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GENERALIZATIONS OF SOME INEQUALITIES OF OSTROWSKI–GRÜSS TYPE

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Abstract. We provide generalizations and improvements of a variety of recent results for the Ostrowski and Simpson inequalities.

1. Introduction

Ostrowski [4, p. 468] proved the following integral inequality which is central to numerical analysis.

THEOREM A. *Suppose* $I \subseteq \mathbf{R}$ *is an interval with* $a, b \in int(I)$ $(a < b)$. *If* $f : I \to \mathbf{R}$ *is differentiable on int(* I *) and* $|f'(x)| \leqslant M$ *for all* $x \in [a, b]$, *then*

$$\left| f(x) - \frac{1}{b-a} \int_a^b f(t)dt \right| \leqslant \left(\frac{1}{4} + \frac{(x - \frac{a+b}{2})^2}{(b-a)^2} \right) (b-a)M, \quad \forall x \in [a, b].$$

There have been several extensions of this result. It has been shown in [3] that if f is differentiable on (a, b) and f' is integrable and satisfies

$$\gamma \leqslant f'(t) \leqslant \Gamma \quad \text{for all} \quad t \in [a, b],$$

then

$$\left| f(x) - \frac{f(b) - f(a)}{b-a} \left(x - \frac{a+b}{2} \right) - \frac{1}{b-a} \int_a^b f(t)dt \right| \leqslant \frac{1}{4\sqrt{3}} (b-a)(\Gamma - \gamma) \quad (1)$$

for all $x \in [a, b]$. A version of this estimate occurs in [1], but without the $\sqrt{3}$ on the right. A more general left-hand side is treated in [2]. Fedotov and Dragomir have shown that if f has a first derivative on (a, b) and $\gamma \leqslant f'(t) \leqslant \Gamma$ for all $t \in (a, b)$, then

$$\left| (C - A)f(a) + (b - a - B + A)f(x) + (B - C)f(b) - \int_a^b f(t)dt \right|$$
$$\leqslant \frac{1}{4}(\Gamma - \gamma)(M_x - m_x)(b - a). \quad (2)$$

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Here $A, B \in \mathbf{R}$, $M_x = \sup\{p_x(t) : t \in (a,b)\}$, $m_x = \inf\{p_x(t) : t \in (a,b)\}$,

$$C = \frac{1}{2(b-a)}\left[(x-a)(x-a+2A) - (x-b)(x-b+2B)\right]$$

and p_x is defined by

$$p_x(t) = \begin{cases} t-a+A, & t \in [a,x] \\ t-b+B, & t \in (x,b] \end{cases}.$$

Here we give generalizations of these results using sequences of harmonic polynomials. We apply our results to some numerical integration rules and derive bounds better than those currently known.

In Section 2 we give a basic identity and inequalities that are used in the sequel. Section 3 treats functions possessing an n-th derivative. In Section 4 we calculate bounds for the error in some numerical integration rules for the case $n = 1$. It is shown that our generalization gives better bounds than some results derived recently in [2]. We conclude with a discussion of Simpson's rule.

2. Preliminaries

Let $\{P_n\}$ and $\{Q_n\}$ be sequences of harmonic polynomials, that is, polynomials satisfying

$$P_n'(x) = P_{n-1}(x), \quad Q_n'(x) = Q_{n-1}(x), \quad \text{with } P_0(x) = Q_0(x) = 1.$$

If f has a derivative of order n, then setting

$$S_n(t,x) = \begin{cases} P_n(t), & t \in [a,x] \\ Q_n(t), & t \in (x,b] \end{cases}$$

and using integration by parts we obtain the identity

$$(-1)^n \int_a^b S_n(t,x) f^{(n)}(t)\,dt = I_n(x),$$

where

$$I_n(x) := \int_a^b f(t)\,dt + \sum_{k=1}^{n} (-1)^k \left[Q_k(b) f^{(k-1)}(b) + (P_k(x) - Q_k(x)) f^{(k-1)}(x) - P_k(a) f^{(k-1)}(a)\right].$$

Various generalizations of Theorem A have been obtained in [5] using such polynomials.

