Journal of
Mathematical
Inequalities
Volume 19, Number 2 (2025), 625–635 doi:10.7153/jmi-2025-19-38

ON A RECURSIVE RELATION AND ITS CONNECTIONS TO NUMBER THEORY

STEVO STEVIĆ

(*Communicated by L. Mihoković*)

Abstract. We give some theoretical explanations for solving the recursive relation

$$1+\sum_{d|n}(-1)^{\frac{n}{d}}a_d=0,$$

by finding its connections with number theory.

1. Introduction and preliminaries

Let $\mathbb{N}=\{1,2,3,\ldots\}$, i.e., the set of positive integers, $\mathbb{Z}$ be the set of whole numbers, $\mathbb{R}$ be the set of real numbers, and $\mathbb{N}_k=\{n\geqslant k|n\in\mathbb{Z}\}$, where $k\in\mathbb{Z}$ is fixed.

Recursive relations of various types have been studied for a long time. One of the basic problems related to the relations is finding their solutions in closed form. First nontrivial results in the topic were obtained in the beginning of the 18th century [6, 8]. For some later sources from the century see, for example, [12, 13, 14], where the investigation of solvability of linear recursive relations, that is, difference equations was predominately done. In the classical books [9, 10, 15] can be found some later presentations of some results in the topic. Since it is not easy to obtain new general methods for dealing with the problem of solvability of recursive relations, the interest in the topic diminished during the 19th century. Nevertheless the fact, solvable recursive relations have occurred from time to time in computational mathematics [7], combinatorics [25], summations of some series [17], many problem books [4, 11, 16], as well as some popular journals for a wide audience, and various other research and educational domains. Solvable recursive relations are also useful in getting some comparison results in the theory of difference equations and systems of difference equations (see, for example, [5, 20, 27, 28]). There have been also some exotic applications of the recursive relations, as it was the case, for example, in [29], where a recursive relation was used in showing the nonexistence of solutions of an integral equation.

Use of the computer algebra packages, among other things, renewed a recent interest in the topic. The use also caused some problems, since some authors do not give any theoretical explanations for the formulas for solutions to the considered recursive

Mathematics subject classification (2020): Primary 39A99; Secondary 11N99, 65Q30.
Keywords and phrases: Recursive relation, sequence of numbers, solution to a recursive relation, Möbius function, Möbius inversion formula.

Paper JMI-19-38

relations in their papers (see, for example, some comments and theoretical explanations of ours given in [31, 36]). For some recent results on finding solutions to recursive relations and systems of recursive relations in closed form, and finding their invariants see, for example, [3, 18, 19, 21, 22, 24, 26, 30, 32, 33, 34, 35, 37, 38] and the related references therein, where many different methods, tricks and ideas can be found.

The following problem appeared in journal Kvant in 1980 (see [1, Problem M624]):

PROBLEM 1. *Find the sequence* $(a_n)_{n\in\mathbb{N}}$, *which is defined by the conditions* $a_1 = 1$,

$$1+\sum_{d|n}(-1)^{\frac{n}{d}}a_d = 0, \tag{1}$$

where the sum is taken over all divisors of the number $n\in\mathbb{N}$, *including* $d=1$ *and* $d=n$. *For example, if* $n=p$ *is a prime number, then* (1) *has the form*

$$1+(-1)^{\frac{p}{1}}a_1+(-1)^{\frac{p}{p}}a_p = 0,$$

from which it follows that $a_p = 2$ *if* $p = 2$, *and* $a_p = 0$ *if* $p > 2$.

In [2] was published one of the typical high-school solutions to such type of problems. Namely, it was noticed therein that the relation in (1) can be written in the form

$$a_n = 1+\sum_{d|n,d<n}(-1)^{\frac{n}{d}}a_d,$$

and were calculated first several members of the sequence $(a_n)_{n\in\mathbb{N}}$.

