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MOMENT CONVERGENCE RATE OF ESTIMATORS IN PARTIALLY LINEAR MODELS UNDER AANA ERRORS

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Abstract. In this paper, we investigate the partially linear regression model based on asymptotically almost negatively associated (AANA) random variables. Under some weak conditions, some results of moment convergence and complete convergence are obtained for the parametric least squares estimator and nonparametric weighted estimator. Our results extend the corresponding ones for negatively associated (NA) errors to AANA errors. In addition, we discuss the selection of design points and weight functions. Last, some simulations are illustrated to show the performance of our results.

1. Introduction

Consider the following heteroscedastic partially linear regression model:

$$y_i = x_i\beta + g(t_i) + \sigma_i e_i, \qquad i \geqslant 1, \tag{1.1}$$

where $\sigma_i^2 = f(u_i)$, (x_i, t_i, u_i) are known and nonrandom design points, β is an unknown parameter to be estimated, $f(\cdot)$ and $g(\cdot)$ are unknown functions defined on a closed interval D of R, and $\{e_i, i \geqslant 1\}$ are random errors.

For the model (1.1) with $1 \leqslant i \leqslant n$, Gao et al. [3] and Baek and Liang [9] respectively defined the least squares (LS) and weighted least squares (WLS) estimator of β and corresponding estimators of $g(\cdot)$:

$$\hat{\beta}_n = \sum_{i=1}^{n} \tilde{x}_i \tilde{y}_i / S_n^2, \qquad \tilde{\beta}_n = \sum_{i=1}^{n} \gamma_i \tilde{x}_i \tilde{y}_i / T_n^2, \tag{1.2}$$

$$\hat{g}_n(t) = \sum_{i=1}^{n} W_{ni}(t)(y_i - x_i\hat{\beta}_n), \qquad \tilde{g}_n(t) = \sum_{i=1}^{n} W_{ni}(t)(y_i - x_i\tilde{\beta}_n), \tag{1.3}$$

where

$$S_n^2 = \sum_{i=1}^{n} \tilde{x}_i^2, \quad T_n^2 = \sum_{i=1}^{n} \gamma_i \tilde{x}_i^2, \tag{1.4}$$

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$\gamma_i = 1/f(u_i)$, $\tilde{x}_i = x_i - \sum_{j=1}^{n} W_{nj}(t_i)x_j$, $\tilde{y}_i = y_i - \sum_{j=1}^{n} W_{nj}(t_i)y_j$ and

$$W_{ni}(t) = W_{ni}(t;t_1,t_2,\ldots,t_n)$$

is a measurable weight function on the closed interval D.

Partially linear model was introduced by Engle et al. [2] to analyse the relationship between temperature and electricity usage. Since then, many statisticians pay attention to studying the asymptotic properties of the estimators in the model (1.1).

Under the case of independent random errors, Hu [6] and Chen et al. [5] established the strong consistency and mean consistency of the estimators; Gao et al. [3] established the asymptotic normality for the estimators of β; and so on. Under the case of dependent random errors, the authors in [7, 10, 26] established the mean consistency and complete consistency of the estimators based on L^q mixingale, linear time series and φ-mixing errors, respectively; Liang and Jing [12] studied the asymptotic normality of estimators with martingale difference errors and linear process errors; Baek and Liang [9] investigated the strong consistency and asymptotic normality of the estimators under negatively associated (NA) errors; Zhou and Hu [14] derived the moment consistency of the estimators with NA errors; Zhang et al. [27] investigated the strong consistency of the estimators under asymptotically almost negatively associated (AANA) errors; and so forth.

Inspired by the literatures above, we devote to investigating the mean consistency and complete consistency of the estimators in model (1.1) based on asymptotically almost negatively associated random variables. Our results in the paper will extend and improve the corresponding ones for independent and identically distributed random errors and some dependent random errors.

Let us begin by recalling the concept of AANA dependence introduced by Chandra and Ghosal [4].

DEFINITION 1.1. A sequence $\{X_n, n \geqslant 1\}$ of random variables is called AANA if there exists a nonnegative sequence $q(n) \to 0$ as $n \to \infty$ such that

$$\mathrm{Cov}(f(X_n), g(X_{n+1},\ldots,X_{n+k})) \leqslant q(n)[\mathrm{Var}(f(X_n))\,\mathrm{Var}(g(X_{n+1},\ldots,X_{n+k}))]^{1/2},$$

for all n, $k \geqslant 1$ and for all coordinatewise nondecreasing continuous functions f and g whenever the variances exist. $\{q(n), n \geqslant 1\}$ are called the mixing coefficients of $\{X_n, n \geqslant 1\}$.

The family of AANA sequence contains NA and independent sequences as special cases. An example of an AANA sequence which is not NA was constructed by Chandra and Ghosal [4]. Many applications of AANA sequences have been found. For the details, one can refer to [4, 16, 18, 24] for the strong laws of large numbers; Ko et al. [8] for the Hájeck-Rènyi type inequalities; Yuan and An [13] for some Rosenthal type inequalities; Wang et al. [17] and Xi et al. [25] for the complete convergence; An [23] for the complete moment convergence; and so on.

The remainder of this paper is organized as follows. Some assumptions and main results are given in Section 2. Simulation is presented in Section 3. We give conclusions in Section 4. We provide preliminary lemmas and proofs of the main results in Section 5. Throughout this paper, let C, C_1, C_2, ..., C_p be positive constants whose values may vary at different places.

2. Main results

2.1. Assumptions

To obtain main results, the following assumptions are needed.

A1. Let $\{e_i, i \geqslant 1\}$ in model (1.1) be an AANA sequence with $Ee_i = 0$ and $Ee_i^2 = 1$, $i \geqslant 1$. Suppose that $\sup_{i \geqslant 1} E|e_i|^p < \infty$ for some $p > 2$ and there exists a positive integer k such that $p \in (2^k, 2^{k+1}]$. In addition, the mixing coefficients $\{q(i), i \geqslant 1\}$ satisfy $\sum_{n=1}^{\infty} q^{\tilde{p}}(n) < \infty$, where $\tilde{p} = (1/2^{k-1} - 2/p)p/(p-1)$.

