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PERTURBED ALGORITHM AND STABILITY FOR STRONGLY NONLINEAR QUASI–VARIATIONAL INCLUSION INVOLVING H–ACCRETIVE OPERATORS

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Abstract. In this paper, a new class of strongly nonlinear quasi-variational inclusions involving H-accretive operator in Banach spaces is studied, which includes many variational inequality(inclusion) and complementarity problems as special cases. By using the resolvent operator technique for H-accretive operator due to Fang and Huang, an existence and uniqueness theorem of solution for strongly nonlinear quasi-variational inclusion is proved. A new perturbed algorithm for finding approximate solution of the strongly nonlinear quasi-variational inclusion is suggested and discussed, the convergence and stability of the iterative sequence generated by new perturbed algorithm is also given. The results presented in this paper improve and generalize some recent results in this field.

1. Introduction

It is known that variational inclusion is an important and useful generalization of variational inequality. Because of the wide applications to optimization and control, economic and transportation equilibrium, and engineering sciences, variational inequalities and variational inclusions have been studied by many authors (see, [1-28, 30, 32] and the references therein). We also know that one of the most important and interesting problems in the theory of variational inequalities and variational inclusions is the development of an efficient and implementable iterative algorithm for solving variational inequalities and inclusions. Among these methods, the resolvent operator techniques for solving variational inequalities and inclusions are interesting and important.

Recently, Fang and Huang [8] introduced a new class of H-accretive operator in Banach spaces, and studied the properties of the resolvent operator associated with the H-accretive operator. They also introduced and studied a class of generalized variational inclusions involving H-accretive operator in Banach spaces.

Inspired and motivated by recent research works in this field, in this paper, we shall introduce and study a new class of strongly nonlinear quasi-variational inclusions involving H-accretive operator in Banach spaces, which includes many variational inequality(inclusion) and complementarity problems as special cases. By using the

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resolvent operator technique for H-accretive operator due to Fang and Huang, an existence and uniqueness theorem of solutions for strongly nonlinear quasi-variational inclusions is proved and a new perturbed algorithm for finding approximate solutions is proposed and discussed. The convergence and stability of the iterative sequence generated by the new perturbed algorithm is also proved. The results presented in this paper improve and generalize many known corresponding results in the literature.

2. Preliminaries

Let X be a real Banach space with dual space X^*, $\langle \cdot, \cdot \rangle$ be the dual pair between X and X^*, and 2^X denote the family of all the nonempty subsets of X. The generalized duality mapping $J_q : X \to 2^{X^*}$ is defined by

$$J_q(x) = \{f^* \in X^* : \langle x, f^* \rangle = \|x\|^q \text{ and } \|f^*\| = \|x\|^{q-1}\}, \quad \forall x \in X,$$

where $q > 1$ is a constant. In particular, J_2 is the usual normalized duality mapping. It is known that, in general, $J_q(x) = \|x\|^{q-2} J_2(x)$ for all $x \neq 0$ and J_q is single-valued if X^* is strictly convex. In the sequel, unless otherwise specified, we always suppose that X is a real Banach space such that J_q is single -valued and $\mathscr{H}$ is a Hilbert space. If $X = \mathscr{H}$, then J_2 becomes the identity mapping of $\mathscr{H}$.

The modulus of smoothness of X is the function $\rho_X : [0, \infty) \to [0, \infty)$ defined by

$$\rho_X(t) = \sup\{\frac{1}{2}(\|x+y\| + \|x-y\|) - 1 : \|x\| \leqslant 1, \|y\| \leqslant t\}.$$

A Banach space X is called uniformly smooth if

$$\lim_{t \to 0} \frac{\rho_X(t)}{t} = 0.$$

X is called q-uniformly smooth if there exists a constant $c > 0$, such that

$$\rho_X(t) \leqslant ct^q, \qquad q > 1.$$

Note that J_q is single -valued if X is uniformly smooth. In the study of characteristic inequalities in q-uniformly smooth Banach spaces, Xu [31] proved the following theorem.

