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GENERALIZED WEIGHTED COMPOSITION OPERATORS FROM BERS–TYPE SPACES INTO BLOCH–TYPE SPACES

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Abstract. New criteria for the boundedness and the compactness of the generalized weighted composition operators from Bers-type spaces into Bloch-type spaces are given in this paper.

1. Introduction

Let $\mathbb{D}$ be the open unit disk in the complex plane $\mathbb{C}$ and $H(\mathbb{D})$ be the space of analytic functions on $\mathbb{D}$. Let $\alpha > 0$. The Bers-type space, denoted by H_α^∞, is the space consisting of all $f \in H(\mathbb{D})$ such that

$$\|f\|_{H_\alpha^\infty} = \sup_{z\in\mathbb{D}}(1-|z|^2)^\alpha |f(z)| < \infty.$$

H_α^∞ is a Banach space under the norm $\|\cdot\|_{H_\alpha^\infty}$. The little Bers-type space, denoted by $H_{\alpha,0}^\infty$, is the subspace of H_α^∞ consisting of those $f \in H_\alpha^\infty$ such that

$$\lim_{|z|\to 1}(1-|z|^2)^\alpha |f(z)| = 0.$$

Let $\beta > 0$. The Bloch-type space $\mathscr{B}^\beta$ is defined as the set of functions $f \in H(\mathbb{D})$ such that

$$B_\beta(f) = \sup_{z\in\mathbb{D}}(1-|z|^2)^\beta |f'(z)| < \infty.$$

$\mathscr{B}^\beta$ becomes a Banach space with the norm $\|f\|_{\mathscr{B}^\beta} = |f(0)| + B_\beta(f)$. When $\beta = 1$, $\mathscr{B}^1 = \mathscr{B}$ is the classical Bloch space. For more information on Bloch-type spaces on the unit disk, see, e.g., [34].

In this paper, φ always denotes a nonconstant analytic self-map of $\mathbb{D}$. The composition operator C_φ, induced by φ, is defined by

$$C_\varphi f = f \circ \varphi$$

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for $f \in H(\mathbb{D})$. A fundamental and interesting problem concerning composition operators is to relate function theoretic properties of φ to operator theoretic properties of C_φ on various spaces (see, e.g., [3]).

Let $u \in H(\mathbb{D})$. The weighted composition operator uC_φ, induced by φ and u, is defined by

$$(uC_\varphi f)(z) = u(z) \cdot f(\varphi(z)), \ \ f \in H(\mathbb{D}).$$

Let D be the differentiation operator and n be a nonnegative integer. Denote

$$Df = f', \ \ D^n f = f^{(n)}, \ \ f \in H(\mathbb{D}).$$

The generalized weighted composition operator $D^n_{\varphi,u}$, introduced by the author of this paper, is defined as follows (see [35, 36, 37]).

$$(D^n_{\varphi,u} f)(z) = u(z) \cdot f^{(n)}(\varphi(z)), \ \ f \in H(\mathbb{D}), \ \ z \in \mathbb{D}.$$

When $n=0$, then $D^n_{\varphi,u} = uC_\varphi$. When $n=0$ and $u(z) \equiv 1$, then we get the composition operator C_φ. When $n=1$, $u(z) = \varphi'(z)$, then $D^n_{\varphi,u} = DC_\varphi$. When $n=1$ and $u(z) = 1$, then $D^n_{\varphi,u} = C_\varphi D$. The operators DC_φ and $C_\varphi D$ were studied, for example, in [7, 9, 12, 13, 19, 20, 24, 27, 31, 33].

Composition operators, weighted composition operators and generalized weighted composition operators between Bers-type spaces and some other spaces in one, as well as, in several complex variables were studied in [4, 5, 6, 17, 18, 19, 21, 22, 30, 32, 35, 38], while composition operators, weighted composition operators and generalized weighted composition operators between Bloch-type spaces and some other spaces in one and several complex variables were studied, for example, in [1, 2, 8, 10, 11, 14, 15, 16, 21, 22, 23, 25, 26, 28, 29, 31, 33, 32, 36, 39].

In this paper, motivated by [1, 2], we give a new criterion for the boundedness or compactness of the operator $D^n_{\varphi,u}$ from Bers-type spaces to Bloch-type spaces, namely we use two families of functions to characterize the generalized weighted composition operators $D^n_{\varphi,u} : H^\infty_\alpha \to \mathscr{B}^\beta$.

Throughout the paper, C denotes a positive constant which may differ from one occurrence to the other. The notation $A \asymp B$ means that there exists a positive constant C such that $B/C \leqslant A \leqslant CB$.