Based on the facts (obtained by the calculations) that

$$a_1 = 1,\ a_2 = 2,\ a_3 = 0,\ a_4 = 4,\ a_5 = a_6 = a_7 = 0,\ a_8 = 8,\ a_9 = 0,$$

it was assumed that

$$a_n = \begin{cases} n, \text{ if } n = 2^m, \\ 0, \text{ if } n \neq 2^m, \end{cases} \tag{2}$$

where $m\in\mathbb{N}_0$, and the relations in (2) were confirmed by the method of mathematical induction.

The fact that in (1) appears a summation over all divisors of a natural number, which frequently appear in number theory [23, 39], has suggested us that the problem has some connections with the multiplicative functions appearing in the theory. Here, among other things, we present our original solution to the problem essentially obtained in 1984, which has not been published so far.

Recall that the Möbius function μ is the multiplicative function on $\mathbb{N}$ defined by:

$$\mu(p) = -1, \quad \mu(p^\alpha) = 0, \text{ if } \alpha > 1,$$

where $p\in\mathbb{N}$ is a prime number. Note that the function takes the values in the set $\{-1,0,1\}$.

The following proposition is one of the basic results related to the function (see, e.g., [39, p. 29]).

PROPOSITION 1. *Let $\theta(n)$ be a multiplicative function and $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ be the canonical representation of an $n \in \mathbb{N}$. Then*

$$\sum_{d|n} \mu(d)\theta(d) = (1-\theta(p_1))\cdots(1-\theta(p_k)), \tag{3}$$

(if $n = 1$ we regard that the right-hand side in (3) is equal to 1).

If we take the function

$$\theta(a) = 1, \quad a \in \mathbb{N},$$

in (3), which is obviously multiplicative, we get:

$$\sum_{d|n} \mu(d) = \begin{cases} 0, \text{ if } n > 1, \\ 1, \text{ if } n = 1. \end{cases} \tag{4}$$

The following result is the Möbius inversion formula (see, e.g., [39, p. 37]).

LEMMA 1. *Let f and g be two arithmetic functions such that*

$$f(n) = \sum_{d|n} g(d), \quad \textit{for} \quad n \in \mathbb{N}.$$

Then

$$g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right), \quad \textit{for} \quad n \in \mathbb{N}.$$

The purpose of this note is to give some theoretical explanations for the formulas in (2). First, we present a detailed solution to Problem 1, which is based on our original idea. Then, we present another solution based on generating functions.

2. First solution to Problem 1

In this section we give our original solution to Problem 1 which gives a theoretical explanation for it. The solution has not been published so far.

First solution to Problem 1. Define a sequence $(b_n)_{n\in\mathbb{N}}$, as follows:

$$b_n := 1, \quad n \in \mathbb{N}. \tag{5}$$

Then the relation in (1) can be written in the form

$$b_n = \sum_{d|n} (-1)^{\frac{n}{d}+1} a_d, \quad n \in \mathbb{N}. \tag{6}$$

Note that each $n \in \mathbb{N}$ can be written in the form

$$n = 2^k m, \tag{7}$$

where $k \in \mathbb{N}_0$ and $m \in \mathbb{N}$ is an odd number.

First, assume that $m = 1$. Then, $n = 2^k$, for some $k \in \mathbb{N}$, and we have

$$b_{2^k} = \sum_{d|2^k} (-1)^{\frac{2^k}{d}+1} a_d = a_{2^k} - \sum_{d|2^k, d<2^k} a_d, \tag{8}$$

that is

$$a_{2^k} = 1 + \sum_{d|2^k, d<2^k} a_d.$$

From this it follows that

$$a_{2^k} - a_{2^{k-1}} = \Bigg(1 + \sum_{d|2^k, d<2^k} a_d\Bigg) - \Bigg(1 + \sum_{d|2^k, d<2^{k-1}} a_d\Bigg) = a_{2^{k-1}},$$