THEOREM X. *Let X be real uniformly smooth Banach space. Then X is q-uniformly smooth if and only if there exists a constant $C_q > 0$, such that for all $x, y \in X$,*

$$\|x+y\|^q \leqslant \|x\|^q + q\langle y, J_q(x) \rangle + C_q \|y\|^q.$$

DEFINITION 2.1. Let S be a selfmap of X, $x_0 \in X$, and let $x_{n+1} = h(S, x_n)$ define an iteration procedure which yields a sequence of points $\{x_n\}_{n=0}^{\infty}$ in X. Suppose that $\{x \in X : Sx = x\} \neq \phi$ and $\{x_n\}_{n=0}^{\infty}$ converges to a fixed point x^* of S. Let $\{u_n\} \subset X$

and let $\epsilon_n = \|u_{n+1} - h(S,u_n)\|$. If $\lim_{n\to\infty} \epsilon_n = 0$ implies that $u_n \to x^*$, then the iteration procedure defined by $x_{n+1} = h(S,x_n)$ is said to be S-stable or stable with respect to S.

LEMMA 2.1. (29) *Let $\{a_n\}$ be a nonnegative real sequence and $\{b_n\}$ be a real sequence in $[0,1]$ such that $\sum_{n=0}^{\infty} b_n = \infty$. If there exists a positive integer n_1 such that*

$$a_{n+1} \leqslant (1-b_n)a_n + b_n c_n, \qquad \forall n \geqslant n_1,$$

where $c_n \geqslant 0$ for all $n \geqslant 0$ and $c_n \to 0 (n \to \infty)$, then $\lim_{n\to\infty} a_n = 0$.

DEFINITION 2.2. Let $H : X \to X$ and $N : X \times X \to X$ be two single-valued operators.

(1) The operator H is said to be

(i) accretive if

$$\langle H(x) - H(y), J_q(x-y)\rangle \geqslant 0, \quad \forall x,y \in X;$$

(ii) strictly accretive if

$$\langle H(x) - H(y), J_q(x-y)\rangle \geqslant 0, \quad \forall x,y \in X,$$

and the equality holds if and only $x = y$;

(iii) α- strongly accretive if there exists some constant $\alpha > 0$ such that

$$\langle H(x) - H(y), J_q(x-y)\rangle \geqslant \alpha\|x-y\|^q, \quad \forall x,y \in X;$$

(iv) β-Lipschitz continuous if there exists some constsnt $\beta > 0$ such that

$$\|H(x) - H(y)\| \leqslant \beta\|x-y\|, \quad \forall x,y \in X.$$

(2) The operator $N(\cdot,\cdot)$ is said to be

(i) r-strongly accretive with respect to H in first argument if there exists some constant $r > 0$ such that

$$\langle N(x,\cdot) - N(y,\cdot), j_q(H(x) - H(y))\rangle \geqslant r\|x-y\|^q, \quad \forall x,y \in X;$$

(ii) s-Lipschitz continuous with respect to the first argument if there exists some constsnt $s > 0$ such that

$$\|N(x,\cdot) - N(y,\cdot)\| \leqslant s\|x-y\|, \quad \forall x,y \in X.$$

In a similar way, we can define Lipschitz continuity of $N(\cdot,\cdot)$ with respect to the second argument.

DEFINITION 2.3. Let $H : X \to X$ be a single-valued operator. A multivalued operator $M : X \to 2^X$ is said to be

(i) accretive if

$$\langle u - v, J_q(x-y)\rangle \geqslant 0, \quad \forall x,y \in X, \quad u \in M(x), \quad v \in M(y);$$

(ii) m-accretive if M is accretive and $(I + \lambda M)(X) = X$, for all $\lambda > 0$, where I denotes the identity mapping on X.

(*iii*) H-accretive if M is accretive and $(H+\lambda M)(X)=X$ holds, for all $\lambda>0$. The example of H-accretive operator can be found in Fang and Huang [8].

REMARK 2.1. If $H=I$, then (iii) of Definition 2.3 reduces to the definition of m-accretive operator, and if $X=\mathscr{H}$ and $H=I$, then (iii) of Definition 2.3 reduces to the definition of maximal monotone operator.

