Journal of
Mathematical
Inequalities
Volume 15, Number 2 (2021), 605–613 doi:10.7153/jmi-2021-15-44

ADDITIVE DOUBLE ρ–FUNCTIONAL INEQUALITIES IN β–HOMOGENEOUS F–SPACES

QI LIU*, SHAOMO ZHUANG AND YONGJIN LI*

(*Communicated by J. Pečarić*)

Abstract. In this paper, we introduce and solve the following additive double ρ-functional inequalities

$$\begin{aligned}\|f(x+y+z)+f(x-y)-f(z)-2f(x)\| \leqslant &\|\rho_1(f(x+y+z)-f(x)-f(y)-f(z))\| \\ &+\|\rho_2(f(x+y+z)-f(x+y)-f(z))\|\end{aligned} \tag{1}$$

where ρ_1,ρ_2 are fixed nonzero complex numbers with $|2\rho_1|^{\beta_2}+|\rho_2|^{\beta_2}<1$, and

$$\begin{aligned}\|f(x+y+z)-f(x)-f(y)-f(z)\| \leqslant &\|\rho_1(f(x+y+z)+f(x-y)-f(z)-2f(x))\| \\ &+\|\rho_2(f(x+y+z)-f(x+y)-f(z))\|\end{aligned} \tag{2}$$

where ρ_1,ρ_2 are fixed nonzero complex numbers with $|\rho_1|^{\beta_2}+|\rho_2|^{\beta_2}<1$.

By adopting the direct method, we have made an attempt to prove the Hyers-Ulam stability of the additive double ρ-functional inequalities in β-homogeneous F-spaces.

1. Introduction

Based on the consideration of the stability of the group homomorphism, the functional equations encounter stability problems that have been originated from the well-known equation of Ulam [18].

Given a group G and a metric group G' with metric $\rho(\cdot,\cdot)$. Given $\varepsilon>0$, does there exist a $\delta>0$ such that if $f:G\to G'$ satisfies $\rho(f(xy),f(x)f(y))<\delta$ for all $x,y\in G$, then a homomorphism $h:G\to G'$ exists with $\rho(f(x),h(x))<\varepsilon$ for all $x\in G$?

In the case of Banach space in equation studied by Ulam, the first affirmative partial answer was published by Hyers [6]. Firstly, for the additive mappings, the Hyers Theorem generalized form was solved by Aoki [1], and further, for linear mapping, it was generalized by Rassias [16] taking an unbounded Cauchy difference in consideration. Găvruta [5] replaced the unbounded Cauchy difference by a general control function to generalize the Rassias Theorem.

Mathematics subject classification (2020): 39B52, 39B62.
Keywords and phrases: Hyers-Ulam stability, additive double ρ-functional inequality, β-homogeneous F-spaces.

* Corresponding author.

Park [13, 14] defined additive ρ-functional inequalities and proved the Hyers-Ulam stability of the additive ρ-functional inequalities in Banach spaces and non-Archimedean normed spaces. A number of studies have been carried out for investigating the stability problems of various functional equations (see [3, 4, 7, 8, 9, 12]).

In [15], Park et al. investigated the following inequalities

$$\|f(x)+f(y)+f(z)\| \leqslant \left\|2f\left(\frac{x+y+z}{2}\right)\right\|,$$

$$\|f(x)+f(y)+f(z)\| \leqslant \|f(x+y+z)\|,$$

$$\|f(x)+f(y)+2f(z)\| \leqslant \left\|2f\left(\frac{x+y}{2}+z\right)\right\|$$

in Banach spaces. In addition to aforementioned literature, a recent study was published by Lu et al. [11], in their findings, they investigated 3-variable Jensen ρ-functional inequalities in complex Banach spaces that are associated with the following functional equations:

$$f(x+y+z)+f(x+y-z)-2f(x)-2f(y)=0,$$

$$f(x+y+z)-f(x-y-z)-2f(y)-2f(z)=0.$$

Although various studies have been successfully conducted in the context of stability problems, however, the non-linear structure of F-spaces (infinite-dimensional) has not yet received considerable attention from the earlier researchers. The nonlinear structure of F-spaces (infinite-dimensional) plays a vital role in functional analysis, consequently, it is essential to re-investigate such structures. As an illustration, the $L^p([0,1])$ for $0<p<1$ equipped with the metric $d(f,g)=\int|f(x)-g(x)|^p dx$ is an F-space instead of a Banach space. Besides these, for F-spaces, several results can be consulted in [2, 10] and the references therein.