The following inequality of Grüss type, established in [3], plays a key role in this paper.

THEOREM B. *Let $f, g : [a,b] \to \mathbf{R}$ be integrable functions such that fg is also integrable. If $\gamma \leqslant g(x) \leqslant \Gamma$ for all $x \in [a,b]$, then*

$$|T(f,g)| \leqslant \frac{1}{2}\sqrt{T(f,f)}(\Gamma - \gamma),$$

where

$$T(f,g) = \frac{1}{b-a}\int_a^b f(x)g(x)dx - \frac{1}{(b-a)^2}\int_a^b f(x)dx\int_a^b g(x)dx.$$

This theorem was proved *via* the following helpful inequality.
If f is integrable on $[a,b]$ and $\varphi \leqslant f(x) \leqslant \phi \ \ \forall x \in [a,b]$, then

$$T(f,f) \leqslant \frac{1}{4}(\phi - \varphi)^2. \tag{3}$$

3. Functions with n-th derivative

THEOREM 1. *Suppose $f : [a,b] \to \mathbf{R}$ is such that $f^{(n)}$ is integrable with*

$$\gamma_n \leqslant f^{(n)}(t) \leqslant \Gamma_n \quad \textit{for all } t \in [a,b].$$

Put

$$U_n(x) := [Q_{n+1}(b) - Q_{n+1}(x) + P_{n+1}(x) - P_{n+1}(a)]/(b-a).$$

Then for any $x \in [a,b]$,

$$\left| I_n(x) - (-1)^n U_n(x)\left[f^{(n-1)}(b) - f^{(n-1)}(a)\right]\right| \leqslant \frac{1}{2}K(\Gamma_n - \gamma_n)(b-a),$$

where

$$K := \left\{\frac{1}{b-a}\left[\int_a^x P_n^2(t)dt + \int_x^b Q_n^2(t)dt\right] - [U_n(x)]^2\right\}^{\frac{1}{2}}.$$

Proof. From the definition of S_n we have

$$\begin{aligned}
&\left| I_n(x) - (-1)^n U_n(x)\left[f^{(n-1)}(b) - f^{(n-1)}(a)\right]\right| \\
&\quad = (b-a)\left|\frac{1}{b-a}\int_a^b S_n(t,x)f^{(n)}(t)dt - \frac{1}{(b-a)^2}\int_a^b S_n(t,x)dt \int_a^b f^{(n)}(t)dt\right| \\
&\quad = (b-a)|T(S_n,f^{(n)})| \\
&\quad \leqslant \frac{1}{2}\sqrt{T(S_n,S_n)}(b-a)(\Gamma_n - \gamma_n).
\end{aligned}$$

The desired result follows, since $K = \sqrt{T(S_n,S_n)}$. □

By using particular harmonic polynomials we can obtain a variety of results which generalize known approximations.

COROLLARY 1. *Under the assumptions of Theorem 1,*

$$\left|\frac{1}{b-a}\int_a^b f(t)dt + \sum_{k=1}^{n}\frac{(-1)^k}{k!(b-a)}\left[(b-B)^k f^{(k-1)}(b)+((x-A)^k-(x-B)^k)f^{(k-1)}(x)-(a-A)^k f^{(k-1)}(a)\right] - \frac{(-1)^n(f^{(n-1)}(b)-f^{(n-1)}(a))}{(n+1)!(b-a)^2}\left[(b-B)^{n+1}-(x-B)^{n+1}+(x-A)^{n+1}-(a-A)^{n+1}\right]\right| \leqslant \frac{1}{2}(\Gamma_n-\gamma_n)K_1$$

for all $x\in[a,b]$ *and* $A,B\in\mathbf{R}$, *where*

$$K_1 := \frac{1}{n!}\left(\frac{(x-A)^{2n+1}-(a-A)^{2n+1}+(b-B)^{2n+1}-(x-B)^{2n+1}}{(2n+1)(b-a)} - \left(\frac{(b-B)^{n+1}-(x-B)^{n+1}+(x-A)^{n+1}-(a-A)^{n+1}}{(n+1)(b-a)}\right)^2\right)^{1/2}.$$

