of heavy-tailed distributions. For two positive functions $a(\cdot)$ and $b(\cdot)$, we write $a(x) \sim b(x)$ if $\lim_{x\to\infty} a(x)/b(x) = 1$, write $a(x) = o(1)b(x)$ if $\lim_{x\to\infty} a(x)/b(x) = 0$. For a proper distribution V, we denote its tail by $\overline{V}(x) = 1 - V(x)$ and its upper Matuszewska index by

$$J_V^+ = -\lim_{y\to\infty} \frac{\log \overline{V}_*(y)}{\log y} \quad \text{with} \quad \overline{V}_*(y) = \liminf_{x\to\infty} \frac{\overline{V}(xy)}{\overline{V}(x)}, \quad y > 1.$$

We say that a distribution V on $[0,\infty)$ belongs to the long-tailed class, denoted by $V \in \mathscr{L}$, if for any $y > 0$,

$$\overline{V}(x+y) \sim \overline{V}(x);$$

belongs to the dominated variation class, denoted by $V \in \mathscr{D}$, if for any $0 < y < 1$,

$$\overline{V}^*(y) < \infty,$$

where $\overline{V}^*(y) = \limsup_{x\to\infty} \overline{V}(xy)/\overline{V}(x)$; belongs to the consistent variation class, denoted by $V \in \mathscr{C}$, if

$$\lim_{y \searrow 1} \overline{V}_*(y) = 1, \quad \text{or equivalently,} \quad \lim_{y \nearrow 1} \overline{V}^*(y) = 1;$$

belongs to the ERV class of extended-regularly-varying-tailed distributions, if there exist some $0 < \alpha \leqslant \beta < \infty$ such that for any $y \geqslant 1$,

$$y^{-\beta} \leqslant \overline{V}_*(y) \leqslant \overline{V}^*(y) \leqslant y^{-\alpha},$$

where we denote $V \in ERV(-\alpha, -\beta)$, and the class ERV is the union of all $ERV(-\alpha,-\beta)$ over the range $0 < \alpha \leqslant \beta < \infty$.

More generally, we say that a distribution V on $(-\infty,\infty)$ belongs to a distribution class if $V(x)\mathbf{1}_{\{x\geqslant 0\}}$ belongs to the class, where $\mathbf{1}_A$ denotes the indicator function of a set A. It is well-known that

$$ERV \subset \mathscr{C} \subset \mathscr{L} \cap \mathscr{D} \subset \mathscr{L}.$$

For more details on heavy-tailed distributions and their applications, we refer to Bingham et al. (1987), Embrechts et al. (1997) and Denisov et al. (2004).

1.3. Dependence structures

In recent years, a study trend of risk theory is to introduce various dependence structures to risk models, among which the widely upper orthant dependence structure was proposed by Wang et al. (2013). Say that r.v.s $\{\xi_i, i \geqslant 1\}$ are widely upper orthant dependent (WUOD), if there exist a sequence of finite and positive numbers $\{g_U(n), n \geqslant 1\}$ such that for each $n \geqslant 1$ and for all $x_i \in (-\infty,\infty)$, $1 \leqslant i \leqslant n$,

$$P\left(\bigcap_{i=1}^{n} \left\{\xi_i > x_i\right\}\right) \leqslant g_U(n) \prod_{i=1}^{n} P(\xi_i > x_i).$$

Adopting the term of Liu et al. (2012), r.v.s $\{\xi_i, i \geqslant 1\}$ are said to be upper tail asymptotically independent (UTAI), if $P(\xi_i > x) > 0$ for all $x \in (-\infty, \infty)$, $i \geqslant 1$, and

$$\lim_{\min\{x_i, x_j\} \to \infty} P(\xi_i > x_i | \xi_j > x_j) = 0, \quad \text{for all } 1 \leqslant i \neq j < \infty.$$

Besides, He et al. (2013) initiated a dependence structure for r.v.s $\{\xi_i, i \geqslant 1\}$ as follows:

$$\lim_{n\to\infty} \sup_{x \geqslant \alpha n} \sup_{1 \leqslant i < j \leqslant n} xP(\xi_i > x | \xi_j > x) = 0, \quad \text{for any } \alpha > 0. \tag{1.2}$$

Clearly, the UTAI r.v.s can properly cover the WUOD r.v.s, see Example 3.1 of Liu et al. (2012), and hence can cover both negatively dependent and positively dependent r.v.s. Also, the dependence structure defined by (1.2) is a special case of UTAI structure. In fact, He et al. (2013) gave an assertion that if $\{X_i, i \geqslant 1\}$ are WUOD, then relation (1.2) follows from

$$\lim_{x\to\infty} x\overline{F}(x) = 0, \tag{1.3}$$

which is slightly weaker than that the corresponding r.v. with distribution F has finite mean. Hence, the dependence structure that satisfies (1.2) at least properly covers the WUOD r.v.s with finite means. He et al. (2013) presented two examples which show that there exist UTAI r.v.s satisfying (1.2), but are not WUOD; and Liu et al. (2017) gave a concrete example to illustrate that there do exist WUOD r.v.s that satisfy (1.2). For some recent literatures on widely upper orthant dependence structure, we refer to Wang et al. (2014), Qiu and Chen (2014), Wang and Hu (2015), and Xi et al. (2018).

