ON SOME NEW INTEGRO–DIFFERENTIAL INEQUALITIES
RELATED TO WEYL'S AND HEISENBERG'S TYPE INEQUALITY

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Abstract. In the present paper some new integro-differential inequalities are obtained, which generalize some results of Pachpatte and Pečarić at ec and derive some new generalizations of Weyl and Heisenberg type inequalities.

1. Introduction

The important Weyl's and Heisonberg's uncertain inequality which describe the uncertain principle of quantum mechanics can be stated as following lemmas respectively:

LEMMA 1. [11] *Let f be a real-valued continuously differentiable function on $(0, \infty)$, then*

$$\int_0^\infty f^2(t)dt \leqslant 2 \left(\int_0^\infty t^2 f^2(t)dt \right)^{\frac{1}{2}} \left(\int_0^\infty f'^2(t)dt \right)^{\frac{1}{2}}, \tag{1}$$

where the integrals on the right side of (1) exist.

LEMMA 2. [6] *Let f be a real-valued continuously differentiable function on $(-\infty, \infty)$, then*

$$\int_{-\infty}^\infty f^2(t)dt \leqslant 2 \left(\int_{-\infty}^\infty t^2 f^2(t)dt \right)^{\frac{1}{2}} \left(\int_{-\infty}^\infty f'^2(t)dt \right)^{\frac{1}{2}}, \tag{2}$$

where the integrals on the right side of (2) exist.

Inequalities (1) and (2) are remarkable in terms of simplicity, the large numbers of results to which they lead, and the variety of applications which can be related to them. In past few years some useful generalizations and similar works of (1) and (2) are established by various investigators, see [1–5,7–10] and the references given therein. In

[9] some interesting integro-differential inequalities were established, which claim their origin to the Weyl's inequality. Using a weight function appearing in the right of this inequality, J. Pečarić and Ž. Hanjš [10] give a more general inequality which unified and improved several inequalities given in [9].

The purpose of the present paper is to establish some new integro-differential inequalities which generalise the main results given in [9,10] and derive some new generalizations of Weyl and Heisenberge type inequalities.

2.   Main results

In what follows, we assume that all integrals appearing below exist on the respective domains of their definition.

THEOREM 1. *Let $a, b, p \geqslant 0$ and $q > 1$ be real constants with $a \leqslant b$. Let φ be a nonnegative continuous function defined on (a, b). Let further that f be a real-valued continuously differentiable function defined on (a, b). Then*
(i)

$$\int_a^b \varphi(t)|f(t)|^{p+q}dt \leqslant \int_a^b \varphi(t)|f(t)|^p \left(m(t)|f'(t)| + n(t)\frac{|f(t)|}{b-a} \right)^q dt; \qquad (3)$$

(ii)

$$\int_a^b \varphi(t)|f(t)|^{p+q}dt \leqslant \int_a^b \varphi(t) \left(m(t)|f'(t)| + n(t)\frac{|f(t)|}{b-a} \right)^{p+q} dt, \qquad (4)$$

where $m(t) = (p+q)\dfrac{\Phi_a(t)}{\varphi(t)}, n(t) = \dfrac{\Phi_a(t)}{\varphi(t)} + t - a, \Phi_a(t) = \int_a^t \varphi(s)ds.$

Proof. Constructing the following definite integral and integrating it by parts, we obtain

$$\int_a^b |f(t)|^{p+q} \left[\varphi(t) - \left(\frac{\Phi_a(t)}{b-a} + \frac{t-a}{b-a}\varphi(t) \right) \right] dt$$

$$= \int_a^b |f(t)|^{p+q} d \left[\left(1 - \frac{t-a}{b-a} \right) \Phi_a(t) \right]$$

$$= -\int_a^b \left(1 - \frac{t-a}{b-a} \right) \Phi_a(t)(p+q)|f(t)|^{p+q-1}|f'(t)|(sgnf(t))dt$$

