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A NEW SIMPLE PROOF OF WILKER'S INEQUALITY

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(*communicated by A. M. Fink*)

Abstract. In this note, we show a new simple proof of Wilker's inequality.

Introduction. J. B. Wilker [1] proposed two open questions and the first one was that:

THEOREM. *If* $0 < x < \pi/2$, *then*

$$\left(\frac{\sin x}{x}\right)^2 + \frac{\tan x}{x} > 2. \tag{1}$$

J. S. Sumner et al. [2] proved the inequality (1). B. N. Guo et al. [3] gave a new proof of the inequality (1). In this paper, we show a new simple proof of Wilker's inequality.

One Lemma.

LEMMA ([4, 5]). *If* $0 < x < \pi/2$, *then*

$$\sin x < x < \tan x. \tag{2}$$

A New Proof of Theorem. Let $f(x) = \sin^2 x \cos x + x \sin x - 2x^2 \cos x$, then

$$f'(x) = 3 \sin x - 3 \sin^3 x - 3x \cos x + 2x^2 \sin x,$$
$$f''(x) = \cos x(2x^2 + 7x \tan x - 9 \sin^2 x) := \cos x \cdot g(x),$$

where, $g(x) = 2x^2 + 7x \tan x - 9 \sin^2 x$. Since

$$g(x) > 2x^2 + 7x^2 - 9x^2 = 0$$

from Lemma, we have $f''(x) > 0$ for $x \in (0, \pi/2)$. That is, $f'(x)$ strictly increases as x increases on $(0, \pi/2)$. Since $f'(0) = 0$, we have $f'(x) > 0$ for $x \in (0, \pi/2)$.Then $f(x)$ is increasing on $(0, \pi/2)$. Now, $f(0) = 0$, so $f(x) > 0$. Therefore, (1) holds and the proof of Theorem is complete.

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