that is,

$$a_{2^k} = 2a_{2^{k-1}}. \tag{9}$$

From (9) it follows that

$$a_{2^k} = 2^{k-1} a_2. \tag{10}$$

From (1) with $p = 2$, we have

$$a_2 = a_1 + 1. \tag{11}$$

Combining (10), (11) and the assumption $a_1 = 1$, we get

$$a_{2^k} = 2^k, \quad k \in \mathbb{N}_0. \tag{12}$$

Now, assume that n is odd, that is, $k = 0$ in (7), so that $n = m$. Then all its divisors are also odd, so from (6) it follows that

$$b_m = \sum_{d|m} a_d, \quad m \in \mathbb{N}.$$

By using Lemma 1 and (5), we get

$$a_m = \sum_{d|m} \mu(d) b_{\frac{m}{d}} = \sum_{d|m} \mu(d), \tag{13}$$

for each odd $m \in \mathbb{N}$.

From (4) and (13), we have

$$a_m = 0, \tag{14}$$

for every odd $m \in \mathbb{N}_3$.

Now assume that n is even and such that (7) holds for some $k \in \mathbb{N}$ and $m \in \mathbb{N}_3$ odd. We have

$$b_{2^k m} = \sum_{d|2^k m} (-1)^{\frac{2^k m}{d}+1} a_d. \tag{15}$$

Let $k=1$ and $m \in \mathbb{N}_3$ is odd. Then, from (15) we have

$$b_{2m} = \sum_{d|2m} (-1)^{\frac{2m}{d}+1} a_d = \sum_{d|2m, d=2l} (-1)^{\frac{2m}{d}+1} a_d + \sum_{d|m} (-1)^{\frac{2m}{d}+1} a_d,$$

from which along with (14) it follows that

$$b_{2m} = \sum_{d|2m, d=2l} (-1)^{\frac{2m}{d}+1} a_d - a_1 = \sum_{d|2m, d=2l} (-1)^{\frac{m}{l}+1} a_d - 1 = \sum_{l|m} a_{2l} - 1.$$

Let

$$c_m^{(2)} := b_{2m} + 1 \quad \text{and} \quad a_l^{(2)} = a_{2l},$$

then we have

$$2 = c_m^{(2)} = \sum_{l|m} a_l^{(2)}.$$

So, by Lemma 1 and (4), we have

$$a_m^{(2)} = \sum_{l|m} \mu(l) c_{\frac{m}{l}}^{(2)} = 2 \sum_{l|m} \mu(l) = 0,$$

for $m \in \mathbb{N}_3$, that is,

$$a_{2m} = 0, \tag{16}$$

for every odd $m \in \mathbb{N}_3$.

Let $k=2$ and $m \in \mathbb{N}_3$. Then, we have

$$\begin{aligned} b_{4m} &= \sum_{d|4m} (-1)^{\frac{4m}{d}+1} a_d \\ &= \sum_{d|4m, d=4l} (-1)^{\frac{4m}{d}+1} a_d + \sum_{d|4m, d=2l} (-1)^{\frac{4m}{d}+1} a_d + \sum_{d|m} (-1)^{\frac{4m}{d}+1} a_d, \end{aligned} \tag{17}$$

where in the first two sums in (17), l is an odd number.

From (17) along with (12), (14) and (16) it follows that

$$\begin{aligned} b_{4m} &= \sum_{d|4m, d=4l} (-1)^{\frac{4m}{d}+1} a_d - a_2 - a_1 \\ &= \sum_{d|4m, d=4l} (-1)^{\frac{m}{l}+1} a_d - 3 = \sum_{l|m} a_{4l} - 3. \end{aligned}$$

Let

$$c_m^{(4)} := b_{4m} + 3 \quad \text{and} \quad a_l^{(4)} = a_{4l},$$

then we have

$$4 = c_m^{(4)} = \sum_{l|m} a_l^{(4)}.$$

So, by Lemma 1 and (4), we have

$$a_m^{(4)} = \sum_{l|m} \mu(l) c_{\frac{m}{l}}^{(4)} = 4 \sum_{l|m} \mu(l) = 0,$$

for each odd $m \in \mathbb{N}_3$, that is,

$$a_{4m} = 0, \tag{18}$$

for every odd $m \in \mathbb{N}_3$.

