Mathematical
Inequalities
& Applications
Volume 10, Number 2 (2007), 403–415

SENSITIVITY ANALYSIS FOR PARAMETRIC GENERAL SET–VALUED MIXED VARITIONAL–LIKE INEQUALITY IN UNIFORMLY SMOOTH BANACH SPACE

K. R. KAZMI AND F. A. KHAN

(*communicated by R. Glowinski*)

Abstract. In this paper, using the concept of P-η-proximal mapping, we study the existence and sensitivity analysis of solution of a parametric general set-valued mixed variational-like inequality problem in uniformly smooth Banach space. The approach used in this paper may be treated as the extension and unification of approaches for studying sensitivity analysis for various important classes of variational inequalities given by many authors, see for example [2, 4, 6-8, 14, 15, 17-19].

1. Introduction

Variational inequality theory has become very effective and powerful tool for studying a wide range of problems arising in mechanics, contact problems in elasticity, optimization and control problems, management science, operation research, general equilibrium problems in economics and transportation, unilateral, obstacle, moving boundary valued problems etc., see for example [3, 9, 12]. Variational inequalities have been generalized and extended in different directions using novel annnovative techniques.

In recent years, much attention has been given to develop general methods for the sensitivity analysis of solution set of various classes of variational inequalities (inclusions). From the mathematical and engineering point of view, sensitivity properties of various classes of variational inequalities can provide new insight concerning the problem being studied and can stimulate ideas for solving problems. The sensitivity analysis of solution set for variational inequalities has been studied extensively by many authors using quite different methods. By using the projection technique, Dafermos [4], Mukherjee and Verma [15], Noor [17] and Yen [21] studied the sensitivity analysis of solution of some classes of variational inequalities with single-valued mappings. By using the implicit function approach that makes use of so-called normal mappings, Robinson [20] studied the sensitivity analysis of solutions for variational inequalities in

Mathematics subject classification (2000): 49J40, 47H05, 47J25, 47J20, 49J53.

Key words and phrases: parametric general variational-like inequality, P-η-proximal mapping, strongly mixed accretive mapping, mixed Lipschitz continuous mapping, generalized mixed pseudocontractive mapping, relaxed mixed Lipschitz mapping, sensitivity analysis.

Paper MIA-10-38

finite-dimensional spaces. By using resolvent operator technique, Adly [1], Noor [18], and Agarwal *et al*. [2] studied the sensitivity analysis of solution of some classes of quasi-variational inclusions with single-valued mappings.

Recently, by using projection and resolvent techniques, Ding and Luo [8], Liu *et al*. [14], Park and Jeong [19] and Ding [7], studied the behaviour and sensitivity analysis of solution set for some classes of generalized variational inequalities (inclusions) with set-valued mappings. It is worth mentioning that most of the results in this direction have been obtained in the setting of *Hilbert space*.

Inspired by recent research works in this area, in this paper, we consider a parametric general set-valued mixed variational-like inequality problem (PGSMVLIP, for short) in *uniformly smooth Banach space*. Further, using P-η-proximal mapping, we study the existence and sensitivity analysis of the solution of PGSMVLIP. The method presented in this paper can be used to generalize and improve the results given by many authors, see for example [2, 4, 6-8, 14, 15, 17-19].

2. Preliminaries

We assume that E is a real Banach space equipped with norm $\|\cdot\|$; E^* is the topological dual space of E; $C(E)$ is the family of all nonempty compact subsets of E; 2^E is the power set of E; $H(\cdot,\cdot)$ is the Hausdroff metric on $C(E)$, defined by

$$H(A,B) = \max\{\sup_{x\in A}\inf_{y\in B} d(x,y), \sup_{y\in B}\inf_{x\in A} d(x,y)\},\ A,B \in C(E);$$

$\langle\cdot,\cdot\rangle$ is the dual pair between E and E^*, and $J : E \longrightarrow 2^{E^*}$ is the *normalized duality mapping* defined by

$$J(x) = \{h \in E^* : \langle x,h\rangle = \|x\|^2, \|x\| = \|h\|\}, \quad x \in E.$$

We observe that if $E \equiv H$, a Hilbert space, then J is the identity map on H. In sequel, we shall denote a *selection* of normalized duality mapping J by j.

Now, we recall the following concepts and results.

DEFINITION 2.1. ([11]) Let $P : E \to E^\star$, $g : E \to E$ and $\eta : E \times E \to E$ be single-valued mappings, then

(i) P is said to be α*-strongly* η*-monotone*, if there exists a constant $\alpha > 0$ such that

$$\langle P(x) - P(y), \eta(x,y)\rangle \geqslant \alpha\|x-y\|^2 \ \ \forall x,y \in E;$$

(ii) g is said to be k*-strongly accretive*, if there exist a constant $k > 0$ and for any $x,y \in E$, $j(x-y) \in J(x-y)$ such that

$$\langle g(x) - g(y), j(x-y)\rangle \geqslant k\|x-y\|^2.$$

DEFINITION 2.2. ([5]) Let $\eta : E \times E \to E$ be a single-valued mapping. A proper functional $\phi : E \to R \cup \{+\infty\}$ is said to be η*-subdifferentiable* at a point $x \in E$ if there exists a point $h \in E^\star$ such that

$$\phi(y) - \phi(x) \geqslant \langle h, \eta(y,x)\rangle \ \ \forall y \in E,$$

where h is called *η-subgradient* of ϕ at x. The set of all η-subgradients of ϕ at x is denoted by $\partial\phi(x)$. The mapping $\partial\phi : E \to 2^{E^\star}$ defined by

$$\partial\phi(x) = \{h \in E^\star : \phi(y) - \phi(x) \geqslant \langle h, \eta(y,x)\rangle \ \ \forall y \in E\}$$

is said to be *η-subdifferential* of ϕ at x.

DEFINITION 2.3. ([5]) A functional $p : E \times E \to R \cup \{+\infty\}$ is said to be *0-diagonally quasi-concave* (0-DQCV, for short) in x, if for any finite set $\{x_1, \cdot, \cdot, \cdot, x_n\} \subset E$ and for any $y = \sum_{i=1}^n \lambda_i x_i$ with $\lambda_i \geqslant 0$ and $\sum_{i=1}^n \lambda_i = 1$, $\min_{1\leqslant i\leqslant n} p(x_i, y) \leqslant 0$ holds.