2. Main results and proofs

In this section we give our main results and proofs. For this purpose, we need two lemmas as follows.

LEMMA 1. *Assume that $0 < \alpha < \infty$. Let $f \in H^\infty_\alpha$. Then there is a positive constant C independent of f such that*

$$|f^{(n)}(z)| \leqslant C \frac{\|f\|_{H^\infty_\alpha}}{(1-|z|^2)^{\alpha+n}}.$$

Proof. Using the fact

$$\sup_{z\in\mathbb{D}}(1-|z|^2)^{\alpha}|f(z)| \asymp |f(0)|+\sup_{z\in\mathbb{D}}(1-|z|^2)^{\alpha+1}|f'(z)|,$$

and the fact that for $f\in\mathscr{B}^{\beta}$ (see [34]),

$$\sup_{z\in\mathbb{D}}(1-|z|^2)^{\beta}|f'(z)| \asymp |f'(0)|+\cdots+|f^{(n-1)}(0)|+\sup_{z\in\mathbb{D}}(1-|z|^2)^{\beta+n-1}|f^{(n)}(z)|,$$

we immediately get the desired result. □

The following criterion follows from standard arguments similar to those outlined in Proposition 3.11 of [3].

LEMMA 2. *Let $u\in H(\mathbb{D})$, $0<\alpha,\beta<\infty$, φ be an analytic self-map of $\mathbb{D}$ and n be a nonnegative integer. The operator $D^n_{\varphi,u}: H^\infty_\alpha($ or $H^\infty_{\alpha,0})\to\mathscr{B}^{\beta}$ is compact if and only if $D^n_{\varphi,u}: H^\infty_\alpha($ or $H^\infty_{\alpha,0})\to\mathscr{B}^{\beta}$ is bounded and for any bounded sequence $(f_k)_{k\in\mathbb{N}}$ in $H^\infty_\alpha($ or $H^\infty_{\alpha,0})$ which converges to zero uniformly on compact subsets of $\mathbb{D}$, we have $\|D^n_{\varphi,u}f_k\|_{\mathscr{B}^{\beta}}\to 0$ as $k\to\infty$.*

For $a\in\mathbb{D}$, set

$$f_a(z)=\frac{(1-|a|^2)}{(1-\bar{a}z)^{\alpha+1}}, \quad \text{and} \quad g_a(z)=\left(\frac{1-|a|^2}{1-\bar{a}z}\right)f_a(z). \tag{1}$$

Next, we will use these two families of functions to characterize the generalized weighted composition operators $D^n_{\varphi,u}: H^\infty_\alpha\to\mathscr{B}^{\beta}$.

THEOREM 1. *Let $u\in H(\mathbb{D})$, $0<\alpha,\beta<\infty$, φ be an analytic self-map of $\mathbb{D}$ and n be a nonnegative integer. Then the following statements are equivalent:*

(a) *The operator $D^n_{\varphi,u}: H^\infty_\alpha\to\mathscr{B}^{\beta}$ is bounded;*

(b) *The operator $D^n_{\varphi,u}: H^\infty_{\alpha,0}\to\mathscr{B}^{\beta}$ is bounded;*

(c) *$u\varphi\in\mathscr{B}^{\beta}$, $u\in\mathscr{B}^{\beta}$,*

$$A:=\sup_{w\in\mathbb{D}}\|D^n_{\varphi,u}f_{\varphi(w)}\|_{\mathscr{B}^{\beta}}<\infty \quad and \quad B:=\sup_{w\in\mathbb{D}}\|D^n_{\varphi,u}g_{\varphi(w)}\|_{\mathscr{B}^{\beta}}<\infty;$$

(d)

$$M_1:=\sup_{z\in\mathbb{D}}\frac{(1-|z|^2)^{\beta}|u'(z)|}{(1-|\varphi(z)|^2)^{\alpha+n}}<\infty \tag{2}$$

and

$$M_2:=\sup_{z\in\mathbb{D}}\frac{(1-|z|^2)^{\beta}|\varphi'(z)||u(z)|}{(1-|\varphi(z)|^2)^{a+n+1}}<\infty. \tag{3}$$