For the case when $Z_i \equiv 1, i \geqslant 1$, namely that the risk model considered in the paper is the standard renewal risk model, Li et al. (2010) introduced a time-dependence structure, which is described by the conditional tail probability of a claim size given the inter-arrival time prior to the claim; Chen and Yuen (2012) introduced a more general dependence structure between the claim size and its corresponding inter-arrival time, called the size-dependence structure and described by the conditional tail probability of the inter-arrival time given the subsequent claim size being large. And Chen and Yuen (2012) investigated the precise large deviations of the aggregate claims in the size-dependence renewal risk model. Inspired by the time-dependence and size-dependence structures, in this paper we adopt a dependence structure between the accident inter-arrival time and the claim-number of the subsequent accident. Precisely, we denote by θ and Z the generic r.v.s of $\{\theta_i, i \geqslant 1\}$ and $\{Z_i, i \geqslant 1\}$, respectively, and assume that θ and Z satisfy that for all $t \in [0, \infty)$ and for any $k \geqslant 1$,

$$P(\theta > t | Z = k) \leqslant P(\theta^* > t), \tag{1.4}$$

where θ^* is a nonnegative r.v. independent of the other sources of randomness.

Note that relation (1.4) means that θ conditional on $(Z = k)$ is stochastically bounded by θ^* for any $k \geqslant 1$, which defines a general dependence structure via the conditional tail probability of the accident inter-arrival time given the claim-number caused by the subsequent accident being fixed, and allows a wide range of dependence structures, also see Wang and Chen (2019). For example, we assume that the nonnegative random pair (θ,Z) has joint tail distribution with the form

$$P(\theta>t,Z>k)=\frac{1}{\max\{1,t\}\max\{1,k\}(1+t+k)}$$

for all $t\in[0,\infty)$ and $k\in\mathbb{N}$. Obviously, the marginal tail distributions of θ and Z are, respectively,

$$P(\theta>t)=\frac{1}{\max\{1,t\}(1+t)},\quad \text{for all } t\in[0,\infty),$$

and

$$P(Z>k)=\frac{1}{\max\{1,k\}(1+k)},\quad \text{for all } k\in\mathbb{N}.$$

Also, if $k=1$,

$$P(Z=1)=1-P(Z>1)=\frac{1}{2},$$

and if $k\geqslant 2$,

$$P(Z=k)=P(Z>k-1)-P(Z>k)=\frac{2}{k(k-1)(1+k)}.$$

Likewise, if $k=1$,

$$P(\theta>t,Z=1)=P(\theta>t)-P(\theta>t,Z>1)=\frac{1}{\max\{1,t\}(1+t)(2+t)},$$

and if $k\geqslant 2$,

$$\begin{aligned}P(\theta>t,Z=k)&=P(\theta>t,Z>k-1)-P(\theta>t,Z>k)\\&=\frac{2k+t}{\max\{1,t\}k(k-1)(t+k)(1+t+k)}.\end{aligned}$$

Now we can verify that random pair (θ,Z) satisfies relation (1.4). In fact, for $k=1$,

$$P(\theta>t|Z=1)=\frac{P(\theta>t,Z=1)}{P(Z=1)}=\frac{2}{\max\{1,t\}(t+1)(2+t)}\leqslant\frac{1}{\max\{1,t\}},$$

and for $k\geqslant 2$,

$$P(\theta>t|Z=k)=\frac{P(\theta>t,Z=k)}{P(Z=k)}=\frac{(2k+t)(k+1)}{2\max\{1,t\}(k+t)(1+k+t)}\leqslant\frac{1}{\max\{1,t\}}.$$

Now take θ^* such that $P(\theta^*>t)=\max\{1,t\}^{-1}$, then relation (1.4) holds.

Let θ_1^* be a positive r.v., independent of all sources of randomness, with the same distribution as θ_1 conditional on $(Z_1=k)$ for any $k\geqslant 1$. Write $\tau_1^*=\theta_1^*,\tau_n^*=\theta_1^*+\sum_{i=2}^n\theta_i$, $n\geqslant 2$, which constitute a delayed renewal counting process

$$N^*(t)=\sup\{n\geqslant 1,\tau_n^*\leqslant t\},\quad t\geqslant 0. \tag{1.5}$$

Note also that the distribution of θ_1^* depends on k through the condition $(Z_1 = k)$, so does the distribution of $\{N^*(t), t \geqslant 0\}$, that is

$$P(N^*(t) = n) = P(N(t) = n | Z_1 = k) \tag{1.6}$$

for all $n \in \mathbb{N}$. Under the dependence structure described by (1.4), it follows that for any $t \geqslant 0$ and all large $x > 0$,

$$\begin{aligned} P\left(\theta_i > t, \sum_{j=1}^{Z_i} X_{ij} > x\right) &= \sum_{k=1}^{\infty} P(\theta_i > t | Z_i = k) P\left(\sum_{j=1}^{k} X_{ij} > x\right) P(Z_i = k) \\ &\leqslant P\left(\sum_{j=1}^{Z_i} X_{ij} > x\right) P(\theta^* > t), \quad i \geqslant 1. \end{aligned}$$

Hence,

$$P\left(\theta_i > t \,\middle|\, \sum_{j=1}^{Z_i} X_{ij} > x\right) \leqslant P(\theta^* > t), \quad i \geqslant 1,$$

which shows that for every $i \geqslant 1$, the accidents' inter-arrival time, θ_i, and the aggregate amount of claims caused by the subsequent accident, $\sum_{j=1}^{Z_i} X_{ij}$, satisfy the size-dependence structure proposed by Chen and Yuen (2012). This result, in insurance practice, seems quite natural because of the fact that the waiting time for a severe accident with large claims and/or their large numbers is dependent on the aggregate amount of claims caused by the corresponding accident. From these arguments above, the dependence structure described by (1.4) has important theoretical and practical significance.