In [8], Fang and Huang showed that $(H+\lambda M)^{-1}$ is a single-valued operator if M is a H-accretive operator and H is a strictly accretive operator, where I is an identity operator and $\lambda>0$ is a constant. Based on this fact, the resolvent operator for a H-accretive operator M can be defined as follows:

$$R_{H,\lambda}^{M}(u)=(H+\lambda M)^{-1},\quad \forall u\in X.$$

LEMMA 2.2. (8) *Let* $H:X\to X$ *be a* r*-strongly accretive operator and* $M:X\to 2^X$ *be an* H*-accretive operator. Then the resolvent operator* $R_{H,\lambda}^{M}:X\to X$ *is Lipschitz continuous with constant* $1/r$*, i. e. ,*

$$\|R_{H,\lambda}^{M}(u)-R_{H,\lambda}^{M}(v)\|\leqslant\frac{1}{r}\|u-v\|,\quad \forall u,v\in X.$$

Let $N:X\times X\to X$ and $H:X\to X$ be two single-valued operators, and $M:X\times X\to 2^X$ be a H-accretive operator with respect to the first argument. Now we consider the following problem:

Find $u\in X$ such that

$$0\in N(u,u)+M(u,u). \tag{2.1}$$

Problem (2.1) is called the strongly nonlinear quasi-variational inclusion involving H-accretive operator.

If $N(u,u)=A(u), M(u,u)=G(u)$ for all $u\in X$, where $A:X\to X$ is a single-valued operator, $G:X\to 2^X$ is a H-accretive operator, then problem (2.1) is equivalent to the following problem:

Find $u\in X$ such that

$$0\in A(u)+G(u). \tag{2.2}$$

which is called the generalized variational inclusion involving H-accretive operator considered by Fang and Huang [8] and has been studied by many authors in the setting of Hilbert spaces when M is maximal monotone and A is strongly monotone. It is easy to see that problem (2.1) includes many variational inequality(inclusion) and complementarity problems as special cases.

3. Main results

LEMMA 3.1. u *is a solution of problem* (2.1) *if and only if there exists* $u\in X$ *such that*

$$u=R_{H,\lambda}^{M(\cdot,u)}(H(u)-\lambda N(u,u)),$$

where $R_{H,\lambda}^{M(\cdot,u)}=(H+\lambda M(\cdot,u))^{-1}$ *and* $\lambda>0$ *is a constant.*

Proof. This directly follows from the definition of $R_{H,\lambda}^{M(\cdot,u)}$.

THEOREM 3.1. *Let X be a q-uniformly smooth Banach space and $H : X \to X$ be r-strongly accretive and s-Lipschitz continuous. Let $N : X \times X \to X$ be Lipschitz continuous in the first and second arguments with constants α and β, respectively, and be γ-strongly accretive with respect to H in the first argument. If for any given $u \in X, M(\cdot,u) : X \to 2^X$ is a H-accretive operator and there exist constant $\delta > 0$ such that for each $u,v,x \in X$,*

$$\|R_{H,\lambda}^{M(\cdot,u)}(x) - R_{H,\lambda}^{M(\cdot,v)}(x)\| \leqslant \delta\|u-v\| \tag{3.1}$$

and

$$\theta = \frac{1}{r}(s^q - q\lambda\gamma + C_q\lambda^q\alpha^q)^{\frac{1}{q}} + \frac{\lambda}{r}\beta + \delta < 1, \tag{3.2}$$

where $C_q > 0$ is the same as in Theorem X. Then the problem (2.1) has a unique solution $u^* \in X$.

Proof. Define $F : X \to X$ as follows:

$$F(u) = R_{H,\lambda}^{M(\cdot,u)}(H(u) - \lambda N(u,u)), \quad \forall u \in X. \tag{3.3}$$

It follows from (3.1), (3.3) and Lemma 2.2 that

$$\begin{aligned}
\|F(u)-F(v)\| &= \|R_{H,\lambda}^{M(\cdot,u)}(H(u)-\lambda N(u,u)) - R_{H,\lambda}^{M(\cdot,v)}(H(v)-\lambda N(v,v))\| \\
&\leqslant \|R_{H,\lambda}^{M(\cdot,u)}(H(u)-\lambda N(u,u)) - R_{H,\lambda}^{M(\cdot,u)}(H(v)-\lambda N(v,v))\| \\
&\quad + \|R_{H,\lambda}^{M(\cdot,u)}(H(v)-\lambda N(v,v)) - R_{H,\lambda}^{M(\cdot,v)}(H(v)-\lambda N(v,v))\| \\
&\leqslant \frac{1}{r}\|H(u)-H(v)-\lambda(N(u,u)-N(v,v)\| + \delta\|u-v\| \\
&\leqslant \frac{1}{r}\|H(u)-H(v)-\lambda(N(u,u)-N(v,u)\| \\
&\quad + \frac{1}{r}\lambda\|N(v,u)-N(v,v)\| + \delta\|u-v\|
\end{aligned} \tag{3.4}$$