Proof. (d) $\Rightarrow$ (a). Suppose that (d) holds. For arbitrary z in $\mathbb{D}$ and $f \in H_\alpha^\infty$, by Lemma 1 we have

$$\begin{aligned}
&(1-|z|^2)^\beta |(D_{\varphi,u}^n f)'(z)| \\
\leqslant\ & (1-|z|^2)^\beta |u'(z)||f^{(n)}(\varphi(z))| + (1-|z|^2)^\beta |f^{(n+1)}(\varphi(z))||u(z)\varphi'(z)| \\
\leqslant\ & C\frac{(1-|z|^2)^\beta |u'(z)|\|f\|_{H_\alpha^\infty}}{(1-|\varphi(z)|^2)^{\alpha+n}} + C\frac{(1-|z|^2)^\beta |u(z)||\varphi'(z)|\|f\|_{H_\alpha^\infty}}{(1-|\varphi(z)|^2)^{\alpha+n+1}} \\
\leqslant\ & C(M_1+M_2)\|f\|_{H_\alpha^\infty}.
\end{aligned} \tag{4}$$

Taking the supremum in (4) over $\mathbb{D}$ and then using the condition in (d) we see that $D_{\varphi,u}^n : H_\alpha^\infty \to \mathscr{B}^\beta$ is bounded.

(a) $\Rightarrow$ (b). This implication is obvious.

(b) $\Rightarrow$ (c). Assume $D_{\varphi,u}^n : H_{\alpha,0}^\infty \to \mathscr{B}^\beta$ is bounded. Taking the functions z^n and z^{n+1} and using the boundedness of $D_{\varphi,u}^n$ we see that

$$u\varphi \in \mathscr{B}^\beta \quad \text{and} \quad u \in \mathscr{B}^\beta.$$

For each $a \in \mathbb{D}$, it is easy to check that $f_a, g_a \in H_{\alpha,0}^\infty$. Moreover $\|f_a\|_{H_\alpha^\infty}$ and $\|g_a\|_{H_\alpha^\infty}$ are bounded by constants independent of a. By the boundedness of $D_{\varphi,u}^n : H_{\alpha,0}^\infty \to \mathscr{B}^\alpha$, we get

$$\sup_{a\in\mathbb{D}} \|D_{\varphi,u}^n f_{\varphi(a)}\|_{\mathscr{B}^\beta} \leqslant \|D_{\varphi,u}^n\| \sup_{a\in\mathbb{D}} \|f_{\varphi(a)}\|_{H_\alpha^\infty} \leqslant C\|D_{\varphi,u}^n\| < \infty$$

and

$$\sup_{a\in\mathbb{D}} \|D_{\varphi,u}^n g_{\varphi(a)}\|_{\mathscr{B}^\beta} \leqslant \|D_{\varphi,u}^n\| \sup_{a\in\mathbb{D}} \|g_{\varphi(a)}\|_{H_\alpha^\infty} \leqslant C\|D_{\varphi,u}^n\| < \infty,$$

as desired.

(c) $\Rightarrow$ (d). Suppose that $u\varphi \in \mathscr{B}^\beta$, $u \in \mathscr{B}^\beta$, A and B are finite. A calculation shows that

$$f_a^{(n)}(a) = \prod_{j=1}^{n}(\alpha+j)\frac{\overline{a}^n}{(1-|a|^2)^{\alpha+n}}, \quad g_a^{(n)}(a) = \prod_{j=2}^{n+1}(\alpha+j)\frac{\overline{a}^n}{(1-|a|^2)^{\alpha+n}}. \tag{5}$$

From (5), for $w \in \mathbb{D}$, we have

$$\begin{aligned}
(D_{\varphi,u}^n f_{\varphi(w)})'(w) =\ & \prod_{j=1}^{n}(\alpha+j)\frac{u'(w)\overline{\varphi(w)}^n}{(1-|\varphi(w)|^2)^{\alpha+n}} \\
& + \prod_{j=1}^{n+1}(\alpha+j)\frac{u(w)\varphi'(w)\overline{\varphi(w)}^{n+1}}{(1-|\varphi(w)|^2)^{1+\alpha+n}}.
\end{aligned} \tag{6}$$