A2. (i) $0 < C_1 \leqslant \liminf_{n\to\infty} \frac{S_n^2}{n} \leqslant \limsup_{n\to\infty} \frac{S_n^2}{n} \leqslant C_2 < \infty$, where S_n^2 is defined in (1.4).

(ii) $0 < m_0 \leqslant \min_{1\leqslant i\leqslant n} f(u_i) \leqslant \max_{1\leqslant i\leqslant n} f(u_i) \leqslant M_0 < \infty$.

(iii) $g(\cdot)$ satisfies the first-order Lipschitz condition on closed interval D.

A3. (i) $\sup_{t\in D} \sum_{j=1}^{n} |W_{nj}(t)| = O(1)$.

(ii) $\max_{1\leqslant j\leqslant n} \sum_{i=1}^{n} |W_{nj}(t_i)| = O(1)$.

(iii) $\sup_{t\in D} \max_{1\leqslant j\leqslant n} |W_{nj}(t)| = O(n^{-\frac{1}{2}})$.

A4. There exists some $a \in (0,1)$ and $k_n, 1 \leqslant k_n \leqslant n$ such that

$$\lim_{n\to\infty} k_n = \infty, \quad \lim_{n\to\infty} \frac{k_n}{n^{1-a}} = 0, \tag{2.1}$$

$$\sup_{t\in D} \sum_{j=1}^{n} |W_{nj}(t)| I\Big(|t - t_j| > \frac{k_n}{n}\Big) \leqslant C_3 \frac{k_n}{n}, \tag{2.2}$$

$$\sup_{t\in D} \Big| \sum_{j=1}^{n} W_{nj}(t) - 1 \Big| \leqslant C_4 \frac{k_n}{n}. \tag{2.3}$$

A5. $\max_{1\leqslant i\leqslant n} |x_i| \leqslant C n^a$, where a is defined by $A4$.

REMARK 2.1. Conditions A2(ii)(iii), A3(i)(iii) are used in [5, 9, 14]; condition A4 (2.1)(2.2), A5 are used in [7, 10, 26]. Therefore, our conditions are quite mild and can be easily satisfied.

2.2. Consistency

THEOREM 2.1. *In the model* (1.1)*, suppose that A1–A5 hold.*
(i) If A4 and A5 hold for some $a \in (0,1)$ *and* $p > 2$*, then*

$$E\left|\hat{\beta}_n - \beta\right|^p = O\Big(\frac{1}{n^{(1-a)p/2}}\Big) + O\Big(\frac{k_n^p}{n^p}\Big), \tag{2.4}$$

$$E\left|\tilde{\beta}_n-\beta\right|^p=O\Big(\frac{1}{n^{(1-a)p/2}}\Big)+O\Big(\frac{k_n^p}{n^p}\Big). \tag{2.5}$$

(ii) If A4 and A5 hold for some $a\in(0,1/3)$ *and* $p>2$*, then*

$$\sup_{t\in D}E\left|\hat{g}_n(t)-g(t)\right|^p=O\Big(\frac{1}{n^{(1-3a)p/2}}\Big)+O\Big(\frac{1}{n^{p/4}}\Big)+O\Big(\frac{k_n^p}{n^{(1-a)p}}\Big), \tag{2.6}$$

$$\sup_{t\in D}E\left|\tilde{g}_n(t)-g(t)\right|^p=O\Big(\frac{1}{n^{(1-3a)p/2}}\Big)+O\Big(\frac{1}{n^{p/4}}\Big)+O\Big(\frac{k_n^p}{n^{(1-a)p}}\Big). \tag{2.7}$$

THEOREM 2.2. *Assume the conditions of Theorem 2.1 hold.*

(i) If A4 and A5 hold for some $a\in(0,1-\frac{2}{p})$*, where* $p>2$*, then* $\hat{\beta}_n$ *converges to* β *completely; similarly,* $\tilde{\beta}_n$ *converges to* β *completely.*

(ii) If A4 and A5 hold for some $a\in(0,\frac{1}{3}-\frac{2}{3p})$*, where* $p>4$*, then* $\hat{g}_n(t)$ *converges to* $g(t)$ *completely; similarly,* $\tilde{g}_n(t)$ *converges to* $g(t)$ *completely.*

2.3. The choice of the design points and the weight functions

We will show that the nearest neighbor weights satisfy the designed assumptions. For simplicity, we assume that $D=[0,1]$ and $t_i=i/n,\ x_i=(-1)^i i/n,\ i=1,\dots,n$. Let $k=k_n=[\frac{n^{1-a}}{\log n}]$, where $\log n=\ln(\max(n,e))$, $a\in(0,\frac{1}{2})$. For $t\in[0,1]$, we rewrite $|t_1-t|,|t_2-t|,\dots,\ |t_n-t|$ as follows:

$$|t_{R_1(t)}-t|\leqslant|t_{R_2(t)}-t|\leqslant\cdots\leqslant|t_{R_n(t)}-t|.$$

If $|t_i-t|=|t_j-t|$, then $|t_i-t|$ is in front of $|t_j-t|$ when $t_i<t_j$. Define the k nearest neighbor weight functions

$$W_{ni}(t)=\begin{cases}\frac{1}{k}, & \text{if } |t_i-t|\leqslant|t_{R_k(t)}-t|,\\ 0, & \text{otherwise.}\end{cases} \tag{2.8}$$

Thus, by (2.8)

$$W_{nR_i(t)}(t)=\begin{cases}\frac{1}{k}, & \text{if } i\leqslant k,\\ 0, & \text{otherwise.}\end{cases}$$

It is easily seen that the conditions A3(i), (2.3) in A4 and A5 are fulfilled. By (2.8) we know that $W_{nj}(t)=0$ if $\left|t-t_j\right|>\frac{k_n}{n}$ and $\sum_{i=1}^n W_{nl}(t_i)\leqslant 2$ for $1\leqslant l\leqslant n$, so that (2.1), (2.2) in A4 and A3(ii)(iii) hold.