In the present paper, we consider a generalized compound renewal risk model with Assumptions H_1, H_2 and the following assumptions.

ASSUMPTION H_3^*. All the claim sizes $\{X_{ij}, i \geqslant 1, j \geqslant 1\}$ are nonnegative r.v.s with common distribution F and satisfying the dependence structure defined by (1.2).

ASSUMPTION H_3^{**}. All the claim sizes $\{X_{ij}, i \geqslant 1, j \geqslant 1\}$ are nonnegative and WUOD r.v.s with common distribution F, and satisfying $EX_1^{\beta} < \infty$ for some $\beta > 1$ and $\sup_{n \geqslant 1} g_U(n) n^{-\varepsilon_0} < \infty$ for some constant $\varepsilon_0 > 0$.

ASSUMPTION H_4^*. The sequence $\{X_{ij}, i \geqslant 1, j \geqslant 1\}$ are independent of $\{\theta_i, i \geqslant 1\}$ and $\{Z_i, i \geqslant 1\}$, but for every $i \geqslant 1$, θ_i and Z_i satisfy the dependence structure defined by relation (1.4).

1.4. Motivation and main results

As is known that the precise large deviations for heavy-tailed random (or non-random) sums and their applications in insurance and finance have been extensively studied in many literatures. See, for example, Cline and Hsing (1991), Klüppelberg and Mikosch (1997), Mikosch and Nagaev (1998), Ng et al. (2004), Tang (2006), Wang and Wang (2007), Liu (2009), and others. For the recent works, we refer the readers to Chen et al. (2011), Konstantinides and Loukissas (2011), Wang and Cheng (2011), Chen and Yuen (2012), Loukissas (2012), Wang et al. (2012), Wang et al. (2013), Yang and Wang (2013), Lu et al. (2014), Jiang et al. (2015), Xiao et al. (2018), Liu et al. (2017, 2020), Wang and Chen (2019), and references therein.

For the compound renewal risk model, Tang et al. (2001) considered the precise large deviations of the aggregate claims under Assumptions H_1-H_4 and $F \in ERV$; Konstantinides and Loukissas (2010) extended the results of Tang et al. (2001) to the case when $F \in \mathscr{C}$. But in practice, the independence structure in Assumptions H_1-H_4 is unrealistic and then limits the usefulness of the obtained results to some extent. Kass and Tang (2005) considered that $\{X_{ij}, i \geqslant 1, j \geqslant 1\}$ are a sequence of i.i.d. r.v.s and $\{Z_i, i \geqslant 1\}$ follow a certain negative dependence structure; Yang et al. (2012) assumed that $\{X_{ij}, i \geqslant 1, j \geqslant 1\}$ are a sequence of i.i.d. r.v.s, but $\{\theta_i, i \geqslant 1\}$ and $\{Z_i, i \geqslant 1\}$ are two sequences of dependent r.v.s, and discussed the precise large deviations if $F \in \mathscr{C}$ and $EZ^p < \infty$ for $p > J_F^+ + 2$. All the references above only considered the case when $\{X_{ij}, i \geqslant 1, j \geqslant 1\}$ are a sequence of i.i.d. r.v.s, and the sequences $\{\theta_i, i \geqslant 1\}$, $\{Z_i, i \geqslant 1\}$ and $\{X_{ij}, i \geqslant 1, j \geqslant 1\}$ are mutually independent.

Motivated by the above references, in the paper we further consider the precise large deviations for the aggregate claims of a generalized compound renewal risk model, in which all the claim sizes satisfy some more general dependence structures, and more importantly, the accidents' inter-arrival times and the claim-numbers of the subsequent accidents satisfy a dependence structure described by relation (1.4).

The main results of this paper are given below, among which the first theorem is concerned with the asymptotic lower bound of the precise large deviations of the aggregate claims in a generalized compound renewal risk model with dependence structures.

THEOREM 1.1. *Consider the aggregate claims* (1.1) *satisfying Assumptions* H_1, H_2, H_3^*, H_4^*. *If* $F \in \mathscr{L} \cap \mathscr{D}$ *and* $EZ_1^{p+1} < \infty$ *for some* $p > J_F^+$, *then for any* $\gamma > 0$,

$$P(S(t) > x) \gtrsim \lambda \nu t \overline{F}(x) \tag{1.7}$$

holds uniformly for all $x \geqslant \gamma t$, *which is equivalent to*

$$\liminf_{t \to \infty} \inf_{x \geqslant \gamma t} \frac{P(S(t) > x)}{\lambda \nu t \overline{F}(x)} \geqslant 1.$$

REMARK 1.1. Let c be any fixed number, then (1.7) is equivalent to the following asymptotic formula, for any $\gamma > \gamma_0 = -c$,

$$P(S(t) > x + c\lambda \nu t) \gtrsim \lambda \nu t \overline{F}(x + c\lambda \nu t), \text{ as } t \to \infty,$$

holds uniformly for all $x \geqslant \gamma t$.

Secondly, we obtain the asymptotic upper bound of the precise large deviations of the aggregate claims with some conditions stronger than those for the asymptotic lower bound.