Assume that we have proved

$$a_{2^s m} = 0, \tag{19}$$

for every odd $m \in \mathbb{N}_3$ and every $1 \leqslant s \leqslant k-1$, for some fixed $k \in \mathbb{N}_3$.

Then, we have

$$\begin{aligned} b_{2^k m} &= \sum_{d|2^k m} (-1)^{\frac{2^k m}{d}+1} a_d \\ &= \sum_{d|2^k m, d=2^k l} (-1)^{\frac{2^k m}{d}+1} a_d + \sum_{d|2^k m, d=2^{k-1} l} (-1)^{\frac{2^k m}{d}+1} a_d + \cdots \\ &\quad \cdots + \sum_{d|2^k m, d=2l} (-1)^{\frac{2^k m}{d}+1} a_d + \sum_{d|m} (-1)^{\frac{2^k m}{d}+1} a_d, \end{aligned} \tag{20}$$

where l in the above sums is an odd number.

From (20) along with (12) and (19) it follows that

$$\begin{aligned} b_{2^k m} &= \sum_{d|2^k m, d=2^k l} (-1)^{\frac{2^k m}{d}+1} a_d - a_{2^{k-1}} - \cdots - a_2 - a_1 \\ &= \sum_{d|2^k m, d=2^k l} (-1)^{\frac{m}{l}+1} a_d - \sum_{j=0}^{k-1} 2^j = \sum_{l|m} a_{2^k l} - 2^k + 1. \end{aligned}$$

Let

$$c_m^{(2^k)} := b_{2^k m} + 2^k - 1 \quad \text{and} \quad a_l^{(2^k)} = a_{2^k l},$$

then we have

$$2^k = c_m^{(2^k)} = \sum_{l|m} a_l^{(2^k)}.$$

From this, Lemma 1 and (4) we get

$$a_m^{(2^k)} = \sum_{l|m} \mu(l) c_{\frac{m}{l}}^{(2^k)} = 2^k \sum_{l|m} \mu(l) = 0,$$

that is,

$$a_{2^k m} = 0, \tag{21}$$

for every odd $m \in \mathbb{N}_3$.

The above inductive argument shows that

$$a_{2^s m} = 0, \tag{22}$$

for every $s \in \mathbb{N}_0$ and odd $m \in \mathbb{N}_3$.

The relations in (12) and (22) give the formulas for the recursive relation (1) with the initial value $a_1 = 1$. □

REMARK 1. The part of the above proof for the case $k = 2$ and $m \in \mathbb{N}_3$, could have been omitted, but we left it for some presentational reasons, and the benefit of the reader.

REMARK 2. Note that instead of the sequence $(b_n)_{n\in\mathbb{N}}$ defined in (5) we can consider the sequence

$$b_n := c, \quad n \in \mathbb{N},$$

where $c \in \mathbb{R} \setminus \{0\}$, that is, we can consider the recursive relation

$$c + \sum_{d|n} (-1)^{\frac{n}{d}} a_d = 0,$$

(the sequence $\widehat{a}_n := a_n/c$ satisfies equation (1)).

REMARK 3. From formulas (10) and (11) we see that the initial value a_1 need not be specified if we assume that (1) holds for $n \in \mathbb{N}_2$. In this case we have

$$a_{2^k} = 2^{k-1}(a_1 + 1),$$

for $k \in \mathbb{N}$.

