Mathematical
Inequalities
& Applications
Volume 10, Number 1 (2007), 165–172

EMBEDDINGS BETWEEN DISCRETE WEIGHTED LEBESGUE SPACES WITH VARIABLE EXPONENTS

ALEŠ NEKVINDA

(*communicated by L. Pick*)

Abstract. Given mappings $p,q,v,w:\mathbb{Z}\to(0,\infty)$ we can consider discrete weighted Lebesgue spaces $\ell^{\{p_n\}}(v_n)$ and $\ell^{\{q_n\}}(w_n)$ with variable exponents. The necessary and sufficient condition to the p, q, v, w for the embedding $\ell^{\{p_n\}}(v_n)\hookrightarrow\ell^{\{q_n\}}(w_n)$ is given.

1. Introduction

The generalized Lebesgue space $\ell^{\{p_n\}}$, $L^{p(x)}$ and the corresponding Sobolev space $W^{1,p(x)}$ have attracted more and more interest in recent years. We refer to [9] for the establishment of the fundamental properties of these spaces, to [4] for some properties of the norm on $L^{p(x)}$, to [6] and [16] for the density of smooth functions in $W^{1,p(x)}$ and to [7] for inequalities of Sobolev type. Further motivation for the study of these spaces is provided in [14, 15] by means of mathematical models of electrorheological fluids which involve nonlinear systems of partial differential equations with coefficients of variable rate of growth.

A crucial difference between $L^{p(x)}$ and the classical Lebesgue spaces is that $L^{p(x)}$ is not, in general, invariant under translation.

The boundedness of Hardy-Littlewood maximal operator plays a very important role in study of spaces $L^{p(x)}$. The basic result was done by L. Diening in [2]. Further results are obtained in [1], [3], [8], [10], [12] and [13].

Consider a discrete analogue $\ell^{\{p_n\}}$ of $L^{p(x)}$. In [5] it is proved that under certain assumptions on $p:\mathbb{Z}\to\mathbb{R}$ the norms of shift operators given by

$$S_ka=\{(S_ka)_n\},(S_ka)_n=a_{n-k},a=\{a_n\},$$

are uniformly bounded on $\ell^{\{p_n\}}$.

In [11] a necessary and sufficient condition to exponent $p,q:\mathbb{Z}\to\mathbb{R}$ is found to guarantee that the norms in spaces $\ell^{\{p_n\}}$ and $\ell^{\{q_n\}}$ are equivalent. Moreover, the norms of S_k are uniformly bounded on $\ell^{\{p_n\}}$ for a bounded p if and only if there exists

Mathematics subject classification (2000): 46E30, 26D15.
Key words and phrases: Lebesgue spaces, variable exponent, embedding.
The work was supported by the research plan of the Ministry of Education of the Czech Republik no. 6840770010.

Paper MIA-10-14

a real number r, $1 \leqslant r < \infty$, such that the norms in $\ell^{\{p_n\}}$ and the classical space ℓ^r are equivalent.

This paper generalizes results from [11]. Given $p,q,v,w : \mathbb{Z} \to \mathbb{R}$ consider weighted discrete Lebesgue spaces $\ell^{\{p_n\}}(v_n), \ell^{\{q_n\}}(w_n)$. We find a necessary and sufficient condition to p,q,v,w to guarantee the embedding $\ell^{\{p_n\}}(v_n) \hookrightarrow \ell^{\{q_n\}}(w_n)$.

2. Preliminaries

Let $\mathbb{Z}$ denote the set of all integers and let $\mathcal{M}$ denote the set of all mappings $a : \mathbb{Z} \to \mathbb{R}$. Given $p \in \mathcal{M}$ denote by $p^* = \sup\{p_n; n \in \mathbb{Z}\}$ and set

$$\mathcal{B} = \{p \in \mathcal{M}; 1 \leqslant p_n \leqslant p^* < \infty\},$$
$$\mathcal{W} = \{v \in \mathcal{M}; 0 < v_n\}.$$

Given $p \in \mathcal{B}$ and $v \in \mathcal{W}$ define for $a \in \mathcal{M}$ a norm

$$\|a\|_{\ell^{\{p_n\}}(v_n)} := \inf\left\{\lambda > 0; \sum_{k\in\mathbb{Z}} \left(\frac{|a_n|}{\lambda}\right)^{p_n} v_n \leqslant 1\right\}$$

and the discrete weighted Lebesgue space $\ell^{\{p_n\}}(v_n)$ as a set of all $a \in \mathcal{M}$ with $\|a\|_{\ell^{\{p_n\}}(v_n)} < \infty$.