$$\leqslant \int_a^b \left(1 - \frac{t-a}{b-a} \right) \Phi_a(t)(p+q)|f(t)|^{p+q-1}|f'(t)|dt,$$

and hence we have

$$\int_a^b \varphi(t)|f(t)|^{p+q}dt \leqslant \int_a^b \left(1 - \frac{t-a}{b-a} \right) \Phi_a(t)(p+q)|f(t)|^{p+q-1}|f'(t)|dt$$

$$+ \int_a^b \left(\frac{\Phi_a(t)}{b-a} + \frac{t-a}{b-a}\varphi(t) \right) |f(t)|^{p+q}dt. \qquad (*)$$

Rewriting the above inequality we have

$$\int_a^b \varphi(t)|f(t)|^{p+q}dt \leqslant \int_a^b \varphi(t)|f(t)|^{p+q-1}\left(m(t)|f'(t)| + n(t)\frac{|f(t)|}{b-a}\right)dt. \quad (5)$$

(i) From the inequality (5) and using the Hölder's inequality with indices $q, q/(q-1)$, we obtain

$$\int_a^b \varphi(t)|f(t)|^{p+q}dt \leqslant \int_a^b \varphi(t)^{1/q}|f(t)|^{p/q}\left(m(t)|f'(t)| + n(t)\frac{|f(t)|}{b-a}\right)$$
$$\cdot \varphi^{(q-1)/q}(t)|f(t)|^{p+q-1-p/q}dt$$
$$\leqslant \left[\int_a^b \varphi(t)|f(t)|^p \left(m(t)|f'(t)| + n(t)\frac{|f(t)|}{b-a}\right)^q dt\right]^{1/q}$$
$$\cdot \left[\int_a^b \varphi(t)|f(t)|^{p+q}dt\right]^{(q-1)/q}. \quad (6)$$

Now we assume that the left member of (6) is positive, otherwise the desired inequality (3) holds trivially. Thus from (6) we derive

$$\left[\int_a^b \varphi(t)|f(t)|^{p+q}dt\right]^{1/q} \leqslant \left[\int_a^b \varphi(t)|f(t)|^p \left(m(t)|f'(t)| + n(t)\frac{|f(t)|}{b-a}\right)^q dt\right]^{1/q},$$

and therefore, the desired inequality (3) follows.

(ii) Similarly, we rewrite (5) and use the Hölder's inequality with indices $p+q$ and $(p+q)/(p+q-1)$. It follows that

$$\int_a^b \varphi(t)|f(t)|^{p+q}dt \leqslant \int_a^b \varphi(t)^{1/(p+q)}\left(m(t)|f'(t)| + n(t)\frac{|f(t)|}{b-a}\right)$$
$$\cdot \varphi^{(p+q-1)/(p+q)}(t)|f(t)|^{p+q-1}dt$$
$$\leqslant \left[\int_a^b \varphi(t)\left(m(t)|f'(t)| + n(t)\frac{|f(t)|}{b-a}\right)^{p+q} dt\right]^{1/(p+q)}$$
$$\cdot \left[\int_a^b \varphi(t)|f(t)|^{p+q}dt\right]^{(p+q-1)/(p+q)}. \quad (7)$$

We now assume that the left member of (7) is positive, otherwise the desired inequality (4) holds trivially. From (7) we derive

$$\left[\int_a^b \varphi(t)|f(t)|^{p+q}dt\right]^{1/(p+q)} \leqslant \left[\int_a^b \varphi(t)\left(m(t)|f'(t)| + n(t)\frac{|f(t)|}{b-a}\right)^q dt\right]^{1/(p+q)}.$$

The last relation is equivalent to inequality (4). $\square$

By letting $a = 0, \varphi(t) = t^\alpha (\alpha \geqslant 0)$ in Theorem 1, we derive the following

COROLLARY 1. *Let f be a real-valued continuously differentiable function defined on $(0, b)$. Then we have*