For definiteness, we assume that $k=k_n$ is even. Through elementary computations, we can get

$$\begin{aligned}\sum_{i=1}^n W_{ni}(t_l)x_i&=\sum_{i=1}^n W_{n,R_i(\frac{l}{n})}\Big(\frac{l}{n}\Big)x_{R_i(\frac{l}{n})}=\frac{1}{k}\sum_{i=1}^k x_{R_i(\frac{l}{n})}=\frac{1}{nk}\sum_{i=1}^k(-1)^{R_l(\frac{l}{n})}R_i(\frac{l}{n})\\ &=\begin{cases}\frac{1}{2n}, & \text{if } l=1,2,\dots,\frac{k}{2}+1,\\ \frac{(-1)^j}{2n}, & \text{if } l=\frac{k}{2}+1+j, 1\leqslant j\leqslant n-k-1,\\ \frac{(-1)^n}{2n}, & \text{if } n-\frac{k}{2}+1\leqslant l\leqslant n.\end{cases}\end{aligned} \tag{2.9}$$

In fact, if $l \in \{1,2,\ldots,\frac{k}{2}+1\}$, then by definition of $R_i(t)$ we know that $(R_1(\frac{l}{n}),\ldots,R_k(\frac{l}{n}))$ is a replacement of $(1,2,\ldots,k)$, thus

$$\frac{1}{nk}\sum_{i=1}^{k}(-1)^{R_i(\frac{l}{n})}R_i\left(\frac{l}{n}\right)=\frac{1}{nk}\sum_{i=1}^{k}(-1)^i i=\frac{1}{2n};$$

if $l=\frac{k}{2}+1+j, 1\leqslant j\leqslant n-k-1$, then $(R_1(\frac{l}{n}),\ldots,R_k(\frac{l}{n}))$ is a replacement of $(j+1,j+2,\ldots,j+k)$, hence

$$\frac{1}{nk}\sum_{i=1}^{k}(-1)^{R_i(\frac{l}{n})}R_i\left(\frac{l}{n}\right)=\frac{1}{nk}\sum_{i=1}^{k}(-1)^{i+j}(i+j)=\frac{(-1)^j}{2n};$$

if $n-\frac{k}{2}+1\leqslant l\leqslant n$, then $(R_1(\frac{l}{n}),\ldots,R_k(\frac{l}{n}))$ is a replacement of $(n,n-1,n-2,\ldots,n-k+1)$, and

$$\begin{aligned}&\frac{1}{nk}\sum_{i=1}^{k}(-1)^{R_i(\frac{l}{n})}R_i\left(\frac{l}{n}\right)\\ &=\frac{1}{nk}\{(-1)^n n+(-1)^{n-1}(n-1)+\cdots+(-1)^{n-k+1}(n-k+1)\}\\ &=\frac{(-1)^n}{2n}.\end{aligned}$$

This proves (2.9). On the one hand, by $k=[\frac{n^{1-a}}{\log n}]$ and $a\in(0,\frac{1}{2})$, it has $\lim\limits_{n\to\infty}\frac{k}{n}=0$ and (2.9),

$$\begin{aligned}S_n^2&=\sum_{l=1}^{n}\left(x_l-\sum_{i=1}^{n}W_{ni}(t_l)x_i\right)^2\\ &\geqslant\sum_{l=\frac{k}{2}+2}^{\frac{k}{2}+1+n-k-1}\left(x_l-\sum_{i=1}^{n}W_{ni}(t_l)x_i\right)^2\\ &=\sum_{j=1}^{n-k-1}\left((-1)^l\frac{l}{n}-\sum_{i=1}^{n}W_{ni}(t_l)x_i\right)^2\Bigg|_{l=\frac{k}{2}+1+j}\\ &=\sum_{j=1}^{n-k-1}\left((-1)^{\frac{k}{2}+1+j}\frac{\frac{k}{2}+1+j}{n}-\frac{(-1)^j}{2n}\right)^2\\ &=\sum_{j=1}^{n-k-1}\left((-1)^{\frac{k}{2}+1}\left(\frac{k}{2n}+\frac{1}{n}+\frac{j}{n}\right)-\frac{1}{2n}\right)^2\\ &\geqslant\sum_{j=1}^{n-k-1}\frac{j^2}{n^2}=\frac{(n-k-1)(n-k)(2n-2k-1)}{6n^2}\sim\frac{n}{3},\quad n\to\infty.\end{aligned}$$

Here $a_n \sim b_n$ means $a_n/b_n \to 1$ as $n \to \infty$. On the other hand,

$$\begin{aligned} S_n^2 &= \sum_{l=1}^{\frac{k}{2}+1} \left((-1)^l \frac{l}{n} - \frac{1}{2n}\right)^2 + \sum_{l=\frac{k}{2}+2}^{\frac{k}{2}+1+n-k-1} \left((-1)^l \frac{l}{n} - \frac{(-1)^j}{2n}\right)^2 \\ &\quad + \sum_{l=n-\frac{k}{2}+1}^{n} \left((-1)^l \frac{l}{n} - \frac{(-1)^n}{2n}\right)^2 \\ &\leqslant 4\sum_{l=1}^{n} \frac{l^2}{n^2} \sim \frac{4n}{3}, \quad n \to \infty. \end{aligned}$$

Thus A2(i) holds.

REMARK 2.2. In Theorem 2.1 (i), let $k = k_n = [\frac{n^{1-a}}{\log n}]$, $a = \frac{1}{3}$. Then, for $p > 2$,

$$E|\hat{\beta}_n - \beta|^p = O(n^{-p/3}), \quad E|\tilde{\beta}_n - \beta|^p = O(n^{-p/3}).$$

So the convergence rates for $\hat{\beta}_n - \beta$ and $\tilde{\beta}_n - \beta$ are

$$\hat{\beta}_n - \beta = O_P(n^{-1/3}), \quad \tilde{\beta}_n - \beta = O_P(n^{-1/3}).$$

Similarly, in Theorem 2.1 (ii), let $k = k_n = [\frac{n^{1-3a}}{\log n}]$, $a \in [1/8, 1/6]$. Then, for $p > 2$

$$\sup_{t\in D} E|\hat{g}_n(t) - g(t)|^p = O(n^{-p/4}),$$

$$\sup_{t\in D} E|\tilde{g}_n(t) - g(t)|^p = O(n^{-p/4}).$$

Thus, the convergence rates for $\hat{g}_n(t) - g(t)$ and $\tilde{g}_n(t) - g(t)$ are

$$\hat{g}_n(t) - g(t) = O_P(n^{-1/4}), \quad \tilde{g}_n(t) - g(t) = O_P(n^{-1/4}).$$