LEMMA 2.1. *Let $p,q \in \mathcal{B}$, $v,w \in \mathcal{W}$. Then the following assertions are equivalent:*
(i) $\ell^{\{p_n\}}(v_n) \hookrightarrow \ell^{\{q_n\}}(w_n)$;
(ii) *the implication*

$$\sum_{k\in\mathbb{Z}} |a_n|^{p_n} v_n \leqslant 1 \Rightarrow \sum_{k\in\mathbb{Z}} |a_n|^{q_n} w_n < \infty$$

holds.

Proof. This lemma is proved in [11] for non-weighted case. It is not difficult to rewrite this proof for weights to obtain our lemma.

3. Key assertions

Set $\mathcal{W}_0 = \{a \in \mathcal{M}; a_n \geqslant 0\}$.

LEMMA 3.1. *Let $v,w \in \mathcal{W}$, $\varepsilon \in \mathcal{W}_0$. Assume $\sup\{v_n^{-1-\varepsilon_n} w_n, n \in \mathbb{Z}\} < \infty$. Then the implication*

$$\sum_{n\in\mathbb{Z}} a_n v_n \leqslant 1 \Rightarrow \sum_{n\in\mathbb{Z}} a_n^{1+\varepsilon_n} w_n < \infty$$

holds for all $a \in \mathcal{W}_0$.

Proof. Assume $\sum_{n\in\mathbb{Z}} a_n v_n \leqslant 1$. Then $a_n \leqslant v_n^{-1}$ and so,

$$\sum_{n\in\mathbb{Z}} a_n^{1+\varepsilon_n} w_n = \sum_{n\in\mathbb{Z}} a_n v_n a_n^{\varepsilon_n} w_n v_n^{-1} \leqslant \sum_{n\in\mathbb{Z}} a_n v_n w_n v_n^{-1-\varepsilon_n} \leqslant \sup v_n^{-1-\varepsilon_n} w_n < \infty,$$

which finishes the proof.

LEMMA 3.2. *Let $v,w \in \mathcal{W}$, $\varepsilon \in \mathcal{W}_0$ and let there exists a positive number ε^* such that $\varepsilon_n \leqslant \varepsilon^*$. Assume that $v_n^{-1-\varepsilon_n} w_n$ is unbounded. Then there is a sequence $a \in \mathcal{W}_0$, such that*

$$\sum_{n\in\mathbb{Z}} a_n v_n \leqslant 1 \ \text{ and } \ \sum_{n\in\mathbb{Z}} a_n^{1+\varepsilon_n} w_n = \infty.$$

Proof. Since $\{v_n^{-1-\varepsilon_n} w_n\}$ is unbounded we can take an infinite set $\mathbb{S} = \{n_1, n_2, \dots\}$ of integers such that

$$v_{n_k}^{-1-\varepsilon_{n_k}} w_{n_k} \geqslant 2^{k(1+\varepsilon^*)}.$$

Set

$$a_n = \begin{cases} 0 & \text{if } n \notin \mathbb{S} \\ 2^{-k} v_{n_k}^{-1} & \text{if } n = n_k. \end{cases}$$

Clearly,

$$\sum_{n\in\mathbb{Z}} a_n v_n = \sum_{n\in\mathbb{S}} a_n v_n = \sum_{k=1}^{\infty} 2^{-k} v_{n_k}^{-1} v_{n_k} = 1$$

and

$$\begin{aligned} \sum_{n\in\mathbb{Z}} a_n^{1+\varepsilon_n} w_n &= \sum_{n\in\mathbb{S}} a_n^{1+\varepsilon_n} w_n = \sum_{k=1}^{\infty} \left(2^{-k} v_{n_k}^{-1}\right)^{1+\varepsilon_{n_k}} w_{n_k} \\ &= \sum_{k=1}^{\infty} 2^{-k(1+\varepsilon_{n_k})} v_{n_k}^{-1-\varepsilon_{n_k}} w_{n_k} \geqslant \sum_{k=1}^{\infty} 2^{-k(1+\varepsilon^*)} 2^{k(1+\varepsilon^*)} \\ &= \sum_{k=1}^{\infty} 1 = \infty. \end{aligned}$$

Thus, the proof follows.