DEFINITION 1.1. Consider X be a linear space. A non-negative valued function $\|\cdot\|$ achieves an F-norm if satisfies the following conditions:

(1) $\|x\|=0$ if and only if $x=0$;

(2) $\|\lambda x\|=\|x\|$ for all λ, $|\lambda|=1$;

(3) $\|x+y\| \leqslant \|x\|+\|y\|$ for all $x,y \in X$;

(4) $\|\lambda_n x\| \to 0$ provided $\lambda_n \to 0$;

(5) $\|\lambda x_n\| \to 0$ provided $x_n \to 0$;

(6) $\|\lambda_n x_n\| \to 0$ provided $\lambda_n \to 0, x_n \to 0$.

Then $(X,\|\cdot\|)$ is called an F^*-space. An F-space is a complete F^*-space.

An F-norm is called β-homogeneous ($\beta > 0$) if $\|tx\| = |t|^{\beta}\|x\|$ for all $x \in X$ and all $t \in \mathbb{C}$ (see [17, 19]).

Considering the current gaps, in this paper, we have made an attempt to investigate the additive double ρ-functional inequalities and successfully proved the Hyers-Ulam stability of the additive double ρ-functional inequalities in β-homogeneous F-spaces.

In order to achieve the proposed objectives of this work, we have organized the paper in the following sections: Section 2 deals with proving the Hyers-Ulam stability of the additive double ρ-functional inequality (1) in β-homogeneous F-spaces.

In Section 3, we prove the Hyers-Ulam stability of the additive double ρ-functional inequality (2) in β-homogeneous F-spaces.

During the entire course of this work, β_1, β_2 are considered as positive real numbers with $\beta_1 \leqslant 1$ and $\beta_2 \leqslant 1$. Furthermore, X is assumed as β_1-homogeneous F-space while Y is a β_2-homogeneous F-space.

2. Additive double ρ-functional inequality (1)

This section solely emphasis an assumption stating that the ρ_1, ρ_2 are considered as fixed complex numbers with $|2\rho_1|^{\beta_2} + |\rho_2|^{\beta_2} < 1$. We investigate the additive double ρ-functional inequality (1) in β-homogeneous F-spaces.

LEMMA 2.1. *A mapping $f : X \to Y$ satisfies*

$$\begin{aligned}\|f(x+y+z)+f(x-y)-f(z)-2f(x)\| \leqslant & \|\rho_1(f(x+y+z)-f(x)-f(y)-f(z))\| \\ & + \|\rho_2(f(x+y+z)-f(x+y)-f(z))\|\end{aligned} \tag{3}$$

for all $x, y, z \in X$ if and only if $f : X \to Y$ is additive.

Proof. Assume that $f : X \to Y$ satisfies (3). Letting $x = y = z = 0$ in (3), we get

$$(1 - |2\rho_1|^{\beta_2} - |\rho_2|^{\beta_2})\|f(0)\| \leqslant 0.$$

Then $f(0) = 0$.

Letting $y = 0$ in (3), we get

$$(1 - |\rho_1|^{\beta_2} - |\rho_2|^{\beta_2})\|f(x+z) - f(x) - f(z)\| \leqslant 0$$

and so

$$f(x+z) = f(x) + f(z)$$

for all $x, z \in X$. Proving that the f is additive and obviously the converse is true. □

The next theorem pertains to presenting the Hyers-Ulam stability of the additive double ρ-functional inequality (3) in β-homogeneous F-spaces.