Proof. This is just the result of Theorem 1 with the polynomial choices $P_n(x) = (x-A)^n/n!$ and $Q_n(x) = (x-B)^n/n!$ □

COROLLARY 2. *Under the assumptions of Theorem 1,*

$$\left|\frac{1}{b-a}\int_a^b f(t)dt + \sum_{k=1}^{n}\frac{(-1)^k(b-a)^{k-1}}{k!(p+q)^k}\times \left[q^k(f^{(k-1)}(b)-(-1)^k f^{(k-1)}(a)) + \left(\frac{p-q}{2}\right)^k[1-(-1)^k]f^{(k-1)}\left(\frac{a+b}{2}\right)\right] - \frac{(-1)^n(b-a)^{n-1}(1+(-1)^n)}{(n+1)!(p+q)^{n+1}}\left[q^{n+1}+\left(\frac{p-q}{2}\right)^{n+1}\right]\left[f^{(n-1)}(b)-f^{(n-1)}(a)\right]\right| \frac{1}{2}K_2(\Gamma_n-\gamma_n) \tag{4}$$

for $p,q\in\mathbf{R}$ *(* $p+q>0$ *), where*

$$K_2 := \frac{(b-a)^n}{n!(p+q)^n}\left(\frac{2(q^{2n+1}+(\frac{p-q}{2})^{2n+1})}{(p+q)(2n+1)} - 2[1+(-1)^n]\frac{(q^{n+1}+(\frac{p-q}{2})^{n+1})^2}{(n+1)^2(p+q)^2}\right)^{1/2}.$$

Proof. This is Corollary 1 with $A=\dfrac{pa+qb}{p+q}$, $x=\dfrac{a+b}{2}$ and $B=\dfrac{qa+pb}{p+q}$, where $p,q\in\mathbf{R}$ and $p+q>0$. □

For $x=b$, Theorem 1 gives the following.

THEOREM 2. *If f satisfies assumptions of Theorem 1, then*

$$\left|\frac{1}{b-a}\int_a^b f(t)dt+\sum_{k=1}^{n}\frac{(-1)^k}{b-a}\left[P_k(b)f^{(k-1)}(b)-P_k(a)f^{(k-1)}(a)\right]\right.$$
$$\left.-\frac{(-1)^n}{(b-a)^2}\big(P_{n+1}(b)-P_{n+1}(a)\big)\big(f^{(n-1)}(b)-f^{(n-1)}(a)\big)\right|$$
$$\leqslant \frac{1}{2}K_3(\Gamma_n-\gamma_n),$$

where

$$K_3:=\left(\frac{1}{b-a}\int_a^b P_n^2(t)dt-\left(\frac{P_{n+1}(b)-P_{n+1}(a)}{b-a}\right)^2\right)^{1/2}.$$

The choice $P_n(x)=[x-(a+b)/2]^n/n!$ provides the following corollary.

COROLLARY 3. *Under the assumptions of Theorem 1,*

$$\left|\frac{1}{b-a}\int_a^b f(t)dt+\sum_{k=1}^{n}\frac{(-1)^k}{2^k k!}(b-a)^{k-1}\left(f^{(k-1)}(b)-(-1)^k f^{(k-1)}(a)\right)\right.$$
$$\left.-\frac{(-1)^n}{2^{n+1}(n+1)!}(b-a)^{n-1}(1+(-1)^n)(f^{(n-1)}(b)-f^{(n-1)}(a))\right|$$
$$\leqslant \frac{1}{2}(\Gamma_n-\gamma_n)K_4,$$

where

$$K_4:=\frac{(b-a)^n}{n!2^n}\left(\frac{1}{2n+1}-\frac{(1+(-1)^n)^2}{(n+1)^2}\right)^{1/2}.$$

4. Functions with first derivatives

In this section we consider the case $n=1$ and compare our results with those given in [2].

On putting $n=1$ in Corollary 1, we obtain the following result.