LEMMA 3.3. *Let $v,w,\varepsilon \in \mathcal{W}$, $\varepsilon_n \leqslant 1$. Assume that there is a positive number c such that*

$$K := \sum_{n\in\mathbb{Z}} \left(\frac{w_n}{v_n} c\right)^{1/\varepsilon_n} v_n < \infty. \tag{3.1}$$

Then the implication

$$\sum_{n\in\mathbb{Z}} a_n v_n \leqslant 1 \Rightarrow \sum_{n\in\mathbb{Z}} a_n^{1-\varepsilon_n} w_n < \infty$$

holds for all $a \in \mathcal{W}_0$.

Proof. Let c satisfies (3.1) and assume $\sum_{n\in\mathbb{Z}} a_n v_n \leqslant 1$. Set

$$\mathbb{Z}_1 = \left\{ n \in \mathbb{Z}; a_n > \left(\frac{w_n}{v_n} c\right)^{1/\varepsilon_n} \right\}$$

$$\mathbb{Z}_2 = \left\{ n \in \mathbb{Z}; a_n \leqslant \left(\frac{w_n}{v_n} c\right)^{1/\varepsilon_n} \right\}.$$

Since $\mathbb{Z}_1$, $\mathbb{Z}_2$ are pairwise disjoint and $\mathbb{Z}_1 \cup \mathbb{Z}_2 = \mathbb{Z}$, we can write

$$\sum_{n\in\mathbb{Z}} a_n^{1-\varepsilon_n} w_n = \sum_{n\in\mathbb{Z}_1} a_n^{1-\varepsilon_n} w_n + \sum_{n\in\mathbb{Z}_2} a_n^{1-\varepsilon_n} w_n = I_1 + I_2.$$

Clearly,

$$I_1 = \sum_{n\in\mathbb{Z}_1} a_n\, a_n^{-\varepsilon_n} w_n \leqslant \sum_{n\in\mathbb{Z}_1} a_n \left(\frac{v_n}{w_n c}\right) w_n = \frac{1}{c} \sum_{n\in\mathbb{Z}_1} a_n v_n \leqslant \frac{1}{c}$$

and

$$\begin{aligned} I_2 &= \sum_{n\in\mathbb{Z}_2} a_n^{1-\varepsilon_n} w_n \leqslant \sum_{n\in\mathbb{Z}_2} \left(\frac{w_n}{v_n} c\right)^{\frac{1-\varepsilon_n}{\varepsilon_n}} w_n \\ &= \sum_{n\in\mathbb{Z}_2} \left(\frac{w_n}{v_n} c\right)^{1/\varepsilon_n} \left(\frac{v_n}{w_n c}\right) w_n = \frac{1}{c} \sum_{n\in\mathbb{Z}_2} \left(\frac{w_n}{v_n} c\right)^{1/\varepsilon_n} v_n < \infty, \end{aligned}$$

which proves the lemma.

LEMMA 3.4. *Let* $v, w, \varepsilon \in \mathcal{W}$, $\varepsilon_n < 1$. *Assume that*

$$\sum_{n\in\mathbb{Z}} \left(\frac{w_n}{v_n} c\right)^{1/\varepsilon_n} v_n = \infty \tag{3.2}$$

holds for each positive c. *Then there is a sequence* $a \in \mathcal{W}_0$, *such that*

$$\sum_{n\in\mathbb{Z}} a_n v_n \leqslant 1 \ \text{ and } \ \sum_{n\in\mathbb{Z}} a_n^{1-\varepsilon_n} w_n = \infty.$$

Proof. Set $N_0 = -1$. We will construct sequences $\{N_k\}_{k\in\mathbb{N}}$, $N_k \in \mathbb{N}$, and $\{c_k\}_{k\in\mathbb{N}}$, $c_k > 0$, satisfying for any $k \in \mathbb{N}$

$$0 < c_k \leqslant \frac{1}{2^k} \quad \text{and} \quad \sum_{N_{k-1}<|n|\leqslant N_k} \left(\frac{w_n}{v_n} c_k\right)^{1/\varepsilon_n} v_n = 1. \tag{3.3}$$

According to (3.2) we can find $N_1 \in \mathbb{N}$ such that

$$\sum_{|n|\leqslant N_1} \left(\frac{w_n}{v_n} \frac{1}{2}\right)^{1/\varepsilon_n} v_n \geqslant 1.$$

Then there exists a number $0 < c_1 \leqslant \frac{1}{2}$ such that

$$\sum_{|n|\leqslant N_1} \left(\frac{w_n}{v_n} c_1\right)^{1/\varepsilon_n} v_n = \sum_{N_0<|n|\leqslant N_1} \left(\frac{w_n}{v_n} c_1\right)^{1/\varepsilon_n} v_n = 1.$$

Assume that we have constructed positive integers $N_1 < N_2 < \cdots < N_k$ and real numbers $c_1, c_2, \ldots, c_k$ such that

$$0 < c_r \leqslant \frac{1}{2^r} \quad \text{and} \quad \sum_{N_{r-1}<|n|\leqslant N_r} \left(\frac{w_n}{v_n} c_r\right)^{1/\varepsilon_n} = 1$$

for $r = 1, 2, \ldots, k$.