DEFINITION 2.4. ([11]) Let $\eta : E \times E \to E$ be a single-valued mapping. Let $\phi : E \to R \cup \{+\infty\}$ be a lower semicontinuous, η-subdifferentiable (may not be convex) and proper functional and $P : E \to E^\star$ be a nonlinear mapping. If for any given point $x^\star \in E^\star$ and $\rho > 0$, there exists a unique point $x \in E$ satisfying

$$\langle P(x) - x^\star, \eta(y,x)\rangle + \rho\phi(y) - \rho\phi(x) \geqslant 0 \ \ \forall y \in E,$$

then the mapping $x^\star \to x$, denoted by $P_\rho^{\partial\phi}(x^\star)$, is called *$P$-$\eta$-proximal mapping* of ϕ. Clearly, we have $x^\star - P(x) \in \rho\partial\phi(x)$ and then it follows that

$$P_\rho^{\partial\phi}(x^*) = (P + \rho\partial\phi)^{-1}(x^\star).$$

LEMMA 2.1. ([11]) *Let E be a real reflexive Banach space; let $\eta : E\times E \to E$ be a continuous mapping such that $\eta(y,y') + \eta(y',y) = 0 \ \forall y,y' \in E$; let $P : E \to E^\star$ be α-strongly η-monotone continuous mapping; let, for any given $x^\star \in E^\star$, the function $h(y,x) = \langle x^\star - P(x), \eta(y,x)\rangle$ be 0-DQCV in y and let $\phi : E \to R \cup \{+\infty\}$ be a lower semicontinuous, η-subdifferentiable and proper functional on E. Then for any given constant $\rho > 0$ and $x^\star \in E^\star$, there exists a unique $x \in E$ such that*

$$\langle P(x) - x^\star, \eta(y,x)\rangle \geqslant \rho\phi(x) - \rho\phi(y) \ \ \forall y \in E, \tag{2.1}$$

that is, $x = P_\rho{}^{\partial\phi}(x^\star)$.

REMARK 2.1. ([11]) Lemma 2.1 shows that for any strongly η-monotone continuous mapping $P : E \to E^\star$ and $\rho > 0$, the P-η-proximal mapping $P_\rho^{\partial\phi} : E^\star \to E$ of a lower semicontinuous, η-subdifferentiable and proper functional ϕ is well defined and for each $x^\star \in E^\star, x = P_\rho^{\partial\phi}(x^\star)$ is the unique solution of the problem (2.1).

LEMMA 2.2. ([11]) *Let E be a real reflexive Banach space and let $\eta : E\times E \to E$ be τ-Lipschitz continuous such that $\eta(y,y') + \eta(y',y) = 0 \ \forall y,y' \in E$; let $P : E \to E^\star$ be α-strongly η-monotone continuous mapping; let, for any given $x^\star \in E^\star$, the function $h(y,x) = \langle x^\star - P(x), \eta(y,x)\rangle$ be 0-DQCV in y; let $\phi : E \to R\cup\{+\infty\}$ be a lower semicontinuous, η-subdifferentiable and proper functional on E and let $\rho > 0$ be any given constant. Then the P-η-proximal mapping $P_\rho^{\partial\phi}$ of ϕ is τ/α-Lipschitz continuous.*

LEMMA 2.3. ([10]) *Let E be a real uniformly smooth Banach space and let $J : E \to E^\star$ be the normalized duality mapping. Then, for all $u, v \in E$, we have*

(a) $\|u+v\|^2 \leqslant \|u\|^2 + 2\langle v, J(u+v)\rangle$;

(b) $\langle u-v, Ju-Jv\rangle \leqslant 2d^2\rho_E(4\|u-v\|/d)$, *where* $d = \sqrt{(\|u\|^2+\|v\|^2)/2}$, $\rho_E(t) = \sup\{\frac{\|u\|+\|v\|}{2} - 1 : \|u\| = 1, \|v\| = t\}$ *is called the modulus of smoothness of* E.

LEMMA 2.4. ([13]) *Let X be a complete metric space, and let $T_1, T_2; X \to C(X)$ be θ - H - contraction mappings, then*

$$H(F(T_1), F(T_2)) \leqslant (1-\theta)^{-1} \sup_{x\in X} H(T_1(x), T_2(x)),$$

where $F(T_1)$ and $F(T_2)$ are the sets of fixed points of T_1 and T_2, respectively.

Let M be a nonempty open subset of E in which the parameter λ takes the values. Let $A, B, C, F, S, T : E\times M \to C(E^\star)$, $D : E\times M \to C(E)$ be set-valued mappings. Let $N, W : E^\star\times E^\star\times E^\star\times M \to E^\star$, $\eta : E\times E \to E$, and $g : E\times M \to E$ be single-valued mappings. Assume that $\phi : E\times E\times M \to R\cup\{+\infty\}$ be such that for each fixed $z\in E, \phi(.,z) : E\times E \to R\cup\{+\infty\}$ be a proper lower semicontinuous, η-subdifferentiable functional in the first argument such that $g(E)\cap dom\ \partial\phi(E,z,\lambda) \neq \emptyset$, $\forall z\in E, \lambda\in M$. We consider the following parametric general set-valued mixed variational-like inclusions problem (PGSMVLIP, for short): Find $x = x(\lambda)\in E, u = u(x,\lambda)\in A(x,\lambda), v = v(x,\lambda)\in B(x,\lambda), w = w(x,\lambda)\in C(x,\lambda), f = f(x,\lambda)\in F(x,\lambda), s = s(x,\lambda)\in S(x,\lambda), t = t(x,\lambda)\in T(x,\lambda)$ and $z = z(x,\lambda)\in D(x,\lambda)$ such that

$$\langle N(u,v,w,\lambda) - W(f,s,t,\lambda), \eta(y, g(x,\lambda))\rangle \geqslant \phi(g(x,\lambda),z,\lambda) - \phi(y,z,\lambda)\ \forall y\in E. \tag{2.2}$$

Now, for each fixed $\lambda\in\Omega$, the solution set $S(\lambda)$ of PGSMVLIP (2.2) is denoted as

$$\begin{aligned} S(\lambda) := \Big\{ & x = x(\lambda)\in E : u = u(x,\lambda)\in A(x,\lambda), v = v(x,\lambda)\in B(x,\lambda),\\ & w = w(x,\lambda)\in C(x,\lambda), f = f(x,\lambda)\in F(x,\lambda), s = s(x,\lambda)\in S(x,\lambda),\\ & t = t(x,\lambda)\in T(x,\lambda) \quad\text{and}\quad z = z(x,\lambda)\in D(x,\lambda) \quad\text{such that}\\ & \langle N(u,v,w,\lambda) - W(f,s,t,\lambda), \eta(y,g(x,\lambda))\rangle \geqslant \phi(g(x,\lambda),z,\lambda) - \phi(y,z,\lambda)\ \forall y\in E\Big\}. \end{aligned} \tag{2.3}$$

The aim of this paper is to study the behaviour and sensitivity analysis of the solution set $S(\lambda)$, and the conditions on these mappings $A, B, C, D, F, S, T, N, W, g, P, \phi$ under which the solution set $S(\lambda)$ of PGSMVLIP (2.2) is nonempty and Lipschitz continuous with respect to parameter $\lambda\in M$.

3. Sensitivity analysis of the solution set $S(\lambda)$

Throughout the rest of paper unless otherwise stated, let E be a real uniformly smooth Banach space with $\rho_E(t) \leqslant ct^2$ for some $c > 0$.

First, we define the following concepts.