THEOREM 1.2. *Consider the aggregate claims* (1.1) *satisfying Assumptions* H_1, H_2, H_3^{**}, H_4^*. *If* $F \in \mathscr{C}$ *and* $EZ_1^{p+1} < \infty$ *for some* $p > \max\{1, J_F^+\}$, *then for any* $\gamma > 0$,

$$P\left(\sum_{i=1}^{N(t)} \sum_{j=1}^{Z_i} (X_{ij} - \mu) > x\right) \lesssim \lambda \nu t \overline{F}(x) \tag{1.8}$$

holds uniformly for all $x \geqslant \gamma t$, *which is equivalent to*

$$\limsup_{t\to\infty} \sup_{x\geqslant \gamma t} \frac{P\left(\sum_{i=1}^{N(t)} \sum_{j=1}^{Z_i} (X_{ij} - \mu) > x\right)}{\lambda \nu t \overline{F}(x)} \leqslant 1.$$

The rest part of this paper is organized as follows: we present some lemmas in Section 2 and prove the main results in Section 3.

2. Some lemmas

Before proving the main results, we now present some lemmas, among which the first one is due to Proposition 2.2.1 of Bingham et al. (1987) and Lemma 3.5 of Tang and Tsitsiashvili (2003).

LEMMA 2.1. *For a distribution* V *supported on* $(-\infty, \infty)$, *the following assertions hold:*
(i) $V \in \mathscr{D} \Leftrightarrow J_V^+ < \infty$;
(ii) if $V \in \mathscr{D}$, *then for any* $p > J_V^+$, $x^{-p} = o(1)\overline{V}(x)$ *as* $x \to \infty$;
(iii) if $V \in \mathscr{D}$, *then for any* $p > J_V^+$, *there exist* $C_1 > 0$ *and* $D > 0$, *such that*

$$\frac{\overline{V}(y)}{\overline{V}(x)} \leqslant C_1 \left(\frac{x}{y}\right)^p, \quad x \geqslant y \geqslant D.$$

In the second lemma, we give some basic properties of WUOD r.v.s (see Proposition 1.1 of Wang et al. (2013)).

LEMMA 2.2. *(i) If* $\{\xi_i, i \geqslant 1\}$ *are WUOD r.v.s and* $\{f_i(\cdot), i \geqslant 1\}$ *are nondecreasing functions, then* $\{f_i(\xi_i), i \geqslant 1\}$ *are still WUOD.*
(ii) If $\{\xi_i, i \geqslant 1\}$ *are WUOD, then for each* $i \geqslant 1$ *and any* $s > 0$,

$$E\exp\left\{s\sum_{i=1}^n \xi_i\right\} \leqslant g_U(n) \prod_{i=1}^n E\exp\{s\xi_i\}.$$

The lemma below establishes the law of large numbers for the delayed renewal counting process $\{N^*(t), t \geqslant 0\}$ defined by (1.5).

LEMMA 2.3. *In addition to (1.4), assume that* $E\theta = \lambda^{-1} > 0$, *then it holds for every* $0 < \delta < 1$ *and every function* $\gamma(\cdot) : [0, \infty) \to (0, \infty)$ *with* $\gamma(t) \uparrow \infty$ *as* $t \to \infty$ *that*

$$\lim_{t\to\infty} \sup_{x \geqslant \gamma(t)} P\left(\left|\frac{N^*(t)}{\lambda t} - 1\right| > \delta\right) = 0.$$

Proof. Follow the proof of Lemma 2.1 of Chen and Yuen (2012) with slight modifications. □

From Theorem 3.1 of Geluk and Tang (2009), we get a lemma as follows.

LEMMA 2.4. *Let* $\{\xi_i, 1 \leqslant i \leqslant n\}$ *be* n *nonnegative and UTAI r.v.s with distributions* $V_i \in \mathscr{L} \cap \mathscr{D}, 1 \leqslant i \leqslant n$, *respectively, then for any* $n \geqslant 1$,

$$P\left(\sum_{i=1}^{n} \xi_i > x\right) \sim \sum_{i=1}^{n} \overline{V}_i(x).$$

In the following lemma, we establish a general inequality for the tail probability of sum of r.v.s with no requirement on any dependence structures, which means that neither independence, nor a special dependence structure, is required among these r.v.s. Also, the inequality is sharp in view of the fact that its bound depends on n, and is useful, particularly when dealing with the tail probability of random sum of heavy-tailed r.v.s.

LEMMA 2.5. *Let* $\{\xi_i, i \geqslant 1\}$ *be a sequence of real-valued r.v.s with common distribution* $V \in \mathscr{D}$. *Then for any* $p > J_V^+$, *there exists some constant* $C_2 > 0$ *such that for all* $x \geqslant 0$ *and* $n \geqslant 1$,

$$P\left(\sum_{i=1}^{n} \xi_i > x\right) \leqslant C_2 n^{p+1} \overline{V}(x). \tag{2.1}$$

Proof. Arbitrarily choose a sufficiently small number $x_0 > 0$. For all $n \geqslant 1$ and $0 \leqslant x \leqslant x_0$,

$$P\left(\sum_{i=1}^{n} \xi_i > x\right) \leqslant \frac{1}{\overline{V}(x_0)} n^{p+1} \overline{V}(x), \tag{2.2}$$

and for all $n \geqslant 1$ and $x > x_0$,

$$P\left(\sum_{i=1}^{n} \xi_i > x\right) \leqslant \sum_{i=1}^{n} P\left(\xi_i > \frac{x}{n}\right) = nP\left(\xi_1 > \frac{x}{n}\right). \tag{2.3}$$

By Lemma 2.1(ii), for x_0 as above, there exists some large positive constant C_3 such that for all $x > x_0$,

$$\frac{x^{-p}}{\overline{V}(x)} \leqslant C_3. \tag{2.4}$$

By $V \in \mathscr{D}$ and Lemma 2.1(iii), for any $p > J_V^+$, there exists some large positive constants C_1 and D such that, for all $x > nD > x_0$,

$$P\left(\xi_1 > \frac{x}{n}\right) \leqslant C_1 n^p \overline{V}(x). \tag{2.5}$$