(i)

$$\int_0^b t^\alpha |f(t)|^{p+q} dt \leqslant N^q \int_0^b t^{\alpha+q} |f(t)|^p \left(|f'(t)| + \frac{|f(t)|}{b} \right)^q dt; \qquad (8)$$

(ii)

$$\int_0^b t^\alpha |f(t)|^{p+q} dt \leqslant N^{p+q} \int_0^b t^{\alpha+p+q} \left(|f'(t)| + \frac{|f(t)|}{b} \right)^{p+q} dt, \qquad (9)$$

where $N = max\left\{ \dfrac{\alpha+2}{\alpha+1}, \dfrac{p+q}{\alpha+1} \right\}$.

REMARK 1. The conclusions of Corollary 1 is the main results of Pachpatte [9] and here the constant N is better than in [9].

COROLLARY 2. *Let $a = 0$ and the conditions of Theorem 1 be satisfied. If $\varphi(t)$ is also nondecreasing, then*

(i)

$$\int_0^b \varphi(t)|f(t)|^{p+q} dt \leqslant N_1^q \int_0^b t^q \varphi(t)|f(t)|^p \left(|f'(t)| + \frac{|f(t)|}{b} \right)^q dt; \qquad (10)$$

(ii)

$$\int_0^b \varphi(t)|f(t)|^{p+q} dt \leqslant N_1^{p+q} \int_0^b t^{p+q} \varphi(t) \left(|f'(t)| + \frac{|f(t)|}{b} \right)^{p+q} dt, \qquad (11)$$

where $N_1 = max\{p+q, 2\}$.

Proof. If $\varphi(t)$ is nondecreasing function, then

$$m(t) = (p+q)\frac{\Phi_0(t)}{\varphi(t)} = (p+q)\frac{\displaystyle\int_0^t \varphi(s)ds}{\varphi(t)} \leqslant (p+q)t,$$

$$n(t) = \frac{\Phi_0(t)}{\varphi(t)} + t \leqslant t + t = 2t.$$

Substituting the last relations into (3) and (4), we obtain (10) and (11), respectively.

REMARK 2. Letting $\varphi(t) = \ln(1+t)$ and $\varphi(t) = \exp(kt)(k > 0)$ in (10) and (11), respectively, we obtain four interesting inequalities, as follows:

$$\int_0^b \ln(1+t)|f(t)|^{p+q} dt \leqslant N_1^q \int_0^b t^q \ln(1+t)|f(t)|^p \left(|f'(t)| + \frac{|f(t)|}{b} \right)^q dt; \quad (12)$$

$$\int_0^b \ln(1+t)|f(t)|^{p+q} dt \leqslant N_1^{p+q} \int_0^b t^{p+q} \ln(1+t) \left(|f'(t)| + \frac{|f(t)|}{b} \right)^{p+q} dt, \quad (13)$$

and

$$\int_0^b \exp(kt)|f(t)|^{p+q}dt \leqslant N_1^q \int_0^b t^q \exp(kt)|f(t)|^p \left(|f'(t)| + \frac{|f(t)|}{b}\right)^q dt; \qquad (14)$$

$$\int_0^b \exp(kt)|f(t)|^{p+q}dt \leqslant N_1^{p+q} \int_0^b t^{p+q} \exp(kt) \left(|f'(t)| + \frac{|f(t)|}{b}\right)^{p+q} dt. \qquad (15)$$

Letting $b \to \infty$ in Theorem 1 then we obtain

COROLLARY 3. *Let $\varphi(t)$ be a nonnegative continuous function defined on (a, ∞).*
Let f be real-valued continuously differentiable function defined (a, ∞). Then
(i)

$$\int_a^\infty \varphi(t)|f(t)|^{p+q}dt \leqslant (p+q)^q \int_a^\infty \varphi^{1-q}(t)\Phi_a^q(t)|f(t)|^p|f'(t)|^q dt; \qquad (16)$$

(ii)

$$\int_a^\infty \varphi(t)|f(t)|^{p+q}dt \leqslant (p+q)^{p+q} \int_a^\infty \varphi^{1-p-q}(t)\Phi_a^{p+q}(t)|f'(t)|^{p+q}dt. \qquad (17)$$

REMARK 3. When $a = 0, \varphi(t) = t^\alpha(\alpha \geqslant 0$ is a constant), from (16) and (17) we derive Pachpatte's another main result in [9].