By assumptions and Theorem X, we obtain

$$\begin{aligned}
&\|H(u)-H(v)-\lambda(N(u,u)-N(v,u)\|^q \\
&\quad \leqslant \|H(u)-H(v)\|^q - q\lambda\langle N(u,u)-N(v,u), J_q(H(u)-H(v)\rangle \\
&\qquad + C_q\lambda^q\|N(u,u)-N(v,u)\|^q \\
&\quad \leqslant (s^q - q\lambda\gamma + C_q\lambda^q\alpha^q)\|u-v\|^q,
\end{aligned} \tag{3.5}$$

$$\|N(v,u)-N(v,v)\| \leqslant \beta\|u-v\| \tag{3.6}$$

Combining (3.4), (3.5) and (3.6), we have

$$\|F(u)-F(v)\| \leqslant \theta\|u-v\|.$$

where

$$\theta = \frac{1}{r}(s^q - q\lambda\gamma + C_q\lambda^q\alpha^q)^{\frac{1}{q}} + \frac{\lambda}{r}\beta + \delta,$$

by (3.2), we know that $\theta < 1$, thus $F(u)$ is a contractive mapping. So there exists a unique point $u^* \in X$ such that

$$u^* = R_{H,\lambda}^{M(\cdot,u^*)}(H(u^*) - \lambda N(u^*, u^*)).$$

It follows from Lemma 3.1 that $u^* \in X$ is a unique solution of problem (2.1).This completes the proof.

REMARK 3.1 If X is 2-uniformly smooth and there exist $\lambda > 0$ such that

$$\begin{cases} |\lambda - \frac{\gamma-\beta l}{C_2\alpha^2-\beta^2}| < \min\{\frac{l}{\beta}, \frac{\sqrt{(\gamma-\beta l)^2-(C_2\alpha^2-\beta^2)(s^2-l^2)}}{C_2\alpha^2-\beta^2}\}, \\ \gamma > \beta l + \sqrt{(C_2\alpha^2 - \beta^2)(s^2 - l^2)}, \quad C_2\alpha^2 > \beta^2, \quad s > l, \\ l = r(1-\delta), 0 < \delta < 1. \end{cases}$$

then (3.2) holds.

Algorithm 3.1 Let $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ be two sequences such that $\alpha_n, \beta_n \in [0,1]$ and $\sum\limits_{n=0}^{\infty} \alpha_n = \infty$. Let $\{e_n\}_{n=0}^{\infty}$ and $\{f_n\}_{n=0}^{\infty}$ be two sequences in X introduced to take into account possible inexact computation. For any given $x_0 \in X$, the perturbed Ishikawa type iterative sequence $\{x_n\}$ is defined by

$$\begin{cases} x_{n+1} = (1-\alpha_n)x_n + \alpha_n R_{H,\lambda}^{M(\cdot,y_n)}(H(y_n) - \lambda N(y_n, y_n)) + \alpha_n e_n, \\ y_n = (1-\beta_n)x_n + \beta_n R_{H,\lambda}^{M(\cdot,x_n)}(H(x_n) - \lambda N(x_n, x_n)) + \beta_n f_n, \end{cases} \tag{3.7}$$

for $n = 0, 1, 2, \cdots$. Let $\{z_n\}$ be any sequence in X and define $\{\epsilon_n\}$ by

$$\begin{cases} \epsilon_n = \|z_{n+1} - \{(1-\alpha_n)z_n + \alpha_n R_{H,\lambda}^{M(\cdot,t_n)}(H(t_n) - \lambda N(t_n, t_n)) + \alpha_n e_n\}\|, \\ t_n = (1-\beta_n)z_n + \beta_n R_{H,\lambda}^{M(\cdot,z_n)}(H(z_n) - \lambda N(z_n, z_n)) + \beta_n f_n, \end{cases} \tag{3.8}$$

for $n = 0, 1, 2, \cdots$.