According to (3.2), we can find N_{k+1} such that

$$\sum_{N_k<|n|\leqslant N_{k+1}} \left(\frac{w_n}{v_n}\frac{1}{2^{k+1}}\right)^{1/\varepsilon_n} \geqslant 1.$$

Then we can take c_{k+1} such that

$$0 < c_{k+1} \leqslant \frac{1}{2^{k+1}} \quad \text{and} \quad \sum_{N_k<|n|\leqslant N_{k+1}} \left(\frac{w_n}{v_n}c_{k+1}\right)^{1/\varepsilon_n} = 1,$$

which proves (3.3).

Define $a \in \mathcal{W}_0$ by

$$a_n = \left(c_k^{1/\varepsilon_n}\right)^{1/(1-\varepsilon_n)} \left(\frac{w_n}{v_n}\right)^{1/\varepsilon_n} \text{ if } N_{k-1} < |n| \leqslant N_k.$$

Using (3.3) we have

$$\begin{aligned}
\sum_{n\in\mathbb{Z}} a_n{}^{1-\varepsilon_n} w_n &= \sum_{k=1}^{\infty} \sum_{N_{k-1}<|n|\leqslant N_k} c_k^{1/\varepsilon_n} \left(\frac{w_n}{v_n}\right)^{(1-\varepsilon_n)/\varepsilon_n} w_n \\
&= \sum_{k=1}^{\infty} \sum_{N_{k-1}<|n|\leqslant N_k} \left(\frac{w_n}{v_n}c_k\right)^{1/\varepsilon_n} v_n \\
&= \sum_{k=1}^{\infty} 1 = \infty.
\end{aligned}$$

Let us estimate $\sum\limits_{n\in\mathbb{Z}} a_n v_n$. Since $c_k \leqslant 1$ and $1-\varepsilon_n^2 < 1$ we obtain

$$\left(c_k^{1/\varepsilon_n}\right)^{1/(1-\varepsilon_n)} \leqslant \left(c_k^{1/\varepsilon_n}\right)^{1+\varepsilon_n},$$

which implies with (3.3)

$$\begin{aligned}
\sum_{n\in\mathbb{Z}} a_n v_n &\leqslant \sum_{k=1}^{\infty} \sum_{N_{k-1}<|n|\leqslant N_k} \left(c_k^{1/\varepsilon_n}\right)^{1/(1-\varepsilon_n)} \left(\frac{w_n}{v_n}\right)^{1/\varepsilon_n} v_n \\
&\leqslant \sum_{k=1}^{\infty} \sum_{N_{k-1}<|n|\leqslant N_k} \left(c_k^{1/\varepsilon_n}\right)^{1+\varepsilon_n} \left(\frac{w_n}{v_n}\right)^{1/\varepsilon_n} v_n \\
&= \sum_{k=1}^{\infty} c_k \sum_{N_{k-1}<|n|\leqslant N_k} \left(\frac{w_n}{v_n}c_k\right)^{1/\varepsilon_n} v_n \\
&\leqslant \sum_{k=1}^{\infty} \frac{1}{2^k} = 1
\end{aligned}$$

and the lemma is proved.

4. Embedding

Let $p,q\in\mathcal{B}$, $v,w\in\mathcal{W}$. Denote $\mathcal{A}=\{n;p_n>q_n\}$, $\mathcal{B}=\{n;p_n\leqslant q_n\}$.

LEMMA 4.1. *Let p,q,v,w satisfy*

$$\inf_{c>0}\sum_{n\in\mathcal{A}}\left(\frac{w_n}{v_n}c\right)^{\frac{p_n}{p_n-q_n}}v_n+\sup_{n\in\mathcal{B}}v_n^{-\frac{q_n}{p_n}}w_n<\infty. \tag{4.1}$$

Then the implication

$$\sum_{n\in\mathbb{Z}}a_n^{p_n}v_n\leqslant 1\Rightarrow\sum_{n\in\mathbb{Z}}a_n^{q_n}w_n<\infty$$

holds for all $a\in\mathcal{W}_0$.