REMARK 4. Note that unlike the recursive relations in other above-mentioned references, relation (1) is of a quite different type. Namely, each member of the sequence defined by (1) is obtained by several previous ones, but their number is not fixed (it can vary from one value of the index n to another, and can also be arbitrary large).

3. Solution to Problem 1 by using Dirichlet series

Here we present our second solution to Problem 1, which is based on a use of generating functions. Namely, we use the Dirichlet series

$$\sum_{n=1}^{\infty} \frac{a_n}{n^s}, \tag{23}$$

which are also useful tools in multiplicative number theory. We may assume that $s > 1$.

Recall that the Dirichlet product of the two Dirichlet series

$$\sum_{k=1}^{\infty} \frac{a_k}{k^s} \quad \text{and} \quad \sum_{l=1}^{\infty} \frac{b_l}{l^s}$$

is

$$\sum_{k=1}^{\infty} \frac{a_k}{k^s} \cdot \sum_{l=1}^{\infty} \frac{b_l}{l^s} = \sum_{n=1}^{\infty} \frac{c_n}{n^s}, \tag{24}$$

where

$$c_n = \sum_{kl=n} a_k b_l = \sum_{d|n} a_d b_{\frac{n}{d}}, \tag{25}$$

(see, e.g., [23, p. 119]).

Second solution to Problem 1. Let $(a_n)_{n\in\mathbb{N}}$ be the sequence defined in Problem 1 and $b_n = (-1)^n$. Then from (24) we have

$$\sum_{k=1}^{\infty} \frac{a_k}{k^s} \cdot \sum_{l=1}^{\infty} \frac{(-1)^l}{l^s} = \sum_{n=1}^{\infty} \frac{\sum_{d|n} a_d (-1)^{\frac{n}{d}}}{n^s}. \tag{26}$$

Note that

$$\sum_{l=1}^{\infty} \frac{(-1)^l}{l^s} = -\left(1 - \frac{1}{2^{s-1}}\right)\zeta(s). \tag{27}$$

By using (1) and (27) in (26), we obtain

$$\left(1 - \frac{1}{2^{s-1}}\right)\zeta(s) \sum_{k=1}^{\infty} \frac{a_k}{k^s} = \zeta(s),$$

from which it follows that

$$\sum_{k=1}^{\infty} \frac{a_k}{k^s} = \left(1 - \frac{1}{2^{s-1}}\right)^{-1}. \tag{28}$$

Using the formula

$$\frac{1}{1-q} = \sum_{j=0}^{\infty} q^j$$

for $|q| < 1$, in (28), we have

$$\sum_{k=1}^{\infty} \frac{a_k}{k^s} = \sum_{l=0}^{\infty} \frac{1}{2^{l(s-1)}} = \sum_{l=0}^{\infty} \frac{2^l}{2^{ls}}, \tag{29}$$

from which the formulas for the sequence $(a_k)_{k\in\mathbb{N}}$ in (2) follow. □

Conclusion

Here we give two solutions to Problem 1, and present some theory which lies behind the recursive relation in (1). The methods and ideas presented here could be useful is solving some related recursive relations, and could motivate some investigations in the direction.

Acknowledgement. I would like to express my sincere thanks to Professor W. Zudilin for a nice conversation on the topic.

Use of AI tools declaration. The author declares he has not used Artificial Intelligence (AI) tools in the creation of this article.

Conflicts of interest. The author declares no conflicts of interest.