THEOREM 3.2. *Let X, H, N, M be the same as in Theorem* 3.1*, and conditions* (3.1) *and* (3.2) *holds. Then*

(*i*) *If* $\lim\limits_{n\to\infty} \|e_n\| = \lim\limits_{n\to\infty} \|f_n\| = 0$*, then the sequence* $\{x_n\}$ *generated by* (3.7) *converges strongly to the unique solution* u^* *of problem* (2.1).

(*ii*) *Moreover, if* $0 < \mu \leqslant \alpha_n$*, then* $\lim\limits_{n\to\infty} z_n = u^*$ *if and only* $\lim\limits_{n\to\infty} \epsilon_n = 0$*, where* ϵ_n *is defined by* (3.8).

Proof. Let $u^* \in X$ be the unique solution of problem (2.1). It follows from Lemma 3.1 that

$$\begin{aligned} u^* &= (1-\alpha_n)u^* + \alpha_n R_{H,\lambda}^{M(\cdot,u^*)}(H(u^*) - \lambda N(u^*, u^*)) \\ &= (1-\beta_n)u^* + \beta_n R_{H,\lambda}^{M(\cdot,u^*)}(H(u^*) - \lambda N(u^*, u^*)). \end{aligned}$$

From (3.1), (3.7) and Lemma 2.2, it follows that

$$
\begin{aligned}
\|x_{n+1}-u^*\| &\leqslant (1-\alpha_n)\|x_n-u^*\| + \alpha_n\|R_{H,\lambda}^{M(\cdot,y_n)}(H(y_n)-\lambda N(y_n,y_n)) \\
&\quad - R_{H,\lambda}^{M(\cdot,u^*)}(H(u^*)-\lambda N(u^*,u^*))\| + \alpha_n\|e_n\| \\
&\leqslant (1-\alpha_n)\|x_n-u^*\| + \alpha_n\|R_{H,\lambda}^{M(\cdot,y_n)}(H(y_n)-\lambda N(y_n,y_n)) \\
&\quad - R_{H,\lambda}^{M(\cdot,y_n)}(H(u^*)-\lambda N(u^*,u^*))\| + \alpha_n\|R_{H,\lambda}^{M(\cdot,y_n)}(H(u^*) \\
&\quad - \lambda N(u^*,u^*)) - R_{H,\lambda}^{M(\cdot,u^*)}(H(u^*)-\lambda N(u^*,u^*))\| + \alpha_n\|e_n\| \\
&\leqslant (1-\alpha_n)\|x_n-u^*\| + \frac{1}{r}\alpha_n\|H(y_n)-H(u^*)-\lambda(N(y_n,y_n) \\
&\quad - N(u^*,u^*))\| + \alpha_n\delta\|y_n-u^*\| + \alpha_n\|e_n\| \\
&\leqslant (1-\alpha_n)\|x_n-u^*\| + \frac{1}{r}\alpha_n\|H(y_n)-H(u^*)-\lambda(N(y_n,y_n)-N(u^*,y_n))\| \\
&\quad + \frac{1}{r}\lambda\alpha_n\|N(u^*,y_n)-N(u^*,u^*)\| + \alpha_n\delta\|y_n-u^*\| + \alpha_n\|e_n\|
\end{aligned} \tag{3.9}
$$

By assumptions and Theorem X, we have

$$\|H(y_n)-H(u^*)-\lambda(N(y_n,y_n)-N(u^*,y_n))\|^q \leqslant (s^q-q\lambda\gamma+C_q\lambda^q\alpha^q)\|y_n-u^*\|^q \tag{3.10}$$

$$\|N(u^*,y_n)-N(u^*,u^*)\| \leqslant \beta\|y_n-u^*\|. \tag{3.11}$$

Substituting (3.10) and (3.11) into (3.9), we obtain

$$\|x_{n+1}-u^*\| \leqslant (1-\alpha_n)\|x_n-u^*\| + \alpha_n\theta\|y_n-u^*\| + \alpha_n\|e_n\|, \tag{3.12}$$

where

$$\theta = \frac{1}{r}(s^q-q\lambda\gamma+C_q\lambda^q\alpha^q)^{\frac{1}{q}} + \frac{1}{r}\lambda\beta + \delta.$$