By (2.4), (2.5) and $V \in \mathscr{D}$, we have that for all $x > x_0$ and $n \geqslant 1$,

$$\begin{aligned} P\left(\xi_1 > \frac{x}{n}\right) &\leqslant \mathbf{1}_{\{x_0 < x \leqslant nD\}} + P\left(\xi_1 > \frac{x}{n}\right)\mathbf{1}_{\{x > nD\}} \\ &\leqslant \frac{(nD)^p}{x^p}\mathbf{1}_{\{x_0 < x \leqslant nD\}} + C_1 n^p \overline{V}(x) \\ &\leqslant D^p \frac{x^{-p}}{\overline{V}(x)}\mathbf{1}_{\{x_0 < x \leqslant nD\}} n^p \overline{V}(x) + C_1 n^p \overline{V}(x) \\ &\leqslant C_4 n^p \overline{V}(x), \end{aligned} \tag{2.6}$$

where $C_4 = D^p C_3 + C_1$. Then by (2.3) and (2.6), it follows that for all $x > x_0$ and $n \geqslant 1$,

$$P\left(\sum_{i=1}^{n} \xi_i > x\right) \leqslant C_4 n^{p+1} \overline{V}(x),$$

which, along with (2.2) and $C_2 = C_4 + 1/\overline{V}(x_0)$, shows that relation (2.1) holds for all $x \geqslant 0$ and $n \geqslant 1$. □

The next lemma implies the closure property of r.v.s satisfying (1.2) under a certain increasing transformation.

LEMMA 2.6. *Consider the compound renewal risk model satisfying Assumptions H_2 and H_3^* with $F \in \mathscr{D}$ and $EZ_1^{p+1} < \infty$ for some $p > J_F^+$. If the sequences $\{Z_i, i \geqslant 1\}$ and $\{X_{ij}, i \geqslant 1, j \geqslant 1\}$ are mutually independent, and $f(\cdot)$ is a continuous and strictly increasing function such that $f(x) \uparrow \infty$ and $x = O(1) f^{-1}(x)$, then $\{Y_i = f(\sum_{j=1}^{Z_i} X_{ij}), i \geqslant 1\}$ also satisfy relation (1.2).*

Proof. For any $1 \leqslant i < j \leqslant n$, $n \geqslant 2$, we have that

$$\begin{aligned} &P(Y_i > x, Y_j > x) \\ &\leqslant \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \sum_{k=1}^{k_1} \sum_{k=1}^{k_2} P\left(X_{ik} > \frac{f^{-1}(x)}{k_1}, X_{jk} > \frac{f^{-1}(x)}{k_2}\right) P(Z_i = k_1) P(Z_j = k_2) \\ &\leqslant \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \sum_{k=1}^{k_1} \sum_{k=1}^{k_2} P\left(X_{ik} > \frac{f^{-1}(x)}{k_1 k_2}, X_{jk} > \frac{f^{-1}(x)}{k_1 k_2}\right) P(Z_i = k_1) P(Z_j = k_2). \end{aligned} \tag{2.7}$$

By $F \in \mathscr{D}$ and (2.6), for any $p > J_F^+$, there exists a positive number C_4 such that for all $x > x_0 > 0$,

$$P(Y_j > x) \geqslant \overline{F}(f^{-1}(x)) \geqslant \frac{1}{C_4 k_1^p k_2^p} \cdot \overline{F}\left(\frac{f^{-1}(x)}{k_1 k_2}\right), \qquad \text{for any } j \geqslant 1. \tag{2.8}$$

Thus by (2.7), (2.8) and $x = O(1)f^{-1}(x)$, for any $p > J_F^+$, we have

$$\lim_{n\to\infty}\sup_{x\geqslant \alpha n}\sup_{1\leqslant i<j\leqslant n} xP(Y_i > x|Y_j > x)$$

$$\leqslant C_4 \lim_{n\to\infty}\sup_{x\geqslant \alpha n}\sup_{1\leqslant i<j\leqslant n}\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty}\sum_{k=1}^{k_1}\sum_{k=1}^{k_2}\frac{xk_1^p k_2^p P\left(X_{ik} > \frac{f^{-1}(x)}{k_1k_2}, X_{jk} > \frac{f^{-1}(x)}{k_1k_2}\right)}{P\left(X_{jk} > \frac{f^{-1}(x)}{k_1k_2}\right)}$$

$$P(Z_i = k_1)P(Z_j = k_2)$$

$$= C_4 \lim_{n\to\infty}\sup_{x\geqslant \alpha n}\sup_{1\leqslant i<j\leqslant n}\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty}\sum_{k=1}^{k_1}\sum_{k=1}^{k_2}\frac{xk_1^{p+1}k_2^{p+1}}{f^{-1}(x)}\frac{f^{-1}(x)}{k_1k_2}$$

$$\times P\left(X_{ik} > \frac{f^{-1}(x)}{k_1k_2}\middle| X_{jk} > \frac{f^{-1}(x)}{k_1k_2}\right) P(Z_i = k_1)P(Z_j = k_2)$$

$$= 0,$$

where the last step is due to the dominated convergence theorem and (1.2). Then, we give the proof of Lemma 2.6. □

For the compound renewal risk model introduced in Section 1, we denote its aggregate claims caused by the first n accidents by

$$S(n) = \sum_{i=1}^{n}\sum_{j=1}^{Z_i} X_{ij}. \tag{2.9}$$

We now deal with the asymptotic upper bound for the precise large deviations of (2.9), which will play an important role to prove Theorem 1.2 and is also of its own value.