REFERENCES

[1] V. ABRAMOVICH, *Zadacha M624*, (5) (1980), 36, (in Russian).
[2] V. ABRAMOVICH, *Reshenie Zadachi M624*, (3) (1981), 27–28, (in Russian).
[3] I. BAJO AND E. LIZ, *Global behaviour of a second-order nonlinear difference equation*, J. Differ. Equat. Appl. **17** (10) (2011), 1471–1486.
[4] M. I. BASHMAKOV, B. M. BEKKER AND V. M. GOL'HOVOI, *Zadachi po Matematike. Algebra i Analiz*, Nauka, Moskva, 1982, (in Russian).
[5] L. BERG AND S. STEVIĆ, *On the asymptotics of the difference equation* $y_n(1+y_{n-1}\cdots y_{n-k+1}) = y_{n-k}$, J. Difference Equ. Appl. **17** (4) (2011), 577–586.
[6] D. BERNOULLI, *Observationes de seriebus quae formantur ex additione vel substractione quacunque terminorum se mutuo consequentium, ubi praesertim earundem insignis usus pro inveniendis radicum omnium aequationum algebraicarum ostenditur*, Commentarii Acad. Petropol. III, 1728 (1732), 85–100, (in Latin).
[7] B. P. DEMIDOVICH AND I. A. MARON, *Computational Mathematics*, Mir Publishers, Moscow, 1973.
[8] A. DE MOIVRE, *Miscellanea analytica de seriebus et quadraturis*, J. Tonson & J. Watts, Londini, 1730, (in Latin).
[9] T. FORT, *Finite Differences and Difference Equations in the Real Domain*, Oxford Univ. Press, London, 1948.
[10] C. JORDAN, *Calculus of Finite Differences*, Chelsea Publishing Company, New York, 1965.
[11] V. A. KRECHMAR, *A Problem Book in Algebra*, Mir Publishers, Moscow, 1974.
[12] S. F. LACROIX, *Traité des Differénces et des Séries*, J. B. M. Duprat, Paris, 1800, (in French).

[13] J.-L. LAGRANGE, *OEuvres*, t. II, Gauthier-Villars, Paris, 1868, (in French).

[14] P. S. LAPLACE, *Recherches sur l'intégration des équations différentielles aux différences finies et sur leur usage dans la théorie des hasards*, Mémoires de l' Académie Royale des Sciences de Paris 1773, t. VII, (1776) (Laplace OEuvres, VIII, 69–197, 1891), (in French).

[15] L. M. MILNE-THOMSON, *The Calculus of Finite Differences*, MacMillan and Co., London, 1933.

[16] D. S. MITRINOVIĆ AND D. D. ADAMOVIĆ, *Nizovi i Redovi*, Naučna Knjiga, Beograd, Serbia, 1980, (in Serbian).

[17] D. S. MITRINOVIĆ AND J. D. KEČKIĆ, *Metodi Izračunavanja Konačnih Zbirova*, Naučna Knjiga, Beograd, 1984, (in Serbian).

[18] G. PAPASCHINOPOULOS AND C. J. SCHINAS, *Invariants for systems of two nonlinear difference equations*, Differ. Equ. Dyn. Syst. **7** (1999), 181–196.

[19] G. PAPASCHINOPOULOS AND C. J. SCHINAS, *Invariants and oscillation for systems of two nonlinear difference equations*, Nonlinear Anal. Theory Methods Appl. **46** (2001), 967–978.

[20] G. PAPASCHINOPOULOS AND C. J. SCHINAS, *On the dynamics of two exponential type systems of difference equations*, Comput. Math. Appl. **64** (2012), 2326–2334.

[21] G. PAPASCHINOPOULOS, C. J. SCHINAS, AND G. STEFANIDOU, *On a k-order system of Lyness-type difference equations*, Adv. Difference Equ. **2007**, Article No. 31272, (2007).

[22] G. PAPASCHINOPOULOS AND G. STEFANIDOU, *Asymptotic behavior of the solutions of a class of rational difference equations*, Inter. J. Difference Equations **5** (2) (2010), 233–249.

[23] G. POLYA AND G. SZEGÖ, *Problems and Theorems in Analysis II*, Springer, Berlin, 1976.

[24] M. H. RHOUMA, *The Fibonacci sequence modulo π, chaos and some rational recursive equations*, J. Math. Anal. Appl. **310** (2005), 506–517.