THEOREM 2. *Let $p, q, \varphi(t), f(t), \Phi_a(t)$ be as defined in Theorem 1, and $k(t)$ be a positive real-valued function on (a, b). Then*

$$\int_a^b \varphi(t)|f(t)|^{p+q}dt \leqslant \left[\int_a^b \Phi_a(t)|f(t)|^p k^{-q}(t) \left(g(t)|f'(t)| + h(t)\frac{|f(t)|}{b-a}\right)^q dt\right]^{1/q}$$

$$\cdot \left[\int_a^b \Phi_a(t)|f(t)|^{p+q}k^{q'}(t)dt\right]^{1/q'}. \qquad (18)$$

where $g(t) = (p+q)\dfrac{b-t}{b-a}, h(t) = 1 + (t-a)\dfrac{\varphi(t)}{\Phi_a(t)}$ and $q' = q/(q-1)$.

Proof. Rewrite inequality $(*)$ in a way:

$$\int_a^b \varphi(t)|f(t)|^{p+q}dt \leqslant \int_a^b \Phi_a(t)|f(t)|^{p+q-1} \left(g(t)|f'(t)| + h(t)\frac{|f(t)|}{b-a}\right) dt$$

$$= \int_a^b \Phi_a^{1/q}(t)|f(t)|^{p/q}k^{-1}(t) \left(g(t)|f'(t)| + h(t)\frac{|f(t)|}{b-a}\right)$$

$$\cdot \Phi_a^{(q-1)/q}|f(t)|^{(q-1)(p+q)/q}k(t)dt,$$

Using Hölder's inequality with indices $q, \dfrac{q}{q-1}$ we get the desired inequality (18). □

REMARK 4. When $a = 0, \varphi(t) = t^\alpha(\alpha \geqslant 0$ is a constant) and $k(t) = t^{-1/q}w(t)$ in Theorem 2, we can derive the main result given in [10] (Theorem 1, (2)).

THEOREM 3. *Let* $f \in C^1((a,\infty), R)$, $\varphi \in C((a,\infty), R_+)$ *and* $k(t) \in C((a,\infty),(0,\infty))$. *Then*

$$\int_a^\infty \varphi(t)|f(t)|^{p+q}dt \leqslant (p+q)\left[\int_a^\infty \Phi_a(t)|f(t)|^p k^{-q}(t)|f'(t)|^q dt\right]^{1/q}$$
$$\cdot \left[\int_a^\infty \Phi_a(t)k^{q'}(t)|f(t)|^{p+q}dt\right]^{1/q'}. \qquad (19)$$

where $q' = \dfrac{q}{q-1}$.

Proof. For any $b > a$, by using Minkowski's inequality, we derive from (18) that

$$\int_a^b \varphi(t)|f(t)|^{p+q}dt \leqslant \left\{\left[\int_a^b \Phi_a(t)|f(t)|^p k^{-q}(t)g^q(t)|f'(t)|^q dt\right]^{1/q} + \right.$$
$$\left. + \left[\int_a^b \Phi_a(t)|f(t)|^{p+q}k^{-q}(t)h^q(t)\frac{1}{(b-a)^q}dt\right]^{1/q}\right\} \cdot \left[\int_a^b \Phi_a(t)|f(t)|^{p+q}k^{q'}(t)dt\right]^{1/q'}$$
$$\leqslant \left[\int_a^\infty \Phi_a(t)|f(t)|^p k^{-q}(t)g^q(t)|f'(t)|^q dt\right]^{1/q} \cdot \left[\int_a^\infty \Phi_a(t)|f(t)|^{p+q}k^{q'}(t)dt\right]^{1/q'} +$$
$$+ \frac{1}{b-a}\left[\int_a^b \Phi_a(t)|f(t)|^{p+q}k^{-q}(t)h^q(t)dt\right]^{1/q} \cdot \left[\int_a^\infty \Phi_a(t)|f(t)|^{p+q}k^{q'}(t)dt\right]^{1/q'}.$$