DEFINITION 3.1. A mapping $g : E \times M \to E$ is said to be

(i) (L_g, l_g)*-mixed Lipschitz continuous*, if there exist constants $L_g, l_g > 0$ such that

$$\|g(x,\lambda) - g(y,\bar{\lambda})\| \leqslant L_g\|x-y\| + l_g\|\lambda - \bar{\lambda}\| \;\; \forall (x,\lambda),(y,\bar{\lambda}) \in E \times M;$$

(ii) ϵ_g*-strongly accretive*, if there exists a constant $\epsilon_g > 0$ such that

$$\langle g(x,\lambda) - g(y,\lambda), x-y\rangle \geqslant \epsilon_g\|x-y\|^2 \;\; \forall (x,\lambda),(y,\bar{\lambda}) \in E \times M.$$

DEFINITION 3.2. A set-valued mapping $A : E\times M \to C(E^\star)$ is said to be (L_A, l_A)*-mixed* H*-Lipschitz continuous*, if there exist constants $L_A, l_A > 0$ such that

$$H(A(x,\lambda), A(y,\bar{\lambda})) \leqslant L_A\|x-y\| + l_A\|\lambda - \bar{\lambda}\| \;\; \forall (x,\lambda),(y,\bar{\lambda}) \in E \times M.$$

DEFINITION 3.3. Let $P : E \to E^\star$, $g : E \times M \to E$ be mappings; let $A, B : E \times M \to 2^E$ be set-valued mappings and let $J^\star : E^\star \to E$ be a normalized duality mapping. A mapping $N : E^\star \times E^\star \times E^\star \times M \to E^\star$ is said to be

(i) $(L_{(N,1)}, L_{(N,2)}, L_{(N,3)}, l_N)$*-mixed Lipschitz continuous*, if there exist constants $L_{(N,1)}, L_{(N,2)}, L_{(N,3)}, l_N > 0$ such that

$$\begin{aligned} &\|N(x_1,y_1,z_1,\lambda_1) - N(x_2,y_2,z_2,\lambda_2)\| \\ &\qquad \leqslant L_{(N,1)}\|x_1-x_2\| + L_{(N,2)}\|y_1-y_2\| + L_{(N,3)}\|z_1-z_2\| + l_N\|\lambda_1-\lambda_2\| \\ &\qquad\qquad \forall (x_i,y_i,z_i,\lambda_i) \in E\times E\times E\times M \quad \text{for} \quad i=1,2; \end{aligned}$$

(ii) ξ*-strongly mixed* $P\circ g$*-accretive with respect to* A *and* B, if there exists a constant $\xi > 0$ such that

$$\begin{aligned} &\langle N(u_1,v_1,\lambda) - N(u_2,v_2,\lambda), J^\star(P\circ g(x,\lambda) - P\circ g(y,\lambda))\rangle \geqslant \xi\|x-y\|^2 \\ &\qquad \forall x,y \in E, \lambda \in M, u_1 \in A(x,\lambda), u_2 \in A(y,\lambda), v_1 \in B(x,\lambda), v_2 \in B(y,\lambda); \end{aligned}$$

(iii) σ*-generalized mixed* $P\circ g$*-pseudocontractive with respect to* A *and* B, if there exists a constant $\sigma > 0$ such that

$$\begin{aligned} &\langle N(u_1,v_1,\lambda) - N(u_2,v_2,\lambda), J^\star(P\circ g(x,\lambda) - P\circ (g-m)(y,\lambda))\rangle \leqslant \sigma\|x-y\|^2 \\ &\qquad \forall x,y \in E, \lambda \in M, u_1 \in A(x,\lambda), u_2 \in A(y,\lambda), v_1 \in B(x,\lambda), v_2 \in B(y,\lambda); \end{aligned}$$

(iv) ν*-relaxed mixed* $P\circ g$*-Lipschitz with respect to* A *and* B, if there exists a constant $\nu > 0$ such that

$$\begin{aligned} &\langle N(u_1,v_1,\lambda) - N(u_2,v_2,\lambda), J^\star(P\circ g(x,\lambda) - P\circ g(y,\lambda))\rangle \leqslant -\nu\|x-y\|^2 \\ &\qquad \forall x,y \in E, \lambda \in M, u_1 \in A(x,\lambda), u_2 \in A(y,\lambda), v_1 \in B(x,\lambda), v_2 \in B(y,\lambda), \end{aligned}$$

where $P\circ g$ denotes P composition g.

First, we prove the following technical lemma.

LEMMA 3.1. $x \in E$ *is a solution of PGSMVLIP* (2.2) *if and only if satisfies*

$$g(x,\lambda) \;=\; P_\rho^{\partial\phi(.,z,\lambda)}[P\circ g(x,\lambda) - \rho(N(u,v,w,\lambda) - W(f,s,t,\lambda))], \tag{3.1}$$

where $P_\rho^{\partial\phi(.,z,\lambda)} = (P + \rho\partial\phi(.,z,\lambda))^{-1}$ *is the* P-η*-proximal mapping of* ϕ *for each fixed* $z \in E$, $\lambda \in M$; $P : E \to E^\star$ *and* $\rho > 0$ *is a constant.*

Proof. Assume that $x \in E$ satisfies (3.1), that is,

$$g(x,\lambda) \ = \ P_\rho^{\partial\phi(.,z,\lambda)}[P\circ g(x,\lambda) - \rho(N(u,v,w,\lambda) - W(f,s,t,\lambda))].$$

Since $P_\rho^{\partial\phi(.,z,\lambda)} = (P + \rho\partial\phi(.,z,\lambda))^{-1}$, the above relation holds if and only if

$$P\circ g(x,\lambda) - \rho(N(u,v,w,\lambda) - W(f,s,t,\lambda)) \in P\circ g(x,\lambda) + \rho\partial\phi(g(x,\lambda),x,\lambda).$$

By the definition of η-subdifferential of $\phi(g(x,\lambda),x,\lambda)$, the above inclusion holds if and only if

$$\phi(y,z,\lambda) - \phi(g(x,\lambda),z,\lambda) \geqslant \langle N(u,v,w,\lambda) - W(f,s,t,\lambda), \eta(y,g(x,\lambda))\rangle \ \forall y \in E,$$

that is, $x \in E$ is a solution of PGSMVLIP (2.2). This completes the proof.

We consider the mapping $G(.,\lambda) : E \times M \to 2^E$ defined by

$$G(x,\lambda) = \bigcup_{\substack{u\in A(x,\lambda), v\in B(x,\lambda),\\ w\in C(x,\lambda), z\in D(x,\lambda),\\ f\in F(x,\lambda), s\in S(x,\lambda),\\ t\in T(x,\lambda)}} \Big[x - g(x,\lambda) + P_\rho^{\partial\phi(.,z,\lambda)}\Big(P\circ g(x,\lambda) - \rho(N(u,v,w,\lambda) - W(f,s,t,\lambda))\Big)\Big]. \tag{3.2}$$

REMARK 3.1. It follows from Lemma 3.1 that the fixed point of mapping G defined by (3.2) is a solution of PGSMVLIP (2.2).

Now, we show that the mapping G defined by (3.2) is a contraction mapping with respect to $x \in E$ uniformly in $\lambda \in M$.