Proof. Let $\sum_{n\in\mathbb{Z}}a_n^{p_n}v_n\leqslant 1$. Set $b_n=a_n^{p_n}$ and $\varepsilon_n=\frac{p_n-q_n}{p_n}$. Thus, $\sum_{n\in\mathbb{Z}}b_nv_n\leqslant 1$. Moreover,

$$\sum_{n\in\mathbb{Z}}a_n^{q_n}w_n=\sum_{n\in\mathcal{A}}a_n^{q_n}w_n+\sum_{n\in\mathcal{B}}a_n^{q_n}w_n=I_1+I_2.$$

Estimate I_1. Since $\varepsilon_n\in(0,1)$ for $n\in\mathcal{A}$ we have by (4.1) for some $c>0$

$$\sum_{n\in\mathcal{A}}\left(\frac{w_n}{v_n}c\right)^{\frac{1}{\varepsilon_n}}v_n=\sum_{n\in\mathcal{A}}\left(\frac{w_n}{v_n}c\right)^{\frac{p_n}{p_n-q_n}}v_n<\infty$$

and by Lemma 3.3 we obtain

$$I_1=\sum_{n\in\mathcal{A}}a_n^{q_n}w_n=\sum_{n\in\mathcal{A}}b_n^{1-\varepsilon_n}w_n<\infty.$$

Estimate I_2. We have by (4.1)

$$\sup_{n\in\mathcal{B}}v_n^{-1-\varepsilon_n}w_n=\sup_{n\in\mathcal{B}}v_n^{-\frac{q_n}{p_n}}w_n<\infty.$$

and since $\varepsilon_n\leqslant 0$ for $n\in\mathcal{B}$ we obtain by Lemma 3.1

$$I_2=\sum_{n\in\mathcal{B}}a_n^{q_n}w_n=\sum_{n\in\mathcal{B}}b_n^{1-\varepsilon_n}w_n<\infty,$$

which finishes the proof.

LEMMA 4.2. *Let p,q,v,w do not satisfy (4.1). Then there exists $a\in\mathcal{W}_0$ such that*

$$\sum_{n\in\mathbb{Z}}a_n^{p_n}v_n\leqslant 1\quad\wedge\quad\sum_{n\in\mathbb{Z}}a_n^{q_n}w_n=\infty.$$

Proof. Since (4.1) is not satisfied then either

$$\inf_{c>0} \sum_{n\in\mathcal{A}} \left(\frac{w_n}{v_n}c\right)^{\frac{p_n}{p_n-q_n}} v_n = \infty \tag{4.2}$$

or

$$\sup_{n\in\mathcal{B}} v_n^{-\frac{q_n}{p_n}} w_n = \infty. \tag{4.3}$$

Assume first (4.2). Denote $\varepsilon_n = \frac{p_n-q_n}{p_n}$. By Lemma 3.4 there exists a sequence b_n such that

$$\sum_{n\in\mathcal{A}} b_n v_n \leqslant 1 \text{ and } \sum_{n\in\mathcal{A}} b_n^{1-\varepsilon_n} w_n = \infty. \tag{4.4}$$

Set

$$a_n = \begin{cases} b_n^{\frac{1}{p_n}} & \text{if } n\in\mathcal{A}; \\ 0 & \text{if } n\in\mathbb{Z}\setminus\mathcal{A}. \end{cases}$$

Thus,

$$\sum_{n\in\mathbb{Z}} a_n^{p_n} v_n = \sum_{n\in\mathcal{A}} b_n v_n \leqslant 1$$

and

$$\sum_{n\in\mathbb{Z}} a_n^{q_n} w_n = \sum_{n\in\mathcal{A}} b_n^{1-\varepsilon_n} w_n = \infty.$$

Assume now (4.3). Denote $\varepsilon_n = \frac{q_n-p_n}{p_n}$. Since

$$\sup_{n\in\mathcal{B}} v_n^{-\frac{q_n}{p_n}} w_n = \sup_{n\in\mathcal{B}} v_n^{-1-\varepsilon_n} w_n = \infty$$

there exists by Lemma 3.2 a sequence b_n such that

$$\sum_{n\in\mathcal{B}} b_n v_n \leqslant 1 \text{ and } \sum_{n\in\mathcal{B}} b_n^{1+\varepsilon_n} w_n = \infty.$$