[25] J. RIORDAN, *Combinatorial Identities*, John Wiley & Sons Inc., New York-London-Sydney, 1968.

[26] C. SCHINAS, *Invariants for difference equations and systems of difference equations of rational form*, J. Math. Anal. Appl. **216** (1997), 164–179.

[27] S. STEVIĆ, *On the recursive sequence $x_{n+1} = \alpha_n + (x_{n-1}/x_n)$ II*, Dyn. Contin. Discrete Impuls. Syst. **10a** (6) (2003), 911–916.

[28] S. STEVIĆ, *Boundedness character of a class of difference equations*, Nonlinear Anal. TMA **70** (2009), 839–848.

[29] S. STEVIĆ, *Difference equations and nonexistence of solutions of an integral equation*, Appl. Math. Comput. **219** (2012), 1998–2003.

[30] S. STEVIĆ, *On the system of difference equations $x_n = c_n y_{n-3}/(a_n + b_n y_{n-1} x_{n-2} y_{n-3})$, $y_n = \gamma_n x_{n-3}/(\alpha_n + \beta_n x_{n-1} y_{n-2} x_{n-3})$*, Appl. Math. Comput. **219** (2013), 4755–4764.

[31] S. STEVIĆ, J. DIBLIK, B. IRIČANIN AND Z. ŠMARDA, *On some solvable difference equations and systems of difference equations*, Abstr. Appl. Anal. Vol. 2012, Article ID 541761, (2012), 11 pp.

[32] S. STEVIĆ, J. DIBLIK, B. IRIČANIN AND Z. ŠMARDA, *On a third-order system of difference equations with variable coefficients*, Abstr. Appl. Anal. Vol. 2012, Article ID 508523, (2012), 22 pages.

[33] S. STEVIĆ, J. DIBLIK, B. IRIČANIN AND Z. ŠMARDA, *On a solvable system of rational difference equations*, J. Difference Equ. Appl. **20** (5–6) (2014), 811–825.

[34] S. STEVIĆ, J. DIBLIK, B. IRIČANIN AND Z. ŠMARDA, *Solvability of nonlinear difference equations of fourth order*, Electron. J. Differential Equations. Vol. 2014, Article No. 264, (2014), 14 pages.

[35] S. STEVIĆ, B. IRIČANIN AND W. KOSMALA, *More on a hyperbolic-cotangent class of difference equations*, Math. Methods Appl. Sci. **42** (2019), 2974–2992.

[36] S. STEVIĆ, B. IRIČANIN, W. KOSMALA AND Z. ŠMARDA, *Note on the bilinear difference equation with a delay*, Math. Methods Appl. Sci. **41** (2018), 9349–9360.

[37] S. STEVIĆ, B. IRIČANIN AND Z. ŠMARDA, *On a product-type system of difference equations of second order solvable in closed form*, J. Inequal. Appl. Vol. 2015, Article No. 327, (2015), 15 pages.

[38] S. STEVIĆ, B. IRIČANIN AND Z. ŠMARDA, *Two-dimensional product-type system of difference equations solvable in closed form*, Adv. Difference Equ. Vol. 2016, Article No. 253, (2016), 20 pages.

[39] I. M. VINOGRADOV, *Osnovy Teorii Chisel*, Izdanie Devyatoe, Nauka, Moskva, 1981, (in Russian). English translation: An Introduction to the Theory of Numbers, Pergamon Press, London & New York, 1955.

(Received January 3, 2025)

Stevo Stević
Mathematical Institute of the Serbian Academy of Sciences and Arts
Knez Mihailova 36/III, 11000 Beograd, Serbia
and
Serbian Academy of Sciences and Arts
Knez Mihailova 35, 11000 Beograd, Serbia
and
Department of Medical Research
China Medical University Hospital, China Medical University
Taichung 40402, Taiwan, Republic of China
e-mail: sscite1@gmail.com