THEOREM 2.1. *Let $r > \frac{\beta_2}{\beta_1}$ and θ be nonnegative real numbers, and let $f : X \to Y$ be a mapping such that*

$$\begin{aligned}\|f(x+y+z)+f(x-y)-f(z)-2f(x)\| \leqslant &\|\rho_1(f(x+y+z)-f(x)-f(y)-f(z))\| \\ &+\|\rho_2(f(x+y+z)-f(x+y)-f(z))\| \\ &+\theta(\|x\|^r+\|y\|^r+\|z\|^r)\end{aligned} \tag{4}$$

for all $x,y,z \in X$. Then there exists a unique additive mapping $A : X \to Y$ such that

$$\|f(x)-A(x)\| \leqslant \frac{2\theta}{(1-|\rho_1|^{\beta_2})(2^{\beta_1 r}-2^{\beta_2})}\|x\|^r \tag{5}$$

for all $x \in X$.

Proof. Letting $x = y = z = 0$ in (4), we get $f(0) = 0$. Letting $x = y$, $z = 0$ in (4), then we obtain

$$\|f(2x)-2f(x)\| \leqslant \frac{2\theta}{1-|\rho_1|^{\beta_2}}\|x\|^r \tag{6}$$

for all $x \in X$. So

$$\left\|f(x)-2f\left(\frac{x}{2}\right)\right\| \leqslant \frac{2\theta}{2^{\beta_1 r}(1-|\rho_1|^{\beta_2})}\|x\|^r$$

for all $x \in X$. Hence

$$\left\|2^l f\left(\frac{x}{2^l}\right)-2^m f\left(\frac{x}{2^m}\right)\right\| \leqslant \sum_{j=l}^{m-1}\frac{2\theta}{(1-|\rho_1|^{\beta_2})2^{\beta_1 r}}\frac{2^{\beta_2 j}}{2^{\beta_1 rj}}\|x\|^r \tag{7}$$

for all nonnegative integers m and l with $m > l$ and all $x \in X$. It follows from (7) that the sequence $\{2^n f(\frac{x}{2^n})\}$ is a Cauchy sequence for all $x \in X$. Since Y is complete, the sequence $\{2^n f(\frac{x}{2^n})\}$ converges. Hence, the mapping $A : X \to Y$ can be defined as:

$$A(x) := \lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right)$$

for all $x \in X$. Furthermore, by considering $l = 0$ and setting the limit $m \to \infty$ in (7), we get (5).

It follows from (4) that

$$\begin{aligned}&\|A(x+y+z)+A(x-y)-A(z)-2A(x)\| \\ &= \lim_{n\to\infty} 2^{\beta_2 n}\left\|f\left(\frac{x+y+z}{2^n}\right)+f\left(\frac{x-y}{2^n}\right)-f\left(\frac{z}{2^n}\right)-2f\left(\frac{x}{2^n}\right)\right\| \\ &\leqslant \lim_{n\to\infty} 2^{\beta_2 n}\left(\left\|\rho_1\left(f\left(\frac{x+y+z}{2^n}\right)-f\left(\frac{x}{2^n}\right)-f\left(\frac{y}{2^n}\right)-f\left(\frac{z}{2^n}\right)\right)\right\|\right. \\ &\quad \left.+\left\|\rho_2\left(f\left(\frac{x+y+z}{2^n}\right)-f\left(\frac{x+y}{2^n}\right)-f\left(\frac{z}{2^n}\right)\right)\right\|\right)+\lim_{n\to\infty}\frac{2^{\beta_2 n}\theta}{2^{\beta_1 nr}}(\|x\|^r+\|y\|^r+\|z\|^r) \\ &= \|\rho_1(A(x+y+z)-A(x)-A(y)-A(z))\|+\|\rho_2(A(x+y+z)-A(x+y)-A(z))\|\end{aligned}$$

for all $x,y,z \in X$. Hence

$$\|A(x+y+z)+A(x-y)-A(z)-2A(x)\| \leqslant \|\rho_1(A(x+y+z)-A(x)-A(y)-A(z))\| + \|\rho_2(A(x+y+z)-A(x+y)-A(z))\|$$

for all $x,y,z \in X$. By Lemma 2.1, the mapping $A: X \to Y$ is additive.