THEOREM 3.1. *Let $A,B,C,F,S,T : E \times M \to C(E^\star)$ and $D : E \times M \to C(E)$ be mixed H-Lipschitz continuous with constants (L_A,l_A), (L_B,l_B), (L_C,l_C), (L_F,l_F), (L_S,l_S), (L_T,l_T), (L_D,l_D), respectively; let $N : E^\star \times E^\star \times E^\star \times M \to E^\star$ be $(L_{(N,1)}, L_{(N,2)}, L_{(N,3)}, l_N)$-mixed Lipschitz continuous; let $W : E^\star \times E^\star \times E^\star \times M \to E^\star$ be ν-mixed Lipschitz in first two arguments with respect to F and S, and σ-generalized mixed peudocontractive in the third argument with respect to T with constants ν and σ, respectively, and $(L_{(W,1)}, L_{(W,2)}, L_{(W,3)}, l_W)$-mixed Lipschitz continuous. Let $g : E \times M \to E$ be ϵ_g-strongly accretive and (L_g,l_g)-mixed Lipschitz continuous, and let $P\circ g$ be $(L_{P\circ g}, l_{P\circ g})$-mixed Lipschitz continuous. Let $\eta : E \times E \to E$ be τ-Lipschitz continuous such that $\eta(y,y') + \eta(y',y) = 0 \ \forall y,y' \in E$; let $P : E \to E^\star$ be α-strongly η-monotone continuous mapping and let, for any given $x^\star \in E^\star$, the function $h(y,x) = \langle x^\star - P(x), \eta(y,x)\rangle$ be 0-DQCV in y. Let $\phi : E \times E \times M \to R\cup\{+\infty\}$ be a proper, lower semicontinuous and, η-subdifferentiable functional such that $g(.,\lambda) \cap \partial\phi(.,y,\lambda) \neq \emptyset \ \forall y \in E, \lambda \in M$. Suppose that there exist constants $\mu_1, \mu_2 > 0$ such that*

$$\|P_\rho^{\partial\phi(.,x,\lambda)}(z) - P_\rho^{\partial\phi(.,y,\bar\lambda)}(z)\| \leqslant \mu_1\|x-y\| + \mu_2\|\lambda - \bar\lambda\| \quad \forall x,y,z \in E,\ \lambda,\bar\lambda \in M, \tag{3.3}$$

and suppose that there is a constant $\rho > 0$ such that

$$\theta := a + \epsilon(\rho); \quad a := \mu_1 L_D + \sqrt{1 - 2\epsilon_g + 64cL_g^2};$$
$$\epsilon(\rho) := r^{-1}\Big[\sqrt{L_{P\circ g}^2 - 2\rho(\nu - \sigma) + 64\rho^2 cL_W^2} + \rho L_N\Big]; \quad r := \frac{\delta}{\tau};$$
$$L_N := [L_A L_{(N,1)} + L_B L_{(N,2)} + L_C L_{(N,3)}];$$
$$L_W := [L_F L_{(W,1)} + L_S L_{(W,2)} + L_T L_{(W,3)}]; \tag{3.4}$$
$$\left|\rho - \frac{(\nu - \sigma) - 2rL_N(1-a)}{64cL_W^2 - L_N^2}\right| < \frac{\sqrt{[(\nu - \sigma) - 2rL_N(1-a)] - [L_{P\circ g}^2 - (1-a)r^2][64cL_W^2 - L_N^2]}}{64cL_W^2 - L_N^2}.$$

Then, the mapping G defined by (3.2) is compact-valued uniform θ-H-contraction with respect to $\lambda \in M$, where θ is given by (3.4). Moreover, for each $\lambda \in M$, the solution set $S(\lambda)$ of PGSMVLIP (2.2) is nonempty and closed.

Proof. Let (x,λ) be an arbitrary element in $E \times M$. Since A,B,C,F,S,T and D are compact-valued mappings then, for any sequences $\{u_n\} \subset A(x,\lambda)$, $\{v_n\} \subset B(x,\lambda)$, $\{w_n\} \subset C(x,\lambda)$, $\{f_n\} \subset F(x,\lambda)$, $\{s_n\} \subset S(x,\lambda)$, $\{t_n\} \subset T(x,\lambda)$, and $\{z_n\} \subset D(x,\lambda)$, there exist subsequences $\{u_{n_i}\} \subset \{u_n\}$, $\{v_{n_i}\} \subset \{v_n\}$, $\{w_{n_i}\} \subset \{w_n\}$, $\{f_{n_i}\} \subset \{f_n\}$, $\{s_{n_i}\} \subset \{s_n\}$, $\{t_{n_i}\} \subset \{t_n\}$, and $\{z_{n_i}\} \subset \{z_n\}$ and elements $x \in A(x,\lambda)$, $v \in B(x,\lambda)$, $w \in C(x,\lambda)$, $f \in F(x,\lambda)$, $s \in S(x,\lambda)$, $t \in T(x,\lambda)$, and $z \in D(x,\lambda)$ such that $u_{n_i} \to u$, $v_{n_i} \to v$, $w_{n_i} \to w$, $f_{n_i} \to f$, $s_{n_i} \to s$, $t_{n_i} \to t$, and $z_{n_i} \to z$, as $i \to \infty$. In view of (3.2) and mixed Lipschitz continuity of N and W, we estimate

$$\begin{aligned}
&\|P_\rho^{\partial\phi(.,z_{n_i},\lambda)}(P\circ g(x,\lambda) - \rho N(u_{n_i},v_{n_i},v_{n_i},\lambda) + \rho W(f_{n_i},s_{n_i},t_{n_i},\lambda))\\
&\quad - P_\rho^{\partial\phi(.,z,\lambda)}(P\circ g(x,\lambda) - \rho N(u,v,w,\lambda) + \rho M(f,s,t,\lambda))\|\\
&\leqslant \|P_\rho^{\partial\phi(.,z_{n_i},\lambda)}(P\circ g(x,\lambda) - \rho N(u_{n_i},v_{n_i},v_{n_i},\lambda) + \rho W(f_{n_i},s_{n_i},t_{n_i},\lambda))\\
&\quad - P_\rho^{\partial\phi(.,z,\lambda)}(P\circ g(x,\lambda) - \rho N(u_{n_i},v_{n_i},v_{n_i},\lambda) + \rho W(f_{n_i},s_{n_i},t_{n_i},\lambda))\\
&\quad + \|P_\rho^{\partial\phi(.,z,\lambda)}(P\circ g(x,\lambda) - \rho N(u_{n_i},v_{n_i},v_{n_i},\lambda) + \rho W(f_{n_i},s_{n_i},t_{n_i},\lambda))\\
&\quad - P_\rho^{\partial\phi(.,z,\lambda)}(P\circ g(x,\lambda) - \rho N(u,v,w,\lambda) + \rho W(f,s,t,\lambda))\| \qquad (3.5)\\
&\leqslant \mu_1\|z_{n_i} - z\| + \rho\frac{\tau}{\delta}\Big[\|N(u_{n_i},v_{n_i},v_{n_i},\lambda) - N(u,v,w,\lambda)\|\\
&\quad + \|W(f_{n_i},s_{n_i},t_{n_i},\lambda) - W(f,s,t,\lambda)\|\Big]\\
&\leqslant \mu_1\|z_{n_i} - z\| + \rho\frac{\tau}{\delta}\Big[L_{(N,1)}\|u_{n_i} - u\| + L_{(N,2)}\|v_{n_i} - v\| + L_{(N,3)}\|w_{n_i} - w\|\\
&\quad + L_{(W,1)}\|f_{n_i} - f\| + L_{(W,2)}\|s_{n_i} - s\| + L_{(W,3)}\|t_{n_i} - t\|\Big] \to 0 \text{ as } i \to \infty.
\end{aligned}$$

Thus, (3.2) and (3.5) yields that $G(x,\lambda) \in C(E)$.