LEMMA 2.7. *Consider the aggregate claims* (2.9) *caused by the first n accidents satisfying Assumptions H_2, H_3^{**} and H_4^*. If $F \in \mathscr{C}$ and $EZ^2 < \infty$, then for any $\gamma > 0$,*

$$P\left(\sum_{i=1}^{n}\sum_{j=1}^{Z_i}(X_{ij} - \mu) > x\right) \lesssim n\nu\overline{F}(x) \tag{2.10}$$

holds uniformly for all $x \geqslant \gamma n$, which is equivalent to

$$\limsup_{n\to\infty}\sup_{x\geqslant\gamma n}\frac{P\left(\sum_{i=1}^{n}\sum_{j=1}^{Z_i}(X_{ij} - \mu) > x\right)}{n\nu\overline{F}(x)} \leqslant 1.$$

Proof. Note that, for any $0 < \delta < 1$,

$$P\left(\sum_{i=1}^{n}\sum_{j=1}^{Z_i}(X_{ij} - \mu) > x\right) = \sum_{k_1=1}^{\infty}\cdots\sum_{k_n=1}^{\infty} P\left(\sum_{i=1}^{n}\sum_{j=1}^{k_i}(X_{ij} - \mu) > x\right)\prod_{i=1}^{n} P(Z_i = k_i)$$

$$= \sum_{k_1=1}^{\infty}\cdots\sum_{k_n=1}^{\infty}\left\{P\left(\sum_{i=1}^{n}\sum_{j=1}^{k_i}(X_{ij} - \mu) > x, \bigcup_{1\leqslant i\leqslant n, 1\leqslant j<k\leqslant k_i}\{X_{ij} > \delta x, X_{ik} > \delta x\}\right)\right.$$

$$
\begin{aligned}
&+P\left(\sum_{i=1}^{n}\sum_{j=1}^{k_i}(X_{ij}-\mu)>x,\bigcup_{1\leqslant i<l\leqslant n,1\leqslant j\leqslant k_i,1\leqslant r\leqslant k_l}\{X_{ij}>\delta x,X_{lr}>\delta x\}\right)\\
&+P\left(\sum_{i=1}^{n}\sum_{j=1}^{k_i}(X_{ij}-\mu)>x,\bigcap_{1\leqslant i\leqslant n,1\leqslant j\leqslant k_i}\{X_{ij}\leqslant \delta x\}\right)\\
&+P\Big(\sum_{i=1}^{n}\sum_{j=1}^{k_i}(X_{ij}-\mu)>x,\bigcup_{1\leqslant i\leqslant n,1\leqslant j\leqslant k_i}\{X_{ij}>\delta x,X_{ik}\leqslant \delta x,X_{lr}\leqslant \delta x,\\
&\qquad 1\leqslant k\neq j\leqslant k_i,1\leqslant l\neq i\leqslant n,1\leqslant r\leqslant k_l\}\Big)\Big\}\prod_{i=1}^{n}P(Z_i=k_i)\\
=&\sum_{i=1}^{4}I_i(x,n). \qquad (2.11)
\end{aligned}
$$

For $I_1(x,n)$, by $F\in\mathscr{C}\subset\mathscr{D}$ and the WUOD property, we have

$$
\begin{aligned}
&\limsup_{n\to\infty}\sup_{x\geqslant\gamma n}\frac{I_1(x,n)}{n\nu\overline{F}(x)}\\
\leqslant&\limsup_{n\to\infty}\sup_{x\geqslant\gamma n}\sum_{k_1=1}^{\infty}\cdots\sum_{k_n=1}^{\infty}\sum_{1\leqslant i\leqslant n,1\leqslant j<k\leqslant k_i}\frac{P(X_{ij}>\delta x,X_{ik}>\delta x)}{n\nu\overline{F}(x)}\prod_{i=1}^{n}P(Z_i=k_i)\\
\leqslant&\frac{g_U(2)}{\nu}\limsup_{n\to\infty}\sup_{x\geqslant\gamma n}\sum_{k_1=1}^{\infty}\cdots\sum_{k_n=1}^{\infty}\sum_{1\leqslant i\leqslant n,1\leqslant j<k\leqslant k_i}\frac{P(X_{ij}>\delta x)P(X_{ik}>\delta x)}{n\overline{F}(x)}\prod_{i=1}^{n}P(Z_i=k_i)\\
\leqslant&\frac{g_U(2)}{\nu}\limsup_{x\to\infty}\frac{EZ^2(\overline{F}(\delta x))^2}{\overline{F}(x)}\\
\leqslant&\frac{g_U(2)EZ^2}{\nu}\limsup_{x\to\infty}\frac{\overline{F}(\delta x)}{\overline{F}(x)}\limsup_{x\to\infty}\overline{F}(\delta x)\\
=&\,0. \qquad (2.12)
\end{aligned}
$$

For $I_2(x,n)$, similarly to (2.12), by $F\in\mathscr{C}\subset\mathscr{D}$, WUOD property and (1.3), we have

$$
\limsup_{n\to\infty}\sup_{x\geqslant\gamma n}\frac{I_2(x,n)}{n\nu\overline{F}(x)}\leqslant\frac{g_U(2)\nu}{\gamma}\limsup_{x\to\infty}\frac{\overline{F}(\delta x)}{\overline{F}(x)}\limsup_{x\to\infty}x\overline{F}(\delta x)=0. \qquad (2.13)
$$