REMARK 2.3. In Theorem 2 of Zhou and Hu [14], the authors considered the moment consistency of parametric least squares estimator in partially linear model (1.1) with NA errors satisfying $\sup_{i\geqslant 1} E|e_i|^p < \infty$ for some $p > 4$, and obtained the results $E|\hat{\beta}_n - \beta|^p = o(n^{-p/4})$ and $\sup_{t\in D} E|\hat{g}_n(t) - g(t)|^p = o(n^{-p/4})$. In this paper, we extend Zhou and Hu [14] to AANA errors with weakly moment condition $\sup_{i\geqslant 1} E|e_i|^p < \infty$ for some $p > 2$, and obtain the convergence rates such as $E|\hat{\beta}_n - \beta|^p = O(n^{-p/3})$ and $\sup_{t\in D} E|\hat{g}_n(t) - g(t)|^p = O(n^{-p/4})$. In addition, we study the complete convergence for $\hat{\beta}_n$ and $\hat{g}_n(t)$ in Theorem 2.2.

3. Simulations

In this section, we will investigate the numerical performance of the moment consistency for the estimators with AANA random errors. An AANA sequence is given:

$$e_i = (1+a_i^2)^{-1/2}(\eta_i + a_i\eta_{i+1}), \ \ i \geqslant 1, \tag{3.1}$$

where $\eta_1, \eta_2, \ldots,$ are independent and identically distributed $N(0,1)$ random variables and $a_i = (1/i^2)$, $i \geqslant 1$. This sequence $\{e_i, i \geqslant 1\}$ has been proved to be an AANA sequence but not a NA sequence (see Chandra and Ghosal [4]).

We will simulate a heteroscedastic partially linear model

$$y_i = x_i\beta + g(t_i) + \sigma_i e_i, \ \ i \geqslant 1$$

where $\beta = 2.5$, $g(t) = \cos(\pi t)$, $\sigma_i = (f(u_i))^{1/2} = (1+0.8\sin(4\pi u_i))^{1/2}$, $x_i = (-1)^i i/n$, $1 \leqslant i \leqslant n$, and the random errors are given by (3.1). Let $D = [0,1]$, $u_i = t_i = i/n$, $1 \leqslant i \leqslant n$. Take $k_n = [n^{0.58}]$ and the nearest neighbor weight defined by (2.8).

The sample sizes are taken as $n =$200, 500, 1000, 1500, 2000 and 2500, respectively, and each case is repeated for 1000 times and the average values of $\hat{\beta}_n$ and $\tilde{\beta}_n$ are calculated as the estimators. Then, we examine the estimation errors of $\hat{\beta}_n$, $\tilde{\beta}_n$ and $\hat{g}_n$, $\tilde{g}_n$, measured by the mean square errors (MSE) defined as $\mathrm{MSE}(\hat{\beta}_n) = E|\hat{\beta}_n - \beta|^2$, $\mathrm{MSE}(\tilde{\beta}_n) = E|\tilde{\beta}_n - \beta|^2$ and the mean integrated squared error(MISE) defined as MISE $(\hat{g}_n) = E\int_D(\hat{g}_n(t) - g(t))^2 dt$, $\mathrm{MISE}(\tilde{g}_n) = E\int_D(\tilde{g}_n(t) - g(t))^2 dt$.

Of interests are the sample means $\overline{\mathrm{MSE}}(\hat{\beta}_n)$ and $\overline{\mathrm{MSE}}(\tilde{\beta}_n)$ of $\mathrm{MSE}(\hat{\beta}_n)$ and $\mathrm{MSE}(\tilde{\beta}_n)$ over the 1000 replications, and similar sample means $\overline{\mathrm{MISE}}(\hat{g}_n)$ and $\overline{\mathrm{MISE}}(\tilde{g}_n)$.

The results are presented in Tables 1 , and the curves of $g(t)$, $\hat{g}_n(t)$, and $\tilde{g}_n(t)$ are provided in Figure 1.

Table 1: *The sample means $\overline{\mathrm{MSE}}$ of LS estimator $\hat{\beta}_n$ and WLS estimator $\tilde{\beta}_n$; $\overline{\mathrm{MISE}}$ of $\hat{g}_n$ and $\tilde{g}_n$ with $\beta = 2.5$.*

n	$\hat{\beta}_n$	$\overline{\mathrm{MSE}}(\hat{\beta}_n)$	$\overline{\mathrm{MISE}}(\hat{g}_n)$	$\tilde{\beta}_n$	$\overline{\mathrm{MSE}}(\tilde{\beta}_n)/\overline{\mathrm{MSE}}(\hat{\beta}_n)$	$\overline{\mathrm{MISE}}(\tilde{g}_n)/\overline{\mathrm{MISE}}(\hat{g}_n)$
200	2.5042	0.0126	0.0477	2.4989	0.5707	0.9988
500	2.4990	0.0052	0.0291	2.4978	0.5658	0.9946
1000	2.4985	0.0024	0.0196	2.4997	0.6159	0.9728
1500	2.5003	0.0015	0.0144	2.5014	0.7148	1.0040
2000	2.5013	0.0012	0.0123	2.5008	0.6131	1.0123
2500	2.5014	0.0010	0.0108	2.5010	0.5544	0.9891

Table 1 shows that as n increases, both $\overline{\mathrm{MSE}}(\hat{\beta}_n)$ and $\overline{\mathrm{MSE}}(\tilde{\beta}_n)$ go to zero as sample n increases. The simulation shows the consistency of $\hat{\beta}_n$, $\tilde{\beta}_n$, $\hat{g}_n(t)$ and $\tilde{g}_n(t)$ in model (1.1) with AANA errors. In addition, we can also see that the $\overline{\mathrm{MSE}}(\tilde{\beta}_n)$ is smaller than $\overline{\mathrm{MSE}}(\hat{\beta}_n)$, which the ratio is less than 1. Thus, the weighted LS estimator

$\tilde{\beta}_n$ is better than LS estimator $\hat{\beta}_n$. On the other hand, by Table 1 and Fig 1, the non-weighted nonparametric function estimator $\hat{g}_n(t)$ is as well as weighted nonparametric function estimator $\tilde{g}_n(t)$.