Now, we show the uniqueness of A. Assuming $T: X \to Y$ as another additive mapping satisfying (5) that yields:

$$\begin{aligned}\|A(x)-T(x)\| &= 2^{\beta_2 n}\left\|A\left(\frac{x}{2^n}\right)-T\left(\frac{x}{2^n}\right)\right\| \\ &\leqslant 2^{\beta_2 n}\left\|A\left(\frac{x}{2^n}\right)-f\left(\frac{x}{2^n}\right)\right\| + 2^{\beta_2 n}\left\|T\left(\frac{x}{2^n}\right)-f\left(\frac{x}{2^n}\right)\right\| \\ &\leqslant \frac{4\cdot 2^{\beta_2 n}\theta}{(1-|\rho_1|^{\beta_2})(2^{\beta_1 r}-2^{\beta_2})2^{\beta_1 nr}}\|x\|^r\end{aligned}$$

which tends to zero as $n \to \infty$ for all $x \in X$. So we can conclude that $A(x) = T(x)$ for all $x \in X$. This proves the uniqueness of A. Thus the mapping $A: X \to Y$ is a unique additive mapping satisfying (5). □

THEOREM 2.2. *Consider $r < \frac{\beta_2}{\beta_1}$ and θ be nonnegative real numbers, and let $f: X \to Y$ be a mapping satisfying (4). Then there exists a unique additive mapping $A: X \to Y$ such that*

$$\|f(x)-A(x)\| \leqslant \frac{2\theta}{(1-|\rho_1|^{\beta_2})(2^{\beta_2}-2^{\beta_1 r})}\|x\|^r \tag{8}$$

for all $x \in X$.

Proof. It follows from (6) that

$$\left\|f(x)-\frac{1}{2}f(2x)\right\| \leqslant \frac{2\theta}{2^{\beta_2}(1-|\rho_1|^{\beta_2})}\|x\|^r$$

for all $x \in X$. Hence

$$\left\|\frac{1}{2^l}f(2^l x)-\frac{1}{2^m}f(2^m x)\right\| \leqslant \sum_{j=l}^{m-1}\frac{2\theta}{(1-|\rho_1|^{\beta_2})2^{\beta_2}}\frac{2^{\beta_1 rj}}{2^{\beta_2 j}}\|x\|^r \tag{9}$$

for all nonnegative integers m and l with $m > l$ and all $x \in X$. It follows from (9) that the sequence $\{\frac{1}{2^n}f(2^n x)\}$ is a Cauchy sequence for all $x \in X$. Since Y is complete, the sequence $\{\frac{1}{2^n}f(2^n x)\}$ converges. Hence, the mapping $A: X \to Y$ can be defined as:

$$A(x) := \lim_{n\to\infty}\frac{1}{2^n}f(2^n x)$$

for all $x \in X$. Moreover, letting $l = 0$ and passing the limit $m \to \infty$ in (9), we get (8).

The rest of the proof is similar to the proof of Theorem 2.1. □

3. Additive double ρ-functional inequality (2)

This section aims at assuming that ρ_1, ρ_2 are fixed complex numbers with $|\rho_1|^{\beta_2} + |\rho_2|^{\beta_2} < 1$. Here we have made considerable efforts in investigating the additive double ρ-functional inequality (2) in β-homogeneous F-spaces.

LEMMA 3.1. *A mapping $f : X \to Y$ satisfies*

$$\begin{aligned}\|f(x+y+z)-f(x)-f(y)-f(z)\| \leqslant &\|\rho_1(f(x+y+z)+f(x-y)-f(z)-2f(x))\| \\ &+\|\rho_2(f(x+y+z)-f(x+y)-f(z))\|\end{aligned} \tag{10}$$

for all $x,y,z \in X$ if and only if $f : X \to Y$ is additive.