Similarly, we can prove that

$$\|y_n-u^*\| \leqslant (1-\beta_n)\|x_n-u^*\| + \beta_n\theta\|x_n-u^*\| + \beta_n\|f_n\|. \tag{3.13}$$

It follows from (3.12), (3.13) and condition (3.2), that

$$
\begin{aligned}
\|x_{n+1}-u^*\| &\leqslant (1-\alpha_n)\|x_n-u^*\| + \alpha_n\theta(1-\beta_n)\|x_n-u^*\| \\
&\quad + \alpha_n\beta_n\theta^2\|x_n-u^*\| + \alpha_n(\theta\beta_n\|f_n\| + \|e_n\|) \\
&\leqslant [1-\alpha_n(1-\theta)]\|x_n-u^*\| + \alpha_n(\theta\|f_n\| + \|e_n\|).
\end{aligned} \tag{3.14}
$$

Letting

$$a_n = \|x_n-u^*\|, \quad b_n = (1-\theta)\alpha_n, \quad c_n = \frac{1}{1-\theta}(\theta\|f_n\| + \|e_n\|),$$

then (3.14) can be written as

$$a_{n+1} = (1-b_n)a_n + b_nc_n.$$

It follows from Lemma 2.1 that $a_n \to 0 (n \to \infty)$, and so $\{x_n\}$ converges strongly to the unique solution u^* of problem (2.1).

Now we prove conclsion (ii). By (3.8), we obtain

$$\begin{aligned} \|z_{n+1}-u^*\| &\leqslant \|z_{n+1}-\{(1-\alpha_n)z_n+\alpha_n R_{H,\lambda}^{M(\cdot,t_n)}(H(t_n)-\lambda N(t_n,t_n))+\alpha_n e_n\}\| \\ &\quad + \|(1-\alpha_n)z_n+\alpha_n R_{H,\lambda}^{M(\cdot,t_n)}(H(t_n)-\lambda N(t_n,t_n))+\alpha_n e_n - u^*\| \\ &\leqslant \|(1-\alpha_n)z_n+\alpha_n R_{H,\lambda}^{M(\cdot,t_n)}(H(t_n)-\lambda N(t_n,t_n))+\alpha_n e_n\}-u^*\|+\epsilon_n. \end{aligned} \tag{3.15}$$

As the proof of in equality (3.14), we have

$$\begin{aligned} &\|(1-\alpha_n)z_n + \alpha_n R_{H,\lambda}^{M(\cdot,t_n)}(H(t_n) - \lambda N(t_n,t_n)) + \alpha_n e_n - u^*\| \\ &\qquad \leqslant [1-(1-\theta)\alpha_n]\|z_n - u^*\| + \alpha_n(\theta\|f_n\| + \|e_n\|). \end{aligned} \tag{3.16}$$

Since $0 < \mu \leqslant \alpha_n$, by (3.15) and (3.16), we have

$$\|z_{n+1} - u^*\| \leqslant [1-(1-\theta)\alpha_n]\|z_n - u^*\| + (1-\theta)\alpha_n\left(\frac{\theta\|f_n\| + \|e_n\|}{1-\theta} + \frac{\epsilon_n}{\mu(1-\theta)}\right).$$

Suppose that $\lim_{n\to\infty} \epsilon_n=0$. Then from $\sum_{n=0}^{\infty} \alpha_n=\infty$ and Lemma 2.1, we have $\lim_{n\to\infty} z_n=u^*$.

Conversely, if $\lim_{n\to\infty} z_n = u^*$, then we get

$$\begin{aligned} \epsilon_n &= \|z_{n+1} - \{(1-\alpha_n)z_n + \alpha_n R_{H,\lambda}^{M(\cdot,t_n)}(H(t_n) - \lambda N(t_n,t_n)) + \alpha_n e_n\}\| \\ &\leqslant \|z_{n+1} - u^*\| + \|(1-\alpha_n)z_n + \alpha_n R_{H,\lambda}^{M(\cdot,t_n)}(H(t_n) - \lambda N(t_n,t_n)) + \alpha_n e_n\} - u^*\| \\ &\leqslant \|z_{n+1} - u^*\| + [1-(1-\theta)\alpha_n]\|z_n - u^*\| + \alpha_n(\theta\|f_n\| + \|e_n\|) \to 0, \end{aligned}$$

as $n \to \infty$. This completes the proof.

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