In the last inequality by letting $b \to \infty$, we obtain (19). $\square$

REMARK 5. Let $a = 0, \varphi(t) = t^\alpha (\alpha \geqslant 0$ is a constant) and $k(t) = t^{-1/q'}w(t)$, then we get another main result appeared in [10] (Theorem 1 (3)).

Let $k(t) = (\Phi_a(t))^{1/q}$, from Theorem 3, we have

COROLLARY 4. *Suppose that the conditions appearing in Theorem 3 are satisfied. Then we have the following Weyl-type inequality*

$$\int_a^\infty \varphi(t)|f(t)|^{p+q}dt \leqslant (p+q)\left[\int_a^\infty |f(t)|^p|f'(t)|^q dt\right]^{1/q}$$
$$\cdot \left[\int_a^\infty \Phi_a^{q'}(t)|f(t)|^{p+q}dt\right]^{1/q'}. \qquad (20)$$

REMARK 6. Let $a = p = 0, q = 2, \varphi(t) = 1$, then $\Phi(t) = \int_0^t 1ds = t$, from (20) we obtain the well-known Weyl inequality

$$\int_0^\infty f^2(t)dt \leqslant 2\left(\int_0^\infty t^2 f^2(t)dt\right)^{\frac{1}{2}}\left(\int_0^\infty f'^2(t)dt\right)^{\frac{1}{2}}.$$

THEOREM 4. *Let* $\alpha \geqslant 2$ *be a real number,* $f \in C^1(R,R)$, $\varphi \in C(R,R_+)$ *be a function with satisfying* $\varphi(-t) = \varphi(t)$, $\Phi_0(t) = \int_0^t \varphi(s)ds$. *Then we have the following Heisenberg type inequality*

$$\int_{-\infty}^{\infty} \varphi(t)|f(t)|^{\alpha}dt \leqslant \alpha \left[\int_{-\infty}^{\infty} |f(t)|^{\alpha-2}f'^2(t)dt\right]^{1/2} \cdot \left[\int_{-\infty}^{\infty} \Phi_0^2(t)|f(t)|^{\alpha}dt\right]^{1/2}. \quad (21)$$

Proof. Setting $p = \alpha - 2$, we have

$$\int_{-\infty}^{\infty} \varphi(t)|f(t)|^{\alpha}dt = \int_{-\infty}^{\infty} \varphi(t)|f(t)|^{2+p}dt$$

$$= \int_{-\infty}^{0} \varphi(t)|f(t)|^{2+p}dt + \int_{0}^{\infty} \varphi(t)|f(t)|^{2+p}dt. \quad (22)$$

Changing the variable $t = -u$ for the first integral in the right side of (22) and using the condition $\varphi(-t) = \varphi(t)$, we get

$$\int_{-\infty}^{0} \varphi(t)|f(t)|^{2+p}dt = \int_{0}^{\infty} \varphi(u)|f(-u)|^{2+p}du. \quad (23)$$

Let $q = 2$ and $a = 0$, from (20) we have

$$\int_{0}^{\infty} \varphi(t)|f(t)|^{2+p}dt \leqslant (2+p)\left[\int_{0}^{\infty} |f(t)|^p|f'(t)|^2dt\right]^{1/2}$$

$$\cdot \left[\int_{0}^{\infty} \Phi_0^2(t)|f(t)|^{2+p}dt\right]^{1/2}. \quad (24)$$