Proof. Assume that $f : X \to Y$ satisfies (10). Letting $x = y = z = 0$ in (10), we get

$$(2^{\beta_2} - |\rho_1|^{\beta_2} - |\rho_2|^{\beta_2})\|f(0)\| \leqslant 0.$$

Then $f(0) = 0$.

Letting $y = 0$ in (10), we get

$$(1 - |\rho_1|^{\beta_2} - |\rho_2|^{\beta_2})\|f(x+z) - f(x) - f(z)\| \leqslant 0.$$

and so

$$f(x+z) = f(x) + f(z)$$

for all $x,z \in X$. Proving that the f is additive and obviously the converse is true. □

In the next paras, we present the Hyers-Ulam stability of the additive double ρ-functional inequality (10) in β-homogeneous F-spaces.

THEOREM 3.1. *Let $r > \frac{\beta_2}{\beta_1}$ and θ be nonnegative real numbers, and let $f : X \to Y$ be a mapping such that*

$$\begin{aligned}\|f(x+y+z)-f(x)-f(y)-f(z)\| \leqslant &\|\rho_1(f(x+y+z)+f(x-y)-f(z)-2f(x))\| \\ &+\|\rho_2(f(x+y+z)-f(x+y)-f(z))\| \\ &+\theta(\|x\|^r+\|y\|^r+\|z\|^r)\end{aligned} \tag{11}$$

for all $x,y,z \in X$. Then there exists a unique additive mapping $A : X \to Y$ such that

$$\|f(x) - A(x)\| \leqslant \frac{2\theta}{(1-|\rho_1|^{\beta_2})(2^{\beta_1 r} - 2^{\beta_2})}\|x\|^r \tag{12}$$

for all $x \in X$.

Proof. Considering $x=y=z=0$ in (11), we get $f(0)=0$. Letting $x=y, z=0$ in (11), then we obtain

$$\|f(2x)-2f(x)\| \leqslant \frac{2\theta}{1-|\rho_1|^{\beta_2}}\|x\|^r \tag{13}$$

for all $x\in X$. So

$$\left\|f(x)-2f\left(\frac{x}{2}\right)\right\| \leqslant \frac{2\theta}{2^{\beta_1 r}(1-|\rho_1|^{\beta_2})}\|x\|^r$$

for all $x\in X$. Hence

$$\left\|2^l f\left(\frac{x}{2^l}\right)-2^m f\left(\frac{x}{2^m}\right)\right\| \leqslant \sum_{j=l}^{m-1}\frac{2\theta}{(1-|\rho_1|^{\beta_2})2^{\beta_1 r}}\frac{2^{\beta_2 j}}{2^{\beta_1 rj}}\|x\|^r \tag{14}$$

for all nonnegative integers m and l with $m>l$ and all $x\in X$. It follows from (14) that the sequence $\{2^n f(\frac{x}{2^n})\}$ is a Cauchy sequence for all $x\in X$. Since Y is complete, the sequence $\{2^n f(\frac{x}{2^n})\}$ converges. So one can define the mapping $A: X\to Y$ by

$$A(x):=\lim_{n\to\infty}2^n f\left(\frac{x}{2^n}\right)$$

for all $x\in X$. Moreover, letting $l=0$ and passing the limit $m\to\infty$ in (14), we get (12).