Now, for each $\lambda \in M$, we prove that $G(x,\lambda)$ is a set-valued contraction mapping. Let (x_1,x_2,λ) be any arbitrary element in $E \times E \times M$ and for any $q_1 \in G(x_1,\lambda)$, there exist $u_1 = u_1(x_1,\lambda) \in A(x_1,\lambda)$, $v_1 = v_1(x_1,\lambda) \in B(x_1,\lambda)$, $w_1 = w_1(x_1,\lambda) \in C(x_1,\lambda)$, $f_1 = f_1(x_1,\lambda) \in F(x_1,\lambda)$, $s_1 = s_1(x_1,\lambda) \in S(x_1,\lambda)$, $t_1 = t_1(x_1,\lambda) \in T(x_1,\lambda)$ and $z_1 = z_1(x_1,\lambda) \in D(x_1,\lambda)$ such that

$$q_1 = x_1 - g(x_1,\lambda) + P_\rho^{\partial\phi(.,z_1,\lambda)}(P\circ g(x_1,\lambda) - \rho N(u_1,v_1,w_1,\lambda) + \rho W(f_1,s_1,t_1,\lambda)). \quad (3.6)$$

It follows from the compactness of $A(x_2,\lambda)$, $B(x_2,\lambda)$, $C(x_2,\lambda)$, $F(x_2,\lambda)$, $S(x_2,\lambda)$, $T(x_2,\lambda)$ and $D(x_2,\lambda)$, and Lipschitz continuity of A,B,C,F,S,T and D that there exist $u_2 = u_2(x_2,\lambda) \in A(x_2,\lambda)$, $v_2 = v_2(x_2,\lambda) \in B(x_2,\lambda)$, $w_2 = w_2(x_2,\lambda) \in C(x_2,\lambda)$, $f_2 = f_2(x_2,\lambda) \in F(x_2,\lambda)$, $s_2 = s_2(x_2,\lambda) \in S(x_2,\lambda)$, $t_2 = t_2(x_2,\lambda) \in T(x_2,\lambda)$ and $z_2 = z_2(x_2,\lambda) \in D(x_2,\lambda)$ satisfying

$$\begin{aligned}
\|u_1 - u_2\| &\leqslant H(A(x_1,\lambda),A(x_2,\lambda)) \leqslant L_A\|x_1 - x_2\|,\\
\|v_1 - v_2\| &\leqslant H(B(x_1,\lambda),B(x_2,\lambda)) \leqslant L_B\|x_1 - x_2\|,\\
\|w_1 - w_2\| &\leqslant H(C(x_1,\lambda),C(x_2,\lambda)) \leqslant L_C\|x_1 - x_2\|,\\
\|f_1 - f_2\| &\leqslant H(F(x_1,\lambda),F(x_2,\lambda)) \leqslant L_F\|x_1 - x_2\|,\\
\|s_1 - s_2\| &\leqslant H(S(x_1,\lambda),S(x_2,\lambda)) \leqslant L_S\|x_1 - x_2\|,\\
\|t_1 - t_2\| &\leqslant H(T(x_1,\lambda),T(x_2,\lambda)) \leqslant L_T\|x_1 - x_2\|,\\
\|z_1 - z_2\| &\leqslant H(D(x_1,\lambda),D(x_2,\lambda)) \leqslant L_D\|x_1 - x_2\|.
\end{aligned} \quad (3.7)$$

Let

$$\begin{aligned}
q_2 = x_2 - g(x_2,\lambda) + P_\rho^{\partial\phi(.,z_2,\lambda)}(P\circ g(x_1,\lambda)&\\
- \rho N(u_2,v_2,w_2,\lambda) + \rho W(f_2,s_2,t_2,\lambda)).&
\end{aligned} \quad (3.8)$$

then we have $q_2 \in G(x_2,\lambda)$.

It follows from (3.2) and Lemma 2.2 that

$$\begin{aligned}
\|q_1 - q_2\| \leqslant\ & \|x_1 - x_2 - (g(x_1,\lambda) - g(x_2,\lambda))\|\\
&+ \|P_\rho^{\partial\phi(.,z_1,\lambda)}[P\circ g(x_1,\lambda) - \rho(N(u_1,v_1,w_1,\lambda) - W(f_1,s_1,t_1,\lambda))]\|\\
&- \|P_\rho^{\partial\phi(.,z_2,\lambda)}[P\circ g(x_1,\lambda) - \rho(N(u_1,v_1,w_1,\lambda) - W(f_1,s_1,t_1,\lambda))]\|\\
&+ \|P_\rho^{\partial\phi(.,z_2,\lambda)}[P\circ g(x_1,\lambda) - \rho(N(u_1,v_1,w_1,\lambda) - W(f_1,s_1,t_1,\lambda))]\|\\
&- \|P_\rho^{\partial\phi(.,z_2,\lambda)}[P\circ g(x_2,\lambda) - \rho(N(u_2,v_2,w_2,\lambda) - W(f_2,s_2,t_2,\lambda))]\|\\
\leqslant\ & \|x_1 - x_2 - (g(x_1,\lambda) - g(x_2,\lambda))\| + \mu_1\|z_1 - z_2\|\\
&+ \frac{\tau}{\delta}\Big[\|P\circ(g-m)(x_1,\lambda) - P\circ(g-m)(x_2,\lambda) + \rho(W(f_1,s_1,t_1,\lambda)\\
&- W(f_2,s_2,t_2,\lambda))\| + \rho\|N(u_1,v_1,w_1,\lambda) - N(u_2,v_2,w_2,\lambda)\|\Big].
\end{aligned} \quad (3.9)$$

Using Lemma 2.3 and ϵ_g-strongly accretiveness and $(L_g,l_g)-$ mixed Lipschitz continuity of g, we have

$$\begin{aligned}
&\|x_1 - x_2 - (g(x_1,\lambda) - g(x_2,\lambda))\|^2 \\
&\quad \leqslant \|x_1 - x_2\|^2 - 2\langle g(x_1,\lambda) - g(x_2,\lambda), J(x_1 - x_2 - (g(x_1,\lambda) - g(x_2,\lambda)))\rangle \\
&\quad \leqslant \|x_1 - x_2\|^2 - 2\langle g(x_1,\lambda) - g(x_2,\lambda), J(x_1 - x_2)\rangle \\
&\qquad + 2\langle g(x_1,\lambda) - g(x_2,\lambda), J(x_1 - x_2) - J(x_1 - x_2 - (g(x_1,\lambda) - g(x_2,\lambda)))\rangle \\
&\quad \leqslant \|x_1 - x_2\|^2 - 2\epsilon_g\|x_1 - x_2\|^2 + 64cL_g^2\|x_1 - x_2\|^2 \\
&\quad \leqslant (1 - 2\epsilon_g + 64cL_g^2)\|x_1 - x_2\|^2.
\end{aligned} \tag{3.10}$$