THEOREM 3. *Let f be differentiable on $[a,b]$ and such that $f'\in L_\infty[a,b]$ with*

$$\gamma\leqslant f'(t)\leqslant\Gamma \quad \textit{for all} \quad t\in[a,b].$$

Set

$$V_n(x):=(b-B)^n-(x-B)^n+(x-A)^n-(a-A)^n.$$

Then for any $x\in[a,b]$,

$$\left|\int_a^b f(t)dt-[(b-B)f(b)+(B-A)f(x)-(a-A)f(a)]+\frac{f(b)-f(a)}{2(b-a)}V_2(x)\right|$$
$$\leqslant \frac{1}{2}(\Gamma-\gamma)(b-a)K_5, \tag{5}$$

where

$$K_5 := \left(\frac{V_3(x)}{3(b-a)} - \left(\frac{V_2(x)}{2(b-a)} \right)^2 \right)^{1/2}.$$

It is interesting to compare this result with (2), which was derived in [2]. In [2] $M_x - m_x$ is expressed in terms of a, b and x and has the following complicated form.

1. If $B-A \leqslant 0$, then $M_x - m_x = (b-a)-(B-A)$.
2. If $B-A > 0$ there are three subcases.
   a) If $0 \leqslant B-A \leqslant (b-a)/2$, then

$$M_x - m_x = \begin{cases} -x+b & \text{for} \quad a \leqslant x \leqslant a+(B-A) \\ (b-a)-(B-A) & \text{for} \quad a+(B-A) < x \leqslant b-(B-A) \\ x-a & \text{for} \quad b-(B-A) < x \leqslant b. \end{cases}$$

   b) If $(b-a)/2 < B-A \leqslant b-a$, then

$$M_x - m_x = \begin{cases} -x+b & \text{for} \quad a \leqslant x < b-(B-A) \\ B-A & \text{for} \quad b-(B-A) \leqslant x < a+(B-A) \\ x-a & \text{for} \quad a+(B-A) \leqslant x \leqslant b. \end{cases}$$

   c) If $B-A > b-a$, then $M_x - m_x = (B-A)-(b-a)$.

If we replace A by $a-A$ and B by $b-B$ in (5), then the left-hand sides of (5) and (2) coincide. The constant K_5 in (5) becomes

$$\left(\frac{B^3-(x-b+B)^3+(x-a+A)^3-A^3}{3(b-a)} - \left(\frac{B^2-(x-b+B)^2+(x-a+A)^2-A^2}{2(b-a)} \right)^2 \right)^{1/2}$$

or

$$\begin{aligned} &\left(\frac{1}{b-a} \left(\int_a^x (t-a+A)^2 dt + \int_x^b (t-b+B)^2 dt \right) \right. \\ &\qquad \left. - \left(\frac{1}{b-a} \left(\int_a^x (t-a+A) dt + \int_x^b (t-b+B) dt \right) \right)^2 \right)^{1/2} \\ &= \left(\frac{1}{b-a} \int_a^b p_x^2(t) dt - \left(\frac{1}{b-a} \int_a^b p_x(t) dt \right)^2 \right)^{1/2} \\ &= \sqrt{T(p_x, p_x)}, \end{aligned}$$

where p_x is as in the Introduction. By (3) this is less or equal to $(M_x - m_x)/2$, the constant on the right-hand side in (2). Hence our result provides an improved and simpler error estimate of the left-hand side of (2).

It is interesting to see the form of these results when $x = (a+b)/2$. Setting $n = 1$ and $x = (a+b)/2$ in Corollary 1 and replacing A by $a-A$ and B by $b-B$ yield the following result.