It follows from (11) that

$$\begin{aligned}
&\|A(x+y+z)-A(x)-A(y)-A(z)\| \\
&=\lim_{n\to\infty}2^{\beta_2 n}\left\|f\left(\frac{x+y+z}{2^n}\right)-f\left(\frac{x}{2^n}\right)-f\left(\frac{y}{2^n}\right)-f\left(\frac{z}{2^n}\right)\right\| \\
&\leqslant \lim_{n\to\infty}2^{\beta_2 n}\left(\left\|\rho_1\left(f\left(\frac{x+y+z}{2^n}\right)+f\left(\frac{x-y}{2^n}\right)-f\left(\frac{z}{2^n}\right)-2f\left(\frac{x}{2^n}\right)\right)\right\|\right. \\
&\quad\left.+\left\|\rho_2\left(f\left(\frac{x+y+z}{2^n}\right)-f\left(\frac{x+y}{2^n}\right)-f\left(\frac{z}{2^n}\right)\right)\right\|\right)+\lim_{n\to\infty}\frac{2^{\beta_2 n}\theta}{2^{\beta_1 nr}}(\|x\|^r+\|y\|^r+\|z\|^r) \\
&=\|\rho_1(A(x+y+z)+A(x-y)-A(z)-2A(x))\|+\|\rho_2(A(x+y+z)-A(x+y)-A(z))\|
\end{aligned}$$

for all $x,y,z\in X$. Hence

$$\begin{aligned}
\|A(x+y+z)-A(x)-A(y)-A(z)\| &\leqslant \|\rho_1(A(x+y+z)+A(x-y)-A(z)-2A(x))\| \\
&\quad+\|\rho_2(A(x+y+z)-A(x+y)-A(z))\|
\end{aligned}$$

for all $x,y,z\in X$. By Lemma 3.1, the mapping $A: X\to Y$ is additive.

Now, we show the uniqueness of A. Assuming $T: X\to Y$ as another additive mapping satisfying (12) that yields:

$$\begin{aligned}
\|A(x)-T(x)\| &= 2^{\beta_2 n}\left\|A\left(\frac{x}{2^n}\right)-T\left(\frac{x}{2^n}\right)\right\| \\
&\leqslant 2^{\beta_2 n}\left\|A\left(\frac{x}{2^n}\right)-f\left(\frac{x}{2^n}\right)\right\|+2^{\beta_2 n}\left\|T\left(\frac{x}{2^n}\right)-f\left(\frac{x}{2^n}\right)\right\| \\
&\leqslant \frac{4\cdot 2^{\beta_2 n}\theta}{(1-|\rho_1|^{\beta_2})(2^{\beta_1 r}-2^{\beta_2})2^{\beta_1 nr}}\|x\|^r
\end{aligned}$$

which tends to zero as $n \to \infty$ for all $x \in X$. So we can conclude that $A(x) = T(x)$ for all $x \in X$. This proves the uniqueness of A. Thus the mapping $A : X \to Y$ is a unique additive mapping satisfying (12). □

THEOREM 3.2. *Consider* $r < \frac{\beta_2}{\beta_1}$ *and* θ *be nonnegative real numbers, and let* $f : X \to Y$ *be a mapping satisfying (11). Then there exists a unique additive mapping* $A : X \to Y$ *such that*

$$\|f(x) - A(x)\| \leqslant \frac{2\theta}{(1-|\rho_1|^{\beta_2})(2^{\beta_2} - 2^{\beta_1 r})}\|x\|^r \tag{15}$$

for all $x \in X$.

Proof. It follows from (13) that

$$\left\|f(x) - \frac{1}{2}f(2x)\right\| \leqslant \frac{2\theta}{2^{\beta_2}(1-|\rho_1|^{\beta_2})}\|x\|^r$$

for all $x \in X$. Hence

$$\left\|\frac{1}{2^l}f(2^l x) - \frac{1}{2^m}f(2^m x)\right\| \leqslant \sum_{j=l}^{m-1} \frac{2\theta}{(1-|\rho_1|^{\beta_2})2^{\beta_2}} \frac{2^{\beta_1 r j}}{2^{\beta_2 j}}\|x\|^r \tag{16}$$

for all nonnegative integers m and l with $m > l$ and all $x \in X$. It follows from (16) that the sequence $\{\frac{1}{2^n}f(2^n x)\}$ is a Cauchy sequence for all $x \in X$. Since Y is complete, the sequence $\{\frac{1}{2^n}f(2^n x)\}$ converges. So we can define the mapping $A : X \to Y$ by

$$A(x) := \lim_{n\to\infty} \frac{1}{2^n}f(2^n x)$$

for all $x \in X$. Moreover, letting $l = 0$ and passing the limit $m \to \infty$ in (16), we get (15).