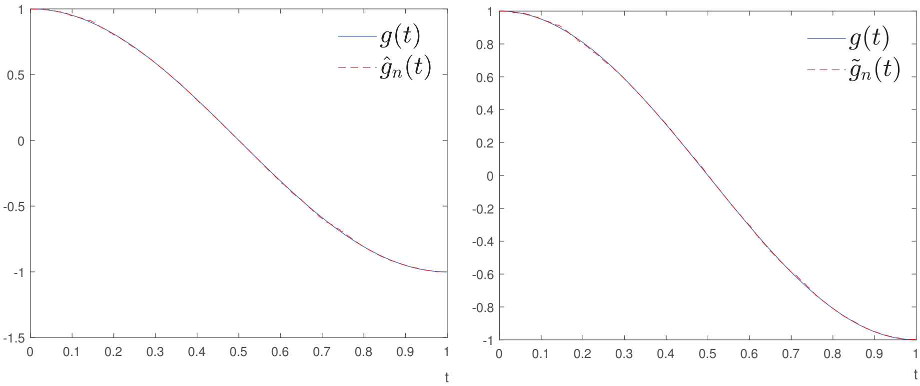

Figure 1: *Curves of* $g(t) = \cos(\pi t)$, $\hat{g}_n(t)$ *and* $\tilde{g}_n(t)$ *with* $\beta = 2.5$ *and* $n = 1500$.

4. Conclusions

In this paper, we study the consistency of parametric least squares estimator and nonparametric weighted estimator in partially linear regression models with AANA errors. Some moment convergence and complete convergence are obtained in Theorems 2.1 and 2.2. In order to illustrate our results, one example of weight functions and some simulations are presented in Sections 2 and 3. Our results extend some results of Zhou and Hu [14] based on NA errors to AANA errors. In future work, it is interesting for researchers to study the limiting distributions of parametric least squares estimator $\hat{\beta}_n$ and nonparametric weighted estimator $\hat{g}_n(t)$ based on AANA errors or other dependent errors.

5. Proofs of main results

LEMMA 5.1. (cf. Yuan and An [13]) *Let* $\{X_i, i \geqslant 1\}$ *be an AANA sequence with mixing coefficients* $\{q(i), i \geqslant 1\}$. *Then* $\{f_i(X_i), i \geqslant 1\}$ *is still an AANA sequence with mixing coefficients* $\{q(i), i \geqslant 1\}$, *where* $f_1, f_2, \ldots$ *are nondecreasing or nonincreasing continuous functions.*

LEMMA 5.2. (cf. Yuan and An [13]) *Let* $\{X_i, i \geqslant 1\}$ *be an AANA sequence of zero mean random variables with mixing coefficients* $\{q(i), i \geqslant 1\}$. *Assume further that* $E|X_n|^p < \infty$ *for all* $n \geqslant 1$ *and some* $p > 2$. *Suppose that there exists a integer number* k *such that* $p \in (2^k, 2^{k+1}]$, *and* $\{q(i), i \geqslant 1\}$ *satisfies* $\sum_{n=1}^{\infty} q^{\tilde{p}}(n) < \infty$, *where*

$\tilde{p} = (1/2^{k-1} - 2/p)p/(p-1)$, *then there exists a positive constant* C_p *depending only on* p *such that*

$$E\Big(\max_{1\leqslant j\leqslant n}\Big|\sum_{i=1}^{j} X_i\Big|^p\Big) \leqslant C_p\Big\{\sum_{i=1}^{n} E\,|X_i|^p + \Big(\sum_{i=1}^{n} EX_i^2\Big)^{p/2}\Big\}. \tag{5.1}$$

LEMMA 5.3. *Let* $\{d_i(z); i \geqslant 1\}$ *be a sequence of real functions defined on closed interval* D, *and the conditions of Lemma 5.2 hold and* $\sup_{i\geqslant 1} E|X_i|^p < \infty$ *for some* $p > 2$. *Then there exists a positive constant* C_p *which only depends on the given number* p *such that*

$$E\Big(\max_{1\leqslant j\leqslant n}\Big|\sum_{i=1}^{j} d_i(z)X_i\Big|^p\Big) \leqslant C_p\Big(\sum_{i=1}^{n}(d_i(z))^2\Big)^{p/2}, \quad n \geqslant 1 \tag{5.2}$$

Proof. Denote by $d_i^+(z) = \max(d_i(z), 0)$, $d_i^-(z) = \max(-d_i(z), 0)$. By Lemma 5.1 we know that $\{d_i^+(z)X_i; 1 \leqslant i \leqslant n\}$ and $\{d_i^-(z)X_i; 1 \leqslant i \leqslant n\}$ are still zero mean AANA random variables with mixing coefficients $\{q(i), i \geqslant 1\}$. By using Lemma 5.2 for these two sequences respectively, we have

$$E\Big(\max_{1\leqslant j\leqslant n}\Big|\sum_{i=1}^{j} d_i(z)X_i\Big|^p\Big) \leqslant C_p\Big\{\sum_{i=1}^{n} E|d_i(z)X_i|^p + \Big(\sum_{i=1}^{n} E\Big(d_i(z)X_i\Big)^2\Big)^{p/2}\Big\}.$$

By $\sup\limits_{i\geqslant 1} E|X_i|^p < \infty$ and $p > 2$, we obtain that

$$\sup_{i\geqslant 1} EX_i^2 \leqslant \Big(\sup_{i\geqslant 1} E|X_i|^p\Big)^{2/p} < \infty.$$

Hence, one can get that

$$E\Big(\max_{1\leqslant j\leqslant n}\Big|\sum_{i=1}^{j} d_i(z)X_i\Big|^p\Big) \leqslant C_p\Big\{\sum_{i=1}^{n} |d_i(z)|^p + \Big(\sum_{i=1}^{n}\Big(d_i(z)\Big)^2\Big)^{p/2}\Big\}. \tag{5.3}$$

Using the fact that

$$\Big(\sum_{i=1}^{n} |d_i(z)|^p\Big)^{1/p} \leqslant \Big(\sum_{i=1}^{n}\Big(d_i(z)\Big)^2\Big)^{1/2}, \quad p \geqslant 2. \tag{5.4}$$

Therefore, the desired result (5.2) follows by (5.3) and (5.4) immediately. This completes the proof. □