THEOREM 4. *Suppose $f : [a,b] \to \mathbf{R}$ is differentiable and such that $\gamma \leqslant f'(t) \leqslant \Gamma$ for all $t \in [a,b]$. Then for any real numbers A and B,*

$$\left| \frac{1}{b-a}\int_a^b f(t)dt - \frac{1}{b-a}\left[\frac{B-A}{2}f(a) + (b-a-B+A)f\left(\frac{a+b}{2}\right) + \frac{B-A}{2}f(b)\right]\right| \leqslant \frac{1}{2}(\Gamma-\gamma)\sqrt{\frac{(b-a)^2}{12} - \frac{(b-a)(B-A)}{4} + \frac{(B-A)^2}{4}}. \tag{6}$$

The left-hand side of this inequality occurs in [2], but with the upper bound

$$\frac{1}{2}(\Gamma-\gamma)(b-a-(B-A)), \tag{7}$$

where $0 \leqslant B-A \leqslant (b-a)/2$.

A simple calculation yields that if $0 \leqslant B-A \leqslant (b-a)/2$, then the right-hand side of (6) is smaller than the constant (7). Our result is valid without any conditions on A and B.

In all the following formulæ we suppose that f satisfies the assumptions of Theorem 3.

Putting $n=1$, $p=1$, $q=0$ in (4) or $A=B$ in (6) provides an error estimate

$$\left|\int_a^b f(t)dt - (b-a)f\left(\frac{a+b}{2}\right)\right| \leqslant \frac{1}{4\sqrt{3}}(\Gamma-\gamma)(b-a)^2$$

for the error in the simple midpoint rule.

Putting in (4) $n=1$, $p=3$, $q=1$ or $B-A=(b-a)/2$ in (6) supplies the estimate

$$\left|\int_a^b f(t)dt - \frac{b-a}{4}\left[f(a) + 2f\left(\frac{a+b}{2}\right) + f(b)\right]\right| \leqslant \frac{1}{8\sqrt{3}}(\Gamma-\gamma)(b-a)^2$$

for the error in the simple quadrature rule.

The estimates for the errors for those rules with the formulæ in [2] give the larger constants $(\Gamma-\gamma)(b-a)^2/4$ and $(\Gamma-\gamma)(b-a)^2/8$ respectively.

5. Simpson's rule

One of the best-known results in numerical integration is Simpson's inequality, which states that if $f^{(4)}$ exists and is bounded on (a,b), then

$$\left|\int_a^b f(t)dt - \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]\right| \leqslant \frac{1}{2880}(b-a)^5\|f^{(4)}\|_\infty. \tag{8}$$

We now give some new estimates for the error in Simpson's rule for calculating $\int_a^b f(t)dt$, which is given by the left-hand side of (8). Our estimates use lower-order derivatives of f.

Consider the two sequences of harmonic polynomials

$$
\begin{aligned}
P_0(x) &= 1, & Q_0(x) &= 1,\\
P_1(x) &= x - \frac{5a+b}{6}, & Q_1(x) &= x - \frac{5b+a}{6},\\
P_2(x) &= \frac{1}{2!}(x-a)\left(x - \frac{2a+b}{3}\right), & Q_2(x) &= \frac{1}{2!}(x-b)\left(x - \frac{2b+a}{3}\right),\\
P_3(x) &= \frac{1}{3!}(x-a)^2\left(x - \frac{a+b}{2}\right), & Q_3(x) &= \frac{1}{3!}(x-b)^2\left(x - \frac{b+a}{2}\right),\\
P_4(x) &= \frac{1}{4!}(x-a)^3\left(x - \frac{a+2b}{3}\right), & Q_4(x) &= \frac{1}{4!}(x-b)^3\left(x - \frac{b+2a}{3}\right),\\
P_5(x) &= \frac{1}{5!}(x-a)^4\left(x - \frac{a+5b}{6}\right), & Q_5(x) &= \frac{1}{5!}(x-b)^4\left(x - \frac{b+5a}{6}\right).
\end{aligned}
$$

Application of Theorem 1 with these polynomials provides the next result.