Using inequality (24) to the right of (23) we derive

$$\int_{0}^{\infty} \varphi(u)|f(-u)|^{2+p}du \leqslant (2+p)\left[\int_{0}^{\infty} |f(-u)|^p|f'(-u)|^2du\right]^{1/2}$$

$$\cdot \left[\int_{0}^{\infty} \Phi_0^2(u)|f(-u)|^{2+p}du\right]^{1/2}.$$

Since

$$\Phi_0(-u) = \int_{0}^{-u} \varphi(s)ds = -\int_{0}^{u} \varphi(-v)dv = -\int_{0}^{u} \varphi(v)dv = -\Phi_0(u),$$

by the last inequality we obtain

$$\int_{0}^{\infty} \varphi(u)|f(-u)|^{2+p}du$$

$$\leqslant (2+p)\left[\int_{0}^{\infty} |f(-u)|^p|f'(-u)|^2du\right]^{1/2} \cdot \left[\int_{0}^{\infty} \Phi_0^2(u)|f(-u)|^{2+p}du\right]^{1/2}$$

$$= (2+p)\left[\int_{-\infty}^{0} |f(t)|^p|f'(t)|^2dt\right]^{1/2} \cdot \left[\int_{-\infty}^{0} \Phi_0^2(t)|f(t)|^{2+p}dt\right]^{1/2}. \quad (25)$$

Substituting (23), (24) and (25) into (22), we obtain

$$
\begin{aligned}
\int_{-\infty}^{\infty} \varphi(t)|f(t)|^{\alpha} dt &\leqslant (2+p) \left[\int_{-\infty}^{0} |f(t)|^{p}|f'(t)|^{2} dt \right]^{1/2} \cdot \left[\int_{-\infty}^{0} \Phi_0^2(t)|f(t)|^{2+p} dt \right]^{1/2} \\
&\quad + (2+p) \left[\int_{0}^{\infty} |f(t)|^{p}|f'(t)|^{2} dt \right]^{1/2} \cdot \left[\int_{0}^{\infty} \Phi_0^2(t)|f(t)|^{2+p} dt \right]^{1/2} \\
&\leqslant (2+p) \left[\int_{-\infty}^{0} |f(t)|^{p}|f'(t)|^{2} dt + \int_{0}^{\infty} |f(t)|^{p}|f'(t)|^{2} dt \right]^{1/2} \\
&\quad \cdot \left[\int_{-\infty}^{0} \Phi_0^2(t)|f(t)|^{2+p} dt + \int_{0}^{\infty} \Phi_0^2(t)|f(t)|^{2+p} dt \right]^{1/2} \\
&= (2+p) \left[\int_{-\infty}^{\infty} |f(t)|^{p}|f'(t)|^{2} dt \right]^{1/2} \cdot \left[\int_{-\infty}^{\infty} \Phi_0^2(t)|f(t)|^{2+p} dt \right]^{1/2}.
\end{aligned}
$$

Letting $\alpha = 2 + p$, from the last inequality, we derive the desired inequality (21). $\square$

REMARK 7. (i) When $\varphi(t) = t^{2n}, n \in \{0, 1, 2, \cdots\}$, from (21) we have the following Heisenberg type inequality

$$
\int_{-\infty}^{\infty} t^{2n}|f(t)|^{\alpha} dt \leqslant \frac{\alpha}{2n+1} \left[\int_{-\infty}^{\infty} |f(t)|^{\alpha-2} f'^2(t) dt \right]^{1/2} \cdot \left[\int_{-\infty}^{\infty} t^{4n+2}|f(t)|^{\alpha} dt \right]^{1/2}. \tag{26}
$$

(ii) Letting $n = 0, \alpha = 2$ in (26), we get Heisenberg inequality

$$
\int_{-\infty}^{\infty} f^2(t) dt \leqslant 2 \left(\int_{-\infty}^{\infty} t^2 f^2(t) dt \right)^{\frac{1}{2}} \left(\int_{-\infty}^{\infty} f'^2(t) dt \right)^{\frac{1}{2}}.
$$

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