Proof of Theorem 2.1. We prove (2.4) first. It follows from (1.1) and (1.2) that

$$\hat{\beta}_n - \beta = \Big\{\sum_{i=1}^{n}\tilde{x}_i\varepsilon_i + \sum_{i=1}^{n}\tilde{x}_i\tilde{g}(t_i) - \sum_{i=1}^{n}\tilde{x}_i\Big(\sum_{j=1}^{n} W_{nj}(t_i)\varepsilon_j\Big)\Big\}/S_n^2. \tag{5.5}$$

where $\tilde{g}(t_i) = g(t_i) - \sum_{j=1}^{n} W_{nj}(t_i) g(t_j)$ and $\varepsilon_i = \sigma_i e_i$. For $p > 2$, one can get by C_r-inequality that

$$E\Big|\hat{\beta}_n - \beta\Big|^p \leqslant 3^{p-1}\Big\{E\Big|\sum_{i=1}^{n} \tilde{x}_i \varepsilon_i / S_n^2\Big|^p + \Big|\sum_{i=1}^{n} \tilde{x}_i \tilde{g}(t_i)/S_n^2\Big|^p + E\Big|\sum_{i=1}^{n} \tilde{x}_i \sum_{j=1}^{n} W_{nj}(t_i)\, \varepsilon_j / S_n^2\Big|^p\Big\}. \tag{5.6}$$

We observe from Lemma 5.3 with $p > 2$ that

$$E\Big|\frac{1}{n}\sum_{i=1}^{n} \tilde{x}_i \varepsilon_i\Big|^p = E\Big|\frac{1}{n}\sum_{i=1}^{n} \tilde{x}_i \sigma_i e_i\Big|^p \leqslant C_p \left(\frac{1}{n^2}\sum_{i=1}^{n} \tilde{x}_i^2 \sigma_i^2\right)^{p/2}. \tag{5.7}$$

We obtain from A2(i)(ii), A3(i) and A5 that for $i = 1, \ldots, n$,

$$|\tilde{x}_i| \leqslant |x_i| + \sup_{t \in D} \sum_{j=1}^{n} \big|W_{nj}(t)\big| \max_{1 \leqslant j \leqslant n} \big|x_j\big| \leqslant C n^a, \tag{5.8}$$

$$\sum_{i=1}^{n} \tilde{x}_i^2 \sigma_i^2 \leqslant C \sum_{i=1}^{n} \tilde{x}_i^2 \leqslant C n.$$

Hence,

$$E\Big|\frac{1}{n}\sum_{i=1}^{n} \tilde{x}_i \varepsilon_i\Big|^p \leqslant \frac{C_p}{n^{p/2}}, \tag{5.9}$$

which combining with A2(i) implies that

$$E\Big|\sum_{i=1}^{n} \tilde{x}_i \varepsilon_i / S_n^2\Big|^p = O\Big(\frac{1}{n^{p/2}}\Big). \tag{5.10}$$

By Lemma 5.3 with $p > 2$, it follows that

$$\begin{aligned} E\Big|\frac{1}{n}\sum_{i=1}^{n} \tilde{x}_i \sum_{j=1}^{n} W_{nj}(t_i)\, \varepsilon_j\Big|^p &= E\Big|\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{n} \tilde{x}_i W_{nj}(t_i)\, \sigma_j e_j\Big|^p \\ &\leqslant C_p \Big(\frac{1}{n^2}\sum_{j=1}^{n}\Big(\sum_{i=1}^{n} \tilde{x}_i W_{nj}(t_i)\, \sigma_j\Big)^2\Big)^{p/2}. \end{aligned} \tag{5.11}$$

By A2(ii), A3(i)(ii) and (5.8), it is easy to see that

$$\begin{aligned} &\sum_{j=1}^{n}\Big(\sum_{i=1}^{n} \tilde{x}_i W_{nj}(t_i)\, \sigma_j\Big)^2 \\ &\leqslant \max_{1 \leqslant j \leqslant n} \sigma_j^2 \max_{1 \leqslant i \leqslant n} |\tilde{x}_i| \max_{1 \leqslant j \leqslant n} \sum_{i=1}^{n} \Big|W_{nj}(t_i)\Big| \sum_{j=1}^{n}\sum_{i=1}^{n} \Big|\tilde{x}_i W_{nj}(t_i)\Big| \\ &\leqslant C n^a \Big(\sup_{t \in D} \sum_{j=1}^{n} \big|W_{nj}(t)\big|\Big)\Big(\sum_{i=1}^{n} |\tilde{x}_i|\Big) \leqslant C n^a \sqrt{\sum_{i=1}^{n} \tilde{x}_i^2}\sqrt{n} \leqslant C n^{a+1}. \end{aligned}$$

Thus

$$E\left|\frac{1}{n}\sum_{i=1}^{n}\tilde{x}_i\sum_{j=1}^{n}W_{nj}(t_i)\varepsilon_j\right|^p \leqslant \frac{C_p}{n^{(1-a)p/2}}, \tag{5.12}$$

which combining with A2(i) yields

$$E\left|\sum_{i=1}^{n}\tilde{x}_i\sum_{j=1}^{n}W_{nj}(t_i)\varepsilon_j/S_n^2\right|^p = O\Big(\frac{1}{n^{(1-a)p/2}}\Big). \tag{5.13}$$

By A2(iii), A3(i) and A4, it can be seen that

$$\begin{aligned}\sup_{t\in D}|\tilde{g}(t)| &= \sup_{t\in D}\left|g(t)-\sum_{j=1}^{n}W_{nj}(t)g(t_j)\right| \\ &= \sup_{t\in D}\left|g(t)(1-\sum_{j=1}^{n}W_{nj}(t))+\sum_{j=1}^{n}W_{nj}(t)(g(t)-g(t_j))\right| \\ &\leqslant C\sup_{t\in D}\left|\sum_{j=1}^{n}W_{nj}(t)-1\right|+C\sup_{t\in D}\sum_{j=1}^{n}\left|W_{nj}(t)\right|\left|t-t_j\right|I\Big(\left|t-t_j\right|>\frac{k_n}{n}\Big) \\ &\quad +C\sup_{t\in D}\sum_{j=1}^{n}\left|W_{nj}(t)\right|\left|t-t_j\right|I\Big(\left|t-t_j\right|\leqslant\frac{k_n}{n}\Big)\leqslant\frac{Ck_n}{n}.\end{aligned} \tag{5.14}$$