Set

$$a_n = \begin{cases} b_n^{\frac{1}{p_n}} & \text{if } n\in\mathcal{B}; \\ 0 & \text{if } n\in\mathbb{Z}\setminus\mathcal{B}. \end{cases}$$

Thus,

$$\sum_{n\in\mathbb{Z}} a_n^{p_n} v_n = \sum_{n\in\mathcal{B}} b_n v_n \leqslant 1$$

and

$$\sum_{n\in\mathbb{Z}} a_n^{q_n} w_n = \sum_{n\in\mathcal{B}} b_n^{1+\varepsilon_n} w_n = \infty,$$

which proves the lemma.

The following theorem is an immediate consequence of Lemmas 2.1, 4.1 and 4.2.

THEOREM 4.3. *The embedding* $\ell^{p_n}(v_n) \hookrightarrow \ell^{q_n}(w_n)$ *holds if and only if* p, q, v, w *satisfy* (4.1).

REFERENCES

[1] D. CRUIZ-URIBE,A. FIORENZA AND C. J. NEUGEBAUER, *The maximal function on variable* L^p *spaces*, Ann. Acad. Sci. Fenn. Math., **28**, (1) (2003), 223–238.

[2] L. DIENING, *Maximal function on generalised Lebesgue spaces* $L^{p(\cdot)}$, Ann. Acad. Sci. Fenn. Math., **7**, (2) (2004), 245–254.

[3] L. DIENING, *Maximal function on Orlicz–Musielak spaces and generalized Lebesgue spaces*, preprint, 2003, (2003).

[4] D. E. EDMUNDS, J. LANG AND A. NEKVINDA, *On* $\ell^{p(x)}$ *norms*, Proc. Roy. Soc. Lond. A, **455**, (1999), 219–225.

[5] D. E. EDMUNDS, A. NEKVINDA, *Averaging operators on* $l^{\{p_n\}}$ *and* $L^{p(x)}$, Math. Inequal. Appl., **5**, (2) (2002), 235–246.

[6] D. E. EDMUNDS, J. RÁKOSNÍK, *Density of smooth functions in* $W^{k,p(x)}(\Omega)$, Proc. Roy. Soc. Lond. A, **437**, (1993), 153–167.

[7] D. E. EDMUNDS, J. RÁKOSNÍK, *Sobolev embeddings with variable exponent*, Studia Math., **143**, (2000), 267–293.

[8] P. HARJULEHTO, P. HÄSTÖ AND M. PERE, *Variable exponent Lebesgue spaces on metric spaces: The Hardy-Littlewood maximal operator*, Real Anal. Exange, **30**, (1) to appeare.

[9] O. KOVÁČIK, J. RÁKOSNÍK, *On spaces* $L^{p(x)}$ *and* $W^{k,p(x)}$, Czechoslovak Math. J., **41**, (1996), 167–177.

[10] A. LERNER, *Some remarks on the Hardy–Littlewood maximal function on variable* L^p *spaces*, preprint, 2004.

[11] A. NEKVINDA, *Equivalence of* $\ell^{\{p_n\}}$ *norms and shift operators*, Math. Inequal. Appl., **5**, (4) (2002), 711–723.

[12] A. NEKVINDA, *Hardy-Littlewood maximal operator on* $L^{p(x)}(\mathbb{R}^n)$, Math. Inequal. Appl., **7**, (2) (2004), 255–265.

[13] A. NEKVINDA, *A note on maximal operator on* $\ell^{\{p_n\}}$ *and* $L^{p(x)}(\mathbb{R})$, submitted to J. Funct. Spaces Appl., (2004)

[14] M. RŮŽIČKA, *Electrorheological fluids: Modeling and mathematical theory* Lecture Notes in Mathematics. 1748. Berlin: Springer, 2000.

[15] M. RŮŽIČKA, *Flow of shear dependent electrorheological fluids*, C.R.Acad. Sci. Paris Série, **I 329**, (1999), 393–398.

[16] S. G. SAMKO, *The density of* $C_0^\infty(\mathbb{R}^n)$ *in generalized Sobolev spaces* $W^{m,p(x)}(\mathbb{R}^n)$, Soviet Math. Doklady, **60**, (1999), 382–385.

(Received January 28, 2006)

Faculty of Civil Engineering
Czech technical University
Thákurova 7
166 29 Praha 6
Czech Republic
e-mail: nekvinda@fsv.cvut.cz