For $I_3(x,n)$, set $\widetilde{X}_i=\min\{X_i,\delta x\}$, $i\geqslant 1$, then by Lemma 2.2 (i), $\{\widetilde{X}_i-E\widetilde{X}_i,i\geqslant 1\}$ are still WUOD. Then by Lemma 2.1 (ii), we prove that

$$
\begin{aligned}
&\limsup_{n\to\infty}\sup_{x\geqslant\gamma n}\frac{I_3(x,n)}{n\nu\overline{F}(x)}\\
=&\limsup_{n\to\infty}\sup_{x\geqslant\gamma n}\sum_{k_1=1}^{\infty}\cdots\sum_{k_n=1}^{\infty}\frac{P\Big(\sum_{i=1}^{n}\sum_{j=1}^{k_i}(\widetilde{X}_{ij}-\mu)>x\Big)}{n\nu\overline{F}(x)}\prod_{i=1}^{n}P(Z_i=k_i)
\end{aligned}
$$

$$
\begin{aligned}
&\leqslant \limsup_{n\to\infty}\sup_{x\geqslant\gamma n}\sum_{k_1=1}^{\infty}\cdots\sum_{k_n=1}^{\infty}\frac{P\Big(\sum_{i=1}^{n}\sum_{j=1}^{k_i}(\widetilde{X}_{ij}-E\widetilde{X}_{ij})>x\Big)}{n\nu\overline{F}(x)}\prod_{i=1}^{n}P(Z_i=k_i)\\
&\leqslant \frac{C_5}{\nu}\lim_{x\to\infty}\frac{x^{-p}}{\overline{F}(x)}\\
&=0,
\end{aligned}
\tag{2.14}
$$

where in the second last step we used the following inequality, for some constant $C_5 > 0$,

$$
P\left(\sum_{i=1}^{n}\sum_{j=1}^{k_i}(\widetilde{X}_{ij}-E\widetilde{X}_{ij})>x\right)\leqslant C_5x^{-p}
$$

resulting from the proof of Lemma 2.3 of Tang (2006), lemma 2.2 (ii) and the WUOD property. For $I_4(x,n)$, we show that, for some $p > J_F^+$

$$
I_4(x,n)\leqslant\sum_{k_1=1}^{\infty}\cdots\sum_{k_n=1}^{\infty}\sum_{i=1}^{n}\sum_{j=1}^{k_i}P\Big(X_{ij}>\delta x\Big)\prod_{i=1}^{n}P(Z_i=k_i)\leqslant n\nu\overline{F}(\delta x). \tag{2.15}
$$

Hence, by (2.15) and $F\in\mathscr{C}$, we have that

$$
\lim_{\delta\uparrow 1}\limsup_{n\to\infty}\sup_{x\geqslant\gamma n}\frac{I_4(x,n)}{n\nu\overline{F}(x)}\leqslant 1. \tag{2.16}
$$

Consequently, substituting (2.12)–(2.14) and (2.16) into (2.11), we get relation (2.10). □

The last lemma is a restatement of Theorem 1(i) of Kočetova et al. (2009).

LEMMA 2.8. *Let the accidents' inter-arrival times $\{\theta_i, i\geqslant 1\}$ be a sequence of positive and i.i.d. r.v.s with common mean λ^{-1}, then it holds for every $a>\lambda$ and some $b>1$ that*

$$
\lim_{t\to\infty}\sum_{n>at}b^nP(\tau_n\leqslant t)=0.
$$

3. Proofs of main results

In this section, we give the proofs of our main results, where all limit relationships are taken as $t\to\infty$ unless stated otherwise. For simplicity, we denote by $\lfloor y\rfloor$ the integer part of a real number y.

Proof of Theorem 1.1. For any, but small, $0<\delta<1$, we have

$$
\begin{aligned}
P(S(t)>x)&\geqslant\sum_{(1-\delta)\lambda t\leqslant n\leqslant(1+\delta)\lambda t}P\left(\sum_{i=1}^{n}\sum_{j=1}^{Z_i}X_{ij}>x,N(t)=n\right)\\
&\geqslant\sum_{(1-\delta)\lambda t\leqslant n\leqslant(1+\delta)\lambda t}\sum_{i=1}^{n}P\left(\sum_{j=1}^{Z_i}X_{ij}>x,N(t)=n\right)
\end{aligned}
$$

$$- \sum_{(1-\delta)\lambda t \leqslant n \leqslant (1+\delta)\lambda t} \sum_{1 \leqslant l < m \leqslant n} P\left(\sum_{j=1}^{Z_l} X_{lj} > x, \sum_{j=1}^{Z_m} X_{mj} > x, N(t) = n\right)$$
$$= I_1(x,t) - I_2(x,t). \tag{3.1}$$

For $I_1(x,t)$, by (1.6), we have that

$$I_1(x,t) = \sum_{(1-\delta)\lambda t \leqslant n \leqslant (1+\delta)\lambda t} \sum_{i=1}^{n} \sum_{k=1}^{\infty} P(N(t) = n | Z_i = k) P\left(\sum_{j=1}^{k} X_{ij} > x\right) P(Z_i = k)$$
$$= \sum_{(1-\delta)\lambda t \leqslant n \leqslant (1+\delta)\lambda t} \sum_{i=1}^{n} P(N^*(t) = n) P\left(\sum_{j=1}^{Z_i} X_{ij} > x\right), \tag{3.2}$$

where $\{N^*(t), t \geqslant 0\}$ is defined by (1.5). By the dominated convergence theorem, and Lemmas 2.4 and 2.5, we obtain that for any $i \geqslant 1$,

$$P\left(\sum_{j=1}^{Z_i} X_{ij} > x\right) = \sum_{k=1}^{\infty} P\left(\sum_{j=1}^{k} X_{ij} > x\right) P(Z_i = k) \sim \nu \overline{F}(x). \tag{3.3}$$