From A2(i), we have

$$\left|\frac{1}{n}\sum_{i=1}^{n}\tilde{x}_i\tilde{g}(t_i)\right| \leqslant \sup_{t\in D}|\tilde{g}(t)|\sum_{i=1}^{n}\frac{|\tilde{x}_i|}{n} \leqslant \frac{Ck_n}{n}, \tag{5.15}$$

which combining with A2(i) implies that

$$\sum_{i=1}^{n}\tilde{x}_i\tilde{g}(t_i)/S_n^2 = O\Big(\frac{k_n}{n}\Big). \tag{5.16}$$

From (5.6), (5.10), (5.13) and (5.16), we obtain

$$E\left|\hat{\beta}_n-\beta\right|^p = O\Big(\frac{1}{n^{(1-a)p/2}}\Big)+O\Big(\frac{k_n^p}{n^p}\Big).$$

Therefore, we prove (2.4).

Now we prove (2.5), which is similar to the proof of (2.4). For $p>2$, one can get

$$\begin{aligned}E\left|\tilde{\beta}_n-\beta\right|^p \leqslant 3^{p-1}E\left|\sum_{i=1}^{n}\gamma_i\tilde{x}_i\varepsilon_i/T_n^2\right|^p+3^{p-1}\left|\sum_{i=1}^{n}\gamma_i\tilde{x}_i\tilde{g}(t_i)/T_n^2\right|^p \\ +3^{p-1}E\left|\sum_{i=1}^{n}\gamma_i\tilde{x}_i\sum_{j=1}^{n}W_{nj}(t_i)\varepsilon_j/T_n^2\right|^p\end{aligned} \tag{5.17}$$

and

$$E\left|\frac{1}{n}\sum_{i=1}^{n}\gamma_i\tilde{x}_i\varepsilon_i\right|^p \leqslant C_p\Big(\frac{1}{n^2}\sum_{i=1}^{n}\gamma_i^2\tilde{x}_i^2\sigma_i^2\Big)^{p/2}. \tag{5.18}$$

Noticing that

$$\max_{1\leqslant i\leqslant n}|\gamma_i\tilde{x}_i\sigma_i|\leqslant Cn^a, \qquad \sum_{i=1}^n \gamma_i^2\tilde{x}_i^2\sigma_i^2\leqslant C\sum_{i=1}^n\tilde{x}_i^2\leqslant Cn,$$

from A2(ii). Then it has

$$E\Big|\frac{1}{n}\sum_{i=1}^n\gamma_i\tilde{x}_i\varepsilon_i\Big|^p\leqslant\frac{C_p}{n^{p/2}}, \tag{5.19}$$

which combining with A2(i) and $T_n^2=\sum_{i=1}^n\gamma_i\tilde{x}_i^2$ imply that

$$0<C_1\leqslant\liminf_{n\to\infty}\frac{T_n^2}{n}\leqslant\limsup_{n\to\infty}\frac{T_n^2}{n}\leqslant C_2<\infty,$$

and

$$E\Big|\sum_{i=1}^n\gamma_i\tilde{x}_i\varepsilon_i/T_n^2\Big|^p=O\Big(\frac{1}{n^{p/2}}\Big). \tag{5.20}$$

The remaining steps of the proof of (2.5) are similar to the proof of (2.4); thus, we omit the details here.

Now we prove (2.6). From (1.3) and model (1.1) and C_r-inequality, for $p>2$, one gets

$$\begin{aligned}&\sup_{t\in D}E\left|\hat{g}_n(t)-g(t)\right|^p\\ &=\sup_{t\in D}E\Big|\sum_{j=1}^nW_{nj}(t)\sigma_je_j-\Big(\hat{\beta}_n-\beta\Big)\sum_{j=1}^nW_{nj}(t)x_j-\tilde{g}(t)\Big|^p\\ &\leqslant 3^{p-1}\Big(\sup_{t\in D}E\Big|\sum_{j=1}^nW_{nj}(t)\sigma_je_j\Big|^p+\sup_{t\in D}E\Big|\Big(\hat{\beta}_n-\beta\Big)\sum_{j=1}^nW_{nj}(t)x_j\Big|^p+\sup_{t\in D}|\tilde{g}(t)|^p\Big).\end{aligned} \tag{5.21}$$

We know from (5.14) that

$$\sup_{t\in D}|\tilde{g}(t)|^p=O\Big(\frac{k_n^p}{n^p}\Big). \tag{5.22}$$

A3(i) and A5 imply that

$$\sup_{t\in D}E\Big|\Big(\hat{\beta}_n-\beta\Big)\sum_{j=1}^nW_{nj}(t)x_j\Big|^p\leqslant CE\Big|n^a\Big(\hat{\beta}_n-\beta\Big)\Big|^p. \tag{5.23}$$

By (5.6), (5.9), (5.12) and (5.15), we have

$$E\Big|n^a\frac{1}{n}\sum_{i=1}^n\tilde{x}_i\varepsilon_i\Big|^p\leqslant\frac{C_p}{n^{(1-2a)p/2}}, \tag{5.24}$$

$$E\Big|n^a\frac{1}{n}\sum_{i=1}^n\tilde{x}_i\sum_{j=1}^nW_{nj}(t_i)\varepsilon_j\Big|^p\leqslant\frac{C_p}{n^{(1-3a)p/2}}, \tag{5.25}$$

$$\left|n^a \frac{1}{n}\sum_{i=1}^{n} \tilde{x}_i \tilde{g}(t_i)\right| \leqslant \frac{Ck_n}{n^{1-a}}. \tag{5.26}$$

Thus by A2(i), (5.5), (5.23)–(5.26) and C_r-inequality, one gets

$$\sup_{t\in D} E\left|\left(\hat{\beta}_n-\beta\right)\sum_{j=1}^{n} W_{nj}(t)x_j\right|^p = O\Big(\frac{1}{n^{(1-3a)p/2}}\Big)+O\Big(\frac{k_n^p}{n^{(1-a)p}}\Big). \tag{5.27}$$