Again, since N is $(L_{(N,1)}, L_{(N,2)}, L_{(N,3)}, l_N)$-mixed Lipschitz continuous and W is $(L_{(W,1)}, L_{(W,2)}, L_{(W,3)}, l_W)$-mixed Lipschitz continuous then we have

$$\begin{aligned}
&\|N(u_1, v_1, w_1, \lambda) - N(u_2, v_2, w_2, \lambda)\| \\
&\quad \leqslant L_{(N,1)}\|u_1 - u_2\| + L_{(N,2)}\|v_1 - v_2\| + L_{(N,3)}\|w_1 - w_2\| \\
&\quad \leqslant [L_A L_{(N,1)} + L_B L_{(N,1)} + L_C L_{(N,3)}]\|x_1 - x_2\|,
\end{aligned} \tag{3.11}$$

and

$$\begin{aligned}
&\|W(f_1, s_1, t_1, \lambda) - W(f_2, s_2, t_2, \lambda)\| \\
&\quad \leqslant [L_{(W,1)}\|f_1 - f_2\| + L_{(W,2)}\|s_1 - s_2\| + L_{(W,3)}\|t_1 - t_2\| \\
&\quad \leqslant [L_F L_{(W,1)} + L_S L_{(W,1)} + L_T L_{(W,3)}]\|x_1 - x_2\|.
\end{aligned} \tag{3.12}$$

Since W is ν-relaxed mixed Lipschitz in first two arguments with respect to F and S and σ-mixed generalized pseudocontractive in the third argument with respect to T then we have, using (3.12),

$$\begin{aligned}
&\|P\circ g(x_1,\lambda) - P\circ g(x_2,\lambda) + \rho(W(f_1, s_1, t_1, \lambda) - W(f_2, s_2, t_2, \lambda))\|^2 \\
&\quad \leqslant \|P\circ g(x_1,\lambda) - P\circ g(x_2,\lambda)\|^2 \\
&\qquad + 2\rho\langle W(f_1, s_1, t_1, \lambda) - W(f_2, s_2, t_1, \lambda), J^\star(P\circ g(x_1,\lambda) - P\circ g(x_2,\lambda))\rangle \\
&\qquad + 2\rho\langle W(f_2, s_2, t_1, \lambda) - W(f_2, s_2, t_2, \lambda), J^\star(P\circ g(x_1,\lambda) - P\circ g(x_2,\lambda))\rangle \\
&\qquad + 2\rho\langle W(f_1, s_1, t_1, \lambda) - W(f_2, s_2, t_2, \lambda), J^\star(P\circ g(x_1,\lambda) - P\circ g(x_2,\lambda) \\
&\qquad + \rho(W(f_1, s_1, t_1, \lambda) - W(f_2, s_2, t_2, \lambda)) - J^\star(P\circ g(x_1,\lambda) - P\circ g(x_2,\lambda))\rangle \\
&\quad \leqslant L_{P\circ g}^2\|x_1 - x_2\|^2 - 2\rho\nu\|x_1 - x_2\|^2 + 2\rho\sigma\|x_1 - x_2\|^2 \\
&\qquad + 64\rho^2 c\|W(f_1, s_1, t_1, \lambda) - W(f_2, s_2, t_2, \lambda)\|^2 \\
&\quad \leqslant \Big(L_{P\circ g}^2 - 2\rho(\nu - \sigma) + 64\rho^2 c[L_F L_{(W,1)} + L_S L_{(W,2)} + L_T L_{(W,3)}]^2\Big)\|x_1 - x_2\|^2.
\end{aligned} \tag{3.13}$$

Now, from (3.9), (3.10), (3.11), (3.12) and (3.13), we have

$$\begin{aligned}
\|q_1 - q_2\| \leqslant \Big[&(1 - 2\epsilon_g + 64cL_g^2)^{\frac{1}{2}} + \mu_1 L_D \\
&+ \frac{\tau}{\delta}\Big(\Big[L_{P\circ g}^2 - 2\rho(\nu - \sigma) + 64\rho^2 c[L_F L_{(W,1)} + L_S L_{(W,2)} + L_T L_{(W,3)}]^2\Big]^{\frac{1}{2}} \\
&+ \rho[L_A L_{(N,1)} + L_B L_{(N,1)} + L_C L_{(N,3)}]\Big)\Big]\|x_1 - x_2\|,
\end{aligned}$$

that is,

$$\|q_1 - q_2\| \leqslant \theta \|x_1 - x_2\|, \tag{3.14}$$

where

$$\theta := \mu_1 L_D + r^{-1}\Big(\sqrt{1-2\epsilon_g+64cL_g^2}+\sqrt{L_{P\circ g}^2-2\rho(\nu-\sigma)+64\rho^2 cL_W^2}+\rho L_N\Big). \tag{3.15}$$

Hence, we have

$$d(q_1, G(x_2,\lambda)) = \inf_{q_2 \in G(x_2,\lambda)} \|q_1 - q_2\| \leqslant \theta\|x_1 - x_2\|. \tag{3.16}$$

Since $q_1 \in G(x_1,\lambda)$ is arbitrary, we obtain

$$\sup_{q_1 \in G(x_1,\lambda)} d(q_1, G(x_2,\lambda)) \leqslant \theta\|x_1 - x_2\|. \tag{3.17}$$

By using same argument, we can prove

$$\sup_{q_2 \in G(x_2,\lambda)} d(q_2, G(x_1,\lambda)) \leqslant \theta\|x_1 - x_2\|. \tag{3.18}$$

By the definition of Hausdorff metric H on $C(E)$, (3.15) and (3.16), we obtain, $\forall (x_1,x_2,\lambda) \in E \times E \times M$,

$$H\Big(G(x_1,\lambda), G(x_2,\lambda)\Big) \leqslant \theta\|x_1 - x_2\|, \tag{3.19}$$

that is, $G(x,\lambda)$ is a uniform θ-H-contraction mapping with respect to $\lambda \in M$.

Let λ be in M and note that condition (3.4) ensure that $\theta < 1$ for $\rho > 0$ satisfying (3.4).