The rest of the proof is similar to the proof of Theorem 3.1. □

Acknowledgement. This work was supported by the National Natural Science Foundation of China (11971493).

REFERENCES

[1] T. AOKI, *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan. 2 (1950), 64–66.
[2] F. ALBIAC, *Nonlinear structure of some classical quasi-Banach spaces and F-spaces*, J. Math. Anal. Appl. 340 (2008), 1312–1325.
[3] Y. CHO, C. PARK AND R. SAADATI, *Functional inequalities in non-Archimedean Banach spaces*, Appl. Math. Lett. 23 (2010), 1238–1242.
[4] P. W. CHOLEWA, *Remarks on the stability of functional equations*, Aequationes Math. 27 (1984), 76–86.
[5] P. GĂVRUTA, *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. Appl. 184 (1994), 431–436.

[6] D. H. HYERS, *On the stability of the linear functional equation*, Proc. Natl. Acad. Sci. USA. 27 (1941), 222–224.
[7] S.-M. JUNG, *On the Hyers-Ulam-Rassias stability of a quadratic functional equation*, J. Math. Anal. Appl. 232 (1999), 384–393.
[8] K. JUN AND H. KIM, *The generalized Hyers-Ulam-Rassias stability of a cubic functional equation*, J. Math. Anal. Appl. 274 (2002), 867–878.
[9] G. KIM AND H. SHIN, *Approximately quadratic mappings in non-Archimedean fuzzy normed spaces*, Nonlinear Funct. Anal. Appl. 23 (2018), 369–380.
[10] N. J. KALTON, *Curves with zero derivative in F-spaces*, Glasg. Math. J. 22 (1981), 19–29.
[11] G. LU, Q. LIU, Y. JIN AND J. XIE, *3-Variable Jensen ρ-functional equations*, J. Nonlinear Sci. Appl. 9 (2016), 5995–6003.
[12] G. LU AND C. PARK, *Hyers-Ulam stability of additive set-valued functional equations*, Appl. Math. Lett. 24 (2011), 1312–1316.
[13] C. PARK, *Additive ρ-functional inequalities and equations*, J. Math. Inequal. 9 (2015), 17–26.
[14] C. PARK, *Additive ρ-functional inequalities in non-Archimedean normed spaces*, J. Math. Inequal. 9 (2015), 397–407.
[15] C. PARK, Y. CHO AND M. HAN, *Functional inequalities associated with Jordan-von-Neumann-type additive functional equations*, J. Inequal. Appl. 2007 (2007), Article ID 41820.
[16] TH. M. RASSIAS, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc. 72 (1978), 297–300.
[17] S. ROLEWICZ, *Metric Linear Spaces*, PWN-Polish Sci. Publ, Reidel and Dordrecht, 1984.
[18] S. M. ULAM, *A Collection of the Mathematical Problems*, Interscience Publ., New York, 1960.
[19] A. WILANSKY, *Modern Methods in Topological Vector Space*, McGraw-Hill International Book Co, New York, 1978.

(Received January 8, 2020)

Qi Liu
Department of Mathematics
Sun Yat-sen University
Guangzhou 510275, P. R. China
e-mail: liuq325@mail2.sysu.edu.cn

Shaomo Zhuang
Department of Mathematics
Sun Yat-sen University
Guangzhou 510275, P. R. China
e-mail: zhuangshm5@mail2.sysu.edu.cn

Yongjin Li
Department of Mathematics
Sun Yat-sen University
Guangzhou 510275, P. R. China
e-mail: stslyj@mail.sysu.edu.cn