By Lemma 5.3, it is clearly that

$$\sup_{t\in D} E\left|\sum_{j=1}^{n} W_{nj}(t)\sigma_j e_j\right|^p \leqslant C_p \sup_{t\in D}\Big(\sum_{j=1}^{n} W_{nj}^2(t)\sigma_j^2\Big)^{p/2}. \tag{5.28}$$

The conditions A3(i)(iii) imply that

$$\sup_{t\in D}\sum_{j=1}^{n} W_{nj}^2(t)\sigma_j^2 \leqslant \max_{1\leqslant j\leqslant n}\sigma_j^2 \sup_{t\in D}\max_{1\leqslant j\leqslant n}\left|W_{nj}(t)\right| \sup_{t\in D}\sum_{j=1}^{n}\left|W_{nj}(t)\right| \leqslant Cn^{-\frac{1}{2}}.$$

Thus

$$\sup_{t\in D} E\left|\sum_{j=1}^{n} W_{nj}(t)\sigma_j e_j\right|^p = O\Big(\frac{1}{n^{p/4}}\Big). \tag{5.29}$$

By (5.21), (5.22), (5.27) and (5.29), it follows

$$\sup_{t\in D} E\left|\hat{g}_n(t)-g(t)\right|^p = O\Big(\frac{1}{n^{(1-3a)p/2}}\Big)+O\Big(\frac{1}{n^{p/4}}\Big)+O\Big(\frac{k_n^p}{n^{(1-a)p}}\Big). \tag{5.30}$$

The proof of (2.7) is similar to that of (2.6); thus, we omit the details here. This completes the proof of theorem 2.1. □

Proof of Theorem 2.2. (i) By $p>2$, $a\in(0,1-2/p)$, Markov inequality, (5.9) and (5.12), for every $\varepsilon>0$, it has

$$\sum_{n=1}^{\infty} P\left(\left|\frac{1}{n}\sum_{i=1}^{n}\tilde{x}_i\varepsilon_i\right|>\varepsilon\right) \leqslant \sum_{n=1}^{\infty} E\left|\frac{1}{n}\sum_{i=1}^{n}\tilde{x}_i\varepsilon_i\right|^p/\varepsilon^p \leqslant \frac{C_p}{\varepsilon^p}\sum_{n=1}^{\infty}\frac{1}{n^{p/2}}<\infty, \tag{5.31}$$

$$\begin{aligned}\sum_{n=1}^{\infty} P\left(\left|\frac{1}{n}\sum_{i=1}^{n}\tilde{x}_i\sum_{j=1}^{n} W_{nj}(t_i)\varepsilon_j\right|>\varepsilon\right) &\leqslant \sum_{n=1}^{\infty} E\left|\frac{1}{n}\sum_{i=1}^{n}\tilde{x}_i\sum_{j=1}^{n} W_{nj}(t_i)\varepsilon_j\right|^p/\varepsilon^p\\ &\leqslant \frac{C_p}{\varepsilon^p}\sum_{n=1}^{\infty}\frac{1}{n^{(1-a)p/2}}<\infty.\end{aligned} \tag{5.32}$$

Combining with A2(i) implies that

$$\sum_{i=1}^{n}\tilde{x}_i\varepsilon_i/S_n^2 \qquad \text{and} \qquad \sum_{i=1}^{n}\tilde{x}_i\sum_{j=1}^{n} W_{nj}(t_i)\varepsilon_j/S_n^2$$

converge to zero completely, which combining with (5.5) and (5.16) yields that $\hat{\beta}_n$ converges to β completely. Similarly, one can easily obtain that $\tilde{\beta}_n$ converges to β completely.

(ii) We will prove that $\hat{g}_n(t)$ converges to $g(t)$ completely.

For $p>4$, $a\in(0,\frac{1}{3}-\frac{2}{3p})$, by (5.29), we obtain

$$\sum_{n=1}^{\infty}P(|\sum_{j=1}^{n}W_{nj}(t)\varepsilon_j|>\varepsilon)\leqslant\sum_{n=1}^{\infty}E|\sum_{j=1}^{n}W_{nj}(t)\varepsilon_j|^p/\varepsilon^p\leqslant\frac{C_p}{\varepsilon^p}\sum_{n=1}^{\infty}\frac{1}{n^{p/4}}<\infty. \tag{5.33}$$

Similar to the (5.31),

$$\sum_{n=1}^{\infty}P(|n^a\frac{1}{n}\sum_{i=1}^{n}\tilde{x}_i\varepsilon_i|>\varepsilon)\leqslant\sum_{n=1}^{\infty}E|n^a\frac{1}{n}\sum_{i=1}^{n}\tilde{x}_i\varepsilon_i|^p/\varepsilon^p\leqslant\frac{C_p}{\varepsilon^p}\sum_{n=1}^{\infty}\frac{1}{n^{(1-2a)p/2}}<\infty, \tag{5.34}$$

$$\begin{aligned}\sum_{n=1}^{\infty}P(|n^a\frac{1}{n}\sum_{i=1}^{n}\tilde{x}_i\sum_{j=1}^{n}W_{nj}(t_i)\varepsilon_j|>\varepsilon)&\leqslant\sum_{n=1}^{\infty}E|n^a\frac{1}{n}\sum_{i=1}^{n}\tilde{x}_i\sum_{j=1}^{n}W_{nj}(t_i)\varepsilon_j|^p/\varepsilon^p\\&\leqslant\frac{C_p}{\varepsilon^p}\sum_{n=1}^{\infty}\frac{1}{n^{(1-3a)p/2}}<\infty.\end{aligned} \tag{5.35}$$

It is easily seen that A2(i), (5.5), (5.26),(5.34) and (5.35) imply that $\sum_{j=1}^{n}W_{nj}(t)(\hat{\beta}_n-\beta)$ converges to zero completely; (5.33) implies that $\sum_{j=1}^{n}W_{nj}(t)\varepsilon_j$ converges to zero completely; hence $\hat{g}_n(t)-g(t)=\sum_{j=1}^{n}W_{nj}(t)\varepsilon_j-\sum_{j=1}^{n}W_{nj}(t)x_j(\hat{\beta}_n-\beta)-\tilde{g}(t)$ converges to zero completely.

In the same way, $\tilde{g}_n(t)-g(t)$ converges to zero completely. This completes the proof. □

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