THEOREM 5. *Suppose f is a function such that $f^{(n)}$ is integrable with $\gamma_n \leqslant f^{(n)}(t) \leqslant \Gamma_n$ for all $t \in [a,b]$ ($n \in \{1,2,3\}$). Then for $n = 1,2,3$ we have*

$$\left| \int_a^b f(t)dt - \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \right| \leqslant C_n(\Gamma_n - \gamma_n)(b-a)^{n+1},$$

where

$$C_1 = \frac{1}{12}, \quad C_2 = \frac{1}{24\sqrt{30}}, \quad C_3 = \frac{1}{96\sqrt{105}}.$$

For $n = 4$ we have

$$\left| \int_a^b f(t)dt - \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] + \frac{(b-a)^4(f^{(3)}(b) - f^{(3)}(a))}{2880} \right|$$
$$\leqslant \frac{1}{5760}\sqrt{\frac{11}{14}}(\Gamma_4 - \gamma_4)(b-a)^5.$$

REMARK 1. The case $n = 1$ can also be treated by putting $n = 1$, $p = 5$, $q = 1$ in (4) or $B - A = (b-a)/3$ in (6).

REMARK 2. An estimate for Simpson's rule for $n = 1$ is also provided by [2], but the constant on the right-hand side is $(\Gamma_1 - \gamma_1)(b-a)^2/6$, that is, twice the value derived here.

REMARK 3. Fedotov and Dragomir applied their estimates of the error bound in the Simpson rule to some special functions to obtain inequalities involving various means of positive real numbers. Our corresponding bounds are tighter by at least a factor of 2. For example, in [2] the inequality

$$\left|\frac{2}{3}HL+\frac{1}{3}AL-AH\right| \leqslant \frac{1}{3}(b-a)^2\frac{A^2HL}{G^4}$$

is derived, where A, G, H and L are respectively the arithmetic, geometric, harmonic and logarithmic means of positive real numbers a and b. Application of Theorem 5 and (8) to $f(x) = 1/x$ on the interval (a,b), $(a,b>0)$ reduces the bound on the right-hand side to the demonstrably tighter

$$\min\left\{\frac{(b-a)^2}{6}\frac{A^2HL}{G^4}, \frac{(b-a)^3}{12\sqrt{30}}\frac{AHL(4A^2-G^2)}{G^6}, \frac{(b-a)^4}{4\sqrt{105}}\frac{A^2HL(2A^2-G^2)}{G^8}, \frac{(b-a)^4}{120a^5}AHL\right\}.$$

As the consequence of the estimates for $n=1,2,3$ we have the following result.

COROLLARY 4. *Suppose f satisfies the assumptions of Theorem 5. Then for $n\in\{1,2,3\}$,*

$$\int_a^b f(t)dt = T(\pi,f)+R_n(\pi,f),$$

where π represents the subdivision $\{a=x_0<x_1,<\ldots<x_m=b\}$ of the interval $[a,b]$, $h_i=x_{i+1}-x_i$ and

$$T(\pi,f)=\frac{1}{6}\sum_{i=0}^{m-1}h_i\left[f(x_i)+4f\left(\frac{x_{i+1}+x_i}{2}\right)+f(x_{i+1})\right].$$

The error R_n satisfies

$$|R_n(\pi,f)| \leqslant C_n(\Gamma_n-\gamma_n)\sum_{i=0}^{m-1}h_i^{n+1},$$

where C_n ($n=1,2,3$) is as in Theorem 5.

Proof. Apply Theorem 5 to the intervals $[x_i,x_{i+1}]$ and sum. □

Finally we have the corresponding result for $n=4$.

COROLLARY 5. *Suppose $f:[a,b]\to\mathbf{R}$ is such that $f^{(4)}$ is integrable. Then*

$$\left|\int_a^b f(t)dt-\frac{b-a}{m}\sum_{i=0}^{m-1}\left[f(x_i)+4f\left(\frac{x_{i+1}+x_i}{2}\right)+f(x_{i+1})\right]+\frac{(b-a)^4}{2880m^4}(f^{(3)}(b)-f^{(3)}(a))\right|$$

$$\leqslant \frac{1}{5760}\sqrt{\frac{11}{14}}(\Gamma_4-\gamma_4)\frac{(b-a)^5}{m^4},$$

where $\{a=x_0<x_1<\ldots<x_m=b\}$ is a uniform subdivision of $[a,b]$.

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