By (3.2), (3.3) and Lemma 2.3, it holds uniformly for all $x \geqslant \gamma t$ that

$$I_1(x,t) \gtrsim (1-\delta)\lambda \nu t \overline{F}(x) P\left(\left|\frac{N^*(t)}{\lambda t} - 1\right| < \delta\right)$$
$$\sim (1-\delta)\lambda \nu t \overline{F}(x). \tag{3.4}$$

For $I_2(x,t)$, by Lemma 2.6, (3.3) and Assumption H_4^*, it holds uniformly for all $x \geqslant \gamma t$ that

$$I_2(x,t) \leqslant \sum_{1 \leqslant l < m \leqslant (1+\delta)\lambda t} \sum_{(1-\delta)\lambda t \leqslant n \leqslant (1+\delta)\lambda t} P\left(\sum_{j=1}^{Z_l} X_{lj} > x, \sum_{j=1}^{Z_m} X_{mj} > x, N(t) = n\right)$$
$$\leqslant \sum_{1 \leqslant l < m \leqslant (1+\delta)\lambda t} P\left(\sum_{j=1}^{Z_l} X_{lj} > x, \sum_{j=1}^{Z_m} X_{mj} > x\right)$$
$$= o(1)((1+\delta)\lambda t)^2 x^{-1} \nu \overline{F}(x) = o(1)\frac{\lambda t}{x} \lambda \nu t \overline{F}(x) = o(1)\lambda \nu t \overline{F}(x). \tag{3.5}$$

So, we substitute (3.4) and (3.5) into (3.1) to obtain that, uniformly for all $x \geqslant \gamma t$,

$$P(S(t) > x) \gtrsim (1-\delta)\lambda \nu t \overline{F}(x),$$

which, along with the arbitrariness of $0 < \delta < 1$, leads to relation (1.7).

Proof of Theorem 1.2. For any, but small, $0 < \delta < 1$, we obtain that

$$P\left(\sum_{i=1}^{N(t)} \sum_{j=1}^{Z_i} (X_{ij} - \mu) > x\right) = P\left(\sum_{i=1}^{N(t)} \sum_{j=1}^{Z_i} (X_{ij} - \mu) > x, N(t) \leqslant (1+\delta)\lambda t\right)$$
$$+ P\left(\sum_{i=1}^{N(t)} \sum_{j=1}^{Z_i} (X_{ij} - \mu) > x, N(t) > (1+\delta)\lambda t\right)$$
$$= I_3(x,t) + I_4(x,t). \tag{3.6}$$

For $I_3(x,t)$, by Lemma 2.7, it holds uniformly for all $x \geqslant \gamma t$ that

$$\begin{aligned} I_3(x,t) &\leqslant P\left(\sum_{i=1}^{(1+\delta)\lambda t}\sum_{j=1}^{Z_i}(X_{ij}-\mu)>x\right) \\ &= P\left(\sum_{i=1}^{\lfloor(1+\delta)\lambda t\rfloor}\sum_{j=1}^{Z_i}(X_{ij}-\mu)>x\right) \\ &\lesssim \lfloor(1+\delta)\lambda t\rfloor \nu\overline{F}(x) \\ &\leqslant (1+\delta)\lambda\nu t\overline{F}(x). \end{aligned} \tag{3.7}$$

For $I_4(x,t)$, by (2.6) with $\xi_1=\sum_{j=1}^{Z_1}X_{1j}$, (3.3) and Lemma 2.8, we get that for any $p>J_F^+$,

$$\begin{aligned} I_4(x,t) &\leqslant \sum_{n>(1+\delta)\lambda t} P\left(\sum_{i=1}^{n}\sum_{j=1}^{Z_i}X_{ij}>x, \tau_n\leqslant t\right) \\ &\leqslant \sum_{n>(1+\delta)\lambda t}\sum_{i=1}^{n} P\left(\sum_{j=1}^{Z_i}X_{ij}>\frac{x}{n}, \tau_n\leqslant t\right) \\ &\leqslant \sum_{n>(1+\delta)\lambda t} nP\left(\sum_{j=1}^{Z_1}X_{1j}>\frac{x}{n}\right)P(\tau_{n-1}\leqslant t) \\ &\leqslant C_6P\left(\sum_{j=1}^{Z_1}X_{1j}>x\right)\sum_{n>(1+\delta)\lambda t} n^{p+1}P(\tau_{n-1}\leqslant t) \\ &\sim C_6\nu\overline{F}(x)\sum_{n>(1+\delta)\lambda t} n^{p+1}P(\tau_{n-1}\leqslant t) \\ &= o(1)\overline{F}(x) \\ &= o(1)t\overline{F}(x) \end{aligned} \tag{3.8}$$

where $C_6>0$ is a constant depending on the distribution of the random sum $\sum_{j=1}^{Z_1}X_{1j}$. Hence, substituting (3.7) and (3.8) into (3.6) yields that, uniformly for all $x\geqslant\gamma t$,

$$P\Big(\sum_{i=1}^{N(t)}\sum_{j=1}^{Z_i}(X_{ij}-\mu)>x\Big)\lesssim(1+\delta)\lambda\nu t\overline{F}(x). \tag{3.9}$$

By (3.9), along with the arbitrariness of $0<\delta<1$, we get relation (1.8).

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