Thus $G(x,\lambda)$ is a set-valued contraction mapping which is uniform with respect to $\lambda \in M$. By a fixed point Theorem of Nadler [16], for each $\lambda \in M$, $G(x,\lambda)$ has a fixed point $x = x(\lambda) \in E$, that is, $x = x(\lambda) \in G(x,\lambda)$, and hence Lemma 3.1 ensure that $S(\lambda) \neq \emptyset$. Further, for any sequences $\{x_n\} \subset S(\lambda)$ with $\lim_{n\to\infty} x_n = x_0$, we have $x_n \in G(x_n,\lambda)$ for all $n \geqslant 1$. By virtue of (3.19), we have that

$$\begin{aligned} d(x_0, G(x_0,\lambda)) &\leqslant \|x_0 - x_n\| + H\Big(G(x_n,\lambda), G(x_0,\lambda)\Big) \\ &\leqslant (1+\theta)\|x_n - x_0\| \to 0 \text{ as } n \to \infty, \end{aligned}$$

that is, $x_0 \in G(x_0,\lambda)$ and $x_0 \in S(\lambda)$. Hence $S(\lambda)$ is closed in E. This completes the proof.

THEOREM 3.2. *Let $\lambda, \bar\lambda$ be in M, and the mappings A,B,C,F,S,T,D,N,W, $g,\phi,\eta,h,P\circ g$, be the same as in Theorem 3.1. If $\bar p = P\circ g(x,\bar\lambda) - \rho N(\bar u,\bar v,\bar w,\bar\lambda)) + \rho W(\bar f,\bar s,\bar t,\bar\lambda)$ and let conditions (3.3) and (3.4) of Theorem 3.1 hold, then for each $\lambda \in M$, the solution set $S(\lambda)$ of PGSMVLIP (2.2) is Lipschitz continuous from M to E.*

Proof. For each $\lambda, \bar{\lambda} \in M$, it follows from Theorem 3.1, $S(\lambda)$ and $S(\bar{\lambda})$ are both nonempty closed subsets of E and $G(x,\lambda)$ and $G(x,\bar{\lambda})$ are both set-valued θ-H-contraction mappings with same contractive constant $\theta \in (0,1)$. By Lemma 2.4, we obtain

$$H(S(\lambda), S(\bar{\lambda})) \leqslant \frac{1}{1-\theta} \sup_{x \in E} H(G(x,\lambda), G(x,\bar{\lambda})). \tag{3.20}$$

Now for any $p_1 \in G(x,\lambda)$, there exist $u = u(x,\lambda) \in A(x,\lambda)$, $v = v(x,\lambda) \in B(x,\lambda)$, $w = w(x,\lambda) \in C(x,\lambda)$, $f = f(x,\lambda) \in F(x,\lambda)$, $s = s(x,\lambda) \in S(x,\lambda)$, $t = t(x,\lambda) \in T(x,\lambda)$ and $z = z(x,\lambda) \in D(x,\lambda)$ satisfying

$$p_1 = x-(g-m)(x,\lambda)+P_\rho^{\partial\phi(.,z,\lambda)}\Big[P\circ g(x,\lambda)-\rho(N(u,v,w,\lambda)-W(f,s,t,\lambda))\Big]. \tag{3.21}$$

It is easy to see that there exist $\bar{u} = u(x,\bar{\lambda}) \in A(x,\bar{\lambda})$, $\bar{v} = v(x,\bar{\lambda}) \in B(x,\bar{\lambda})$, $\bar{w} = w(x,\bar{\lambda}) \in C(x,\bar{\lambda})$, $\bar{f} = f(x,\bar{\lambda}) \in F(x,\bar{\lambda})$, $\bar{s} = s(x,\bar{\lambda}) \in S(x,\bar{\lambda})$, $\bar{t} = t(x,\bar{\lambda}) \in T(x,\bar{\lambda})$ and $\bar{z} = z(x,\bar{\lambda}) \in D(x,\bar{\lambda})$ such that

$$\begin{aligned}
\|u-\bar{u}\| &\leqslant H(A(x,\lambda),A(x,\bar{\lambda})) \leqslant l_A\|\lambda-\bar{\lambda}\|,\\
\|v-\bar{v}\| &\leqslant H(B(x,\lambda),B(x,\bar{\lambda})) \leqslant l_B\|\lambda-\bar{\lambda}\|,\\
\|w-\bar{w}\| &\leqslant H(C(x,\lambda),C(x,\bar{\lambda})) \leqslant l_C\|\lambda-\bar{\lambda}\|,\\
\|f-\bar{f}\| &\leqslant H(F(x,\lambda),F(x,\bar{\lambda})) \leqslant l_F\|\lambda-\bar{\lambda}\|,\\
\|s-\bar{s}\| &\leqslant H(S(x,\lambda),S(x,\bar{\lambda})) \leqslant l_S\|\lambda-\bar{\lambda}\|,\\
\|t-\bar{t}\| &\leqslant H(T(x,\lambda),T(x,\bar{\lambda})) \leqslant l_T\|\lambda-\bar{\lambda}\|,\\
\|z-\bar{z}\| &\leqslant H(D(x,\lambda),D(x,\bar{\lambda})) \leqslant l_D\|\lambda-\bar{\lambda}\|.
\end{aligned} \tag{3.22}$$

Let

$$p_2 = x-(g-m)(x,\bar{\lambda})+P_\rho^{\partial\phi(.,\bar{z},\bar{\lambda})}\Big[P\circ g(x,\bar{\lambda})-\rho(N(\bar{u},\bar{v},\bar{w},\bar{\lambda})-W(\bar{f},\bar{s},\bar{t},\bar{\lambda}))\Big]. \tag{3.23}$$

Clearly, $p_2 \in G(x,\bar{\lambda})$.

Since N and W are mixed Lipschitz continuous and in view of (3.3) and (3.21)-(3.23) and with $\bar{p} = P\circ g(x,\bar{\lambda}) - \rho N(\bar{u},\bar{v},\bar{w},\bar{\lambda})) + \rho W(\bar{f},\bar{s},\bar{t},\bar{\lambda})$, we have

$$\begin{aligned}
\|p_1-p_2\| \leqslant{} & \|g(x,\lambda)-g(x,\bar{\lambda})\|\\
&+\|P_\rho^{\partial\phi(.,z,\lambda)}\Big(P\circ g(x,\lambda)-\rho(N(u,v,w,\lambda)-W(f,s,t,\lambda))\Big)-P_\rho^{\partial\phi(.,z,\lambda)}(\bar{p})\|\\
&+\|P_\rho^{\partial\phi(.,z,\lambda)}(\bar{p})-P_\rho^{\partial\phi(.,\bar{z},\lambda)}(\bar{p})\|+\|P_\rho^{\partial\phi(.,\bar{z},\lambda)}(\bar{p})-P_\rho^{\partial\phi(.,\bar{z},\bar{\lambda})}(\bar{p})\|\\
\leqslant{} & \|g(x,\lambda)-g(x,\bar{\lambda})\|+\frac{\tau}{\delta}\|P\circ g(x,\lambda)-P\circ g(x,\bar{\lambda})\|\\
&+\frac{\tau}{\delta}\rho\Big[\|N(u,v,w,\lambda)-N(\bar{u},v,w,\lambda)\|+\|N(\bar{u},v,w,\lambda)-N(\bar{u},\bar{v},w,\lambda)\|\\
&+\|N(\bar{u},\bar{v},w,\lambda)-N(\bar{u},\bar{v},\bar{w},\lambda)\|+\|N(\bar{u},\bar{v},\bar{w},\lambda)-N(\bar{u},\bar{v},\bar{w},\bar{\lambda})\|\\
&+\|W(f,s,t,\lambda)-W(\bar{f},s,t,\lambda)\|+\|W(\bar{f},s,t,\lambda)-W(\bar{f},\bar{s},t,\lambda)\|
\end{aligned}$$

