Journal of Mathematical Inequalities
Volume 16, Number 1 (2022), 345–346
doi:10.7153/jmi-2022-16-24

A NOTE ON THE PAPER "SINGULAR INTEGRAL OPERATORS IN GENERALIZED MORREY SPACES ON CURVES IN THE COMPLEX PLANE", MEDITERR. J. MATH. (2017) 14: 203. DOI.ORG/10.1007/S00009–017–1004–9, BY E. BURTSEVA

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(*Communicated by M. Krnić*)

Abstract. We show that the results of the paper mentioned in the title are known or follow from the known results.

The paper "Singular integral operators in generalized Morrey spaces on curves in the complex plane" attracted our attention by its title because of the words "Morrey spaces" in that title. Since we are involved by the studies of Morrey spaces and operators in such spaces, we started to read that paper with expectation to find new results there. A big surprise is that this is not the case.

Two tasks are considered in that paper:

1) the boundedness of the singular operator S with Cauchy kernel in generalized Morrey spaces with radial weights of special type;

2) an application of this boundedness to solvability of singular integral equations $af + bSf = g$ in such spaces.

The statements given in the paper either are not new; they are merely examples of already known theory, or are derived immediately also from known facts by known methods. After we read the paper wholy, we noticed that the content of the paper is mainly a compilation of texts of earlier published studies in [5], [6], [7] and some others.

More precisely:

1) Since the weights are of special type, the weighted boundedness of singular operators is immediately reduced to that of non-weighted singular operator, as it was done in various known studies, see for instance [5] and [7], from where big portions of text are in fact copied. As regards the non-weighted boundedness it is known for generalized Morrey spaces even in the general setting of quasi-metric measure spaces, see [3], and also [1, 2]. Since the author admits only curves with the arc-cord condition,

Mathematics subject classification (2020): 46E15, 45E05.
Keywords and phrases: Singular operators, function spaces, integral equations, Morrey spaces, weights.

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the non-weighted case considered by the author is a particular case of what is known in a much more general setting.

2) It is well known that in the case of continuous coefficients a and b and closed curves, fredholmness in fact does not depend on the choice of space, see for instance [4], where it is shown that the picture of normal solvability is the same in all the spaces where the singular operator is bounded. This means that results on fredholmness in this paper is an example contained in the known theory.

Acknowledgements. The research of authors was partially supported by the RUDN University Strategic Academic Leadership Program.

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(Received February 14, 2021)

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