$$+\|W(\bar{f},\bar{s},t,\lambda)-W(\bar{f},\bar{s},\bar{t},\lambda)\|+\|W(\bar{f},\bar{s},\bar{t},\lambda)-W(\bar{f},\bar{s},\bar{t},\bar{\lambda})\|\Big]$$
$$+\mu_1\|z-\bar{z}\|+\mu_2\|\lambda-\bar{\lambda}\|$$
$$\leqslant l_g\|\lambda-\bar{\lambda}\|+\frac{\tau}{\delta}l_{P\circ g}\|\lambda-\bar{\lambda}\|+\frac{\tau}{\delta}\rho\Big[l_AL_{(N,1)}+l_BL_{(N,2)}+l_CL_{(N,3)}+l_N \tag{3.24}$$
$$+l_FL_{(W,1)}+l_SL_{(W,2)}+l_TL_{(W,3)}+l_W\Big]\|\lambda-\bar{\lambda}\|+\mu_1 l_D+\mu_2\|\lambda-\bar{\lambda}\|$$
$$=\theta_1\|\lambda-\bar{\lambda}\|,$$

where
$$\theta_1:=l_g+\mu_2+\mu_1 l_D+\frac{\tau}{\delta}\Big[l_{P\circ g}+\rho(L_N+L_W)\Big]; \tag{3.25}$$
$L_N:=l_AL_{(N,1)}+l_BL_{(N,2)}+l_CL_{(N,3)}+l_N$ and $L_W:=l_FL_{(W,1)}+l_SL_{(W,2)}+l_TL_{(W,3)}+l_W$.

Hence, we obtain
$$\sup_{p_1\in G(x,\lambda)} d(p_1,G(x,\bar{\lambda}))\ \leqslant\ \theta_1\|\lambda-\bar{\lambda}\|.$$

By using similar argument, we have
$$\sup_{p_2\in G(x,\bar{\lambda})} d(p_2,G(x,\lambda))\ \leqslant\ \theta_1\|\lambda-\bar{\lambda}\|.$$

It follows that
$$H(G(x,\lambda),G(x,\bar{\lambda}))\ \leqslant\ \theta_1\|\lambda-\bar{\lambda}\|\ \ \forall(x,\lambda),(x,\bar{\lambda})\in E\times M. \tag{3.26}$$

By Lemma 2.4, we obtain
$$H(S(\lambda),S(\bar{\lambda}))\ \leqslant\ \frac{\theta_1}{1-\theta}\|\lambda-\bar{\lambda}\|. \tag{3.27}$$

This proves that $S(\lambda)$ is Lipschitz continuous in $\lambda\in M$.

REFERENCES

[1] S. ADLY, *Perturbed algorithms and sensitivity analysis for a general class of variational inclusions*, J. Math. Anal. Appl., **201**, (3) (1996), 609–630.
[2] R. P. AGARWAL, Y. J. CHO AND N. J. HUANG, *Sensitivity analysis for strongly nonlinear quasi-variational inclusions*, Appl. Math. Lett., **13**, (2002), 807–828.
[3] C. BAIOCCHI, A. CAPELO, *Variational and Quasi-variational Inequalities: Applications to Free Boundary Problems*, John Wiley and Sons, New York, 1984.
[4] S. DAFERMOS, *Sensitivity analysis in variational inequalities*, Math. Oper. Res., **13**, (1998), 421–434.
[5] X. P. DING, *Generalized quasi-variational-like inclusions with nonconvex functional*, Appl. Math. Comput., **122**, (2001), 267–282.
[6] X. P. DING, *Sensitivity analysis for generalized nonlinear implicit quasi-variational inclusions*, Appl. Math. Lett., **17**, (2) (2004), 225–235.
[7] X. P. DING, *Parametric completely generalized mixed implicit quasi-variational inclusions involving h-maximal monotone mappings*, J. Comput. Appl. Math., **182**, (2) (2005), 252–269.
[8] X. P. DING, C. L. LUO, *On parametric generalized quasi-variational inequalities*, J. Optim. Theory Appl., **100**, (1999), 195–205.
[9] F. GIANNESSI, A. MAUGERI, *Variational Inequalities and Network Equilibrium Problems*, Plenum Press, New York, 1995.

[10] K. GOEBEL, S. REICH, *Uniform Convexity, Hyperbolic Geometry and Nonexpansive Mappings*, Marcel Dekker, New York, 1995.

[11] K. R. KAZMI, M. I. BHAT, *Convergence and stability of iterative algorithms of generalized set-valued variational-like inclusions in Banach spaces*, Appl. Math. Comput., **166**, (2005), 164–180.

[12] N. KIKUCHI, J. T. ODEN, *Contact Problems in Elasticity, A Study of Variational Inequalities and Finite Element Methods*, SIAM, Philadalphia, 1988.

[13] T. C. LIM, *On fixed point stability for set-valued contractive mappings with applications to generalized differential equation*, J. Math. Anal. Appl., **110**, (1985), 436–441.

[14] Z. LIU, L. DEBNATH, S. M. KANG AND J. S. UME, *Sensitivity analysis for parametric completely generalized nonlinear implicit quasi-variational inclusions*, J. Math. Anal. Appl., **277**, (1) (2003), 142–154.

[15] R. N. MUKHERJEE, H. L. VERMA, *Sensitivity analysis of generalized variational inequalities*, J. Math. Anal. Appl., **167**, (1992), 299–304.

[16] S. B. NADLER JR, *Multivalued contractive mappings*, Pacific J. Math., **30**, (1969), 475–488.

[17] M. A. NOOR, *Sensitivity analysis framework for general quasi-variational inclusions*, Comput. Math. Appl., **44**, (2002), 1175–1181.

[18] M. A. NOOR, *Sensitivity analysis for quasivariational inclusions*, J. Math. Anal. Appl., **236**, (1999), 290–299.

[19] J. Y. PARK, J. U. JEONG, *Parametric generalized mixed variational inequalities*, Appl. Math. Lett., **17**, (2004), 43–48.

[20] S. M. ROBINSON, *Sensitivity analysis for variational inequalities by normal-map technique*, in: F. Giannessi, A. Maugeri (Eds.), Variational Inequalities and Network Equilibrium problems, Plenum Press, New York, (1995), 257–269.

[21] N. D. YEN, *Hölder continuity of solutions to a parametric variational inequality*, Appl. Math. Optim., **31**, (1995), 245–255.

(Received February 17, 2006)

Corresponding author: K. R. Kazmi
Department of Mathematics
Aligarh Muslim University
Aligarh-202002
India
e-mail: kr_kazmi@lycos.com

F. A. Khan
Department of Mathematics
Aligarh Muslim University
Aligarh-202002
India