Therefore

$$\begin{aligned}
&\frac{(1-|w|^2)^\beta|u'(w)||\varphi(w)|^n}{(1-|\varphi(w)|^2)^{\alpha+n}}\\
&\leqslant \frac{(1-|w|^2)^\beta|(D^n_{\varphi,u}f_{\varphi(w)})'(w)|}{\prod_{j=1}^n(\alpha+j)}+\frac{(\alpha+n+1)(1-|w|^2)^\beta|u(w)\varphi'(w)||\varphi(w)|^{n+1}}{(1-|\varphi(w)|^2)^{1+\alpha+n}}\\
&\leqslant \frac{\|(D^n_{\varphi,u}f_{\varphi(w)}\|_{\mathscr{B}^\beta}}{\prod_{j=1}^n(\alpha+j)}+\frac{(\alpha+n+1)(1-|w|^2)^\beta|u(w)\varphi'(w)||\varphi(w)|^{n+1}}{(1-|\varphi(w)|^2)^{1+\alpha+n}}\\
&\leqslant \frac{A}{\prod_{j=1}^n(\alpha+j)}+\frac{(\alpha+n+1)(1-|w|^2)^\beta|u(w)\varphi'(w)||\varphi(w)|^{n+1}}{(1-|\varphi(w)|^2)^{1+\alpha+n}}. \qquad (7)
\end{aligned}$$

In addition,

$$\begin{aligned}
(D^n_{\varphi,u}g_{\varphi(w)})'(w) &= \prod_{j=2}^{n+1}(\alpha+j)\frac{u'(w)\overline{\varphi(w)}^n}{(1-|\varphi(w)|^2)^{\alpha+n}}\\
&\quad+\prod_{j=2}^{n+2}(\alpha+j)\frac{u(w)\varphi'(w)\overline{\varphi(w)}^{n+1}}{(1-|\varphi(w)|^2)^{1+\alpha+n}}. \qquad (8)
\end{aligned}$$

Therefore, by multiplying (6) by $\alpha+n+1$ and (8) by $\alpha+1$, then subtracting such obtained equalities and using the triangle inequality, we obtain

$$\begin{aligned}
&\frac{|u(w)\varphi'(w)||\varphi(w)|^{n+1}}{(1-|\varphi(w)|^2)^{1+\alpha+n}}\\
&\leqslant \frac{\alpha+1+n}{\prod_{j=1}^{n+1}(\alpha+j)}|(D^n_{\varphi,u}f_{\varphi(w)})'(w)|+\frac{\alpha+1}{\prod_{j=1}^{n+1}(\alpha+j)}|(D^n_{\varphi,u}g_{\varphi(w)})'(w)|, \qquad (9)
\end{aligned}$$

which implies

$$\frac{(1-|w|^2)^\beta|u(w)\varphi'(w)||\varphi(w)|^{n+1}}{(1-|\varphi(w)|^2)^{1+\alpha+n}}\leqslant \frac{\alpha+1+n}{\prod_{j=1}^{n+1}(\alpha+j)}A+\frac{\alpha+1}{\prod_{j=1}^{n+1}(\alpha+j)}B. \qquad (10)$$

From (7) and (10), we get

$$\frac{(1-|w|^2)^\beta|u'(w)||\varphi(w)|^n}{(1-|\varphi(w)|^2)^{\alpha+n}}\leqslant \frac{\alpha+2+n}{\prod_{j=1}^{n}(\alpha+j)}A+\frac{\alpha+1}{\prod_{j=1}^{n}(\alpha+j)}B. \qquad (11)$$

Fix $r\in(0,1)$. If $|\varphi(w)|>r$, then from (10) we obtain

$$\frac{(1-|w|^2)^\beta|u(w)\varphi'(w)|}{(1-|\varphi(w)|^2)^{1+\alpha+n}}\leqslant \frac{1}{r^{n+1}}\Big(\frac{\alpha+1+n}{\prod_{j=1}^{n+1}(\alpha+j)}A+\frac{\alpha+1}{\prod_{j=1}^{n+1}(\alpha+j)}B\Big). \qquad (12)$$

On the other hand, if $|\varphi(w)|\leqslant r$, by the fact that

$$(1-|w|^2)^\beta|u(w)\varphi'(w)|\leqslant \|u\varphi\|_{\mathscr{B}^\beta}+\|u\|_{\mathscr{B}^\beta},$$

we get

$$\frac{(1-|w|^2)^\beta|u(w)\varphi'(w)|}{(1-|\varphi(w)|^2)^{1+\alpha+n}} \leqslant \frac{1}{(1-r^2)^{1+\alpha+n}}\Big(\|u\varphi\|_{\mathscr{B}^\beta}+\|u\|_{\mathscr{B}^\beta}\Big). \tag{13}$$

From (12) and (13) we see that M_2 is finite. Using similar arguments and (11) we can obtain that M_1 is finite as well. The proof of this theorem is finished. □

THEOREM 2. *Let $u\in H(\mathbb{D})$, $0<\alpha,\beta<\infty$, φ be an analytic self-map of $\mathbb{D}$ and n be a nonnegative integer. Suppose that the operator $D^n_{\varphi,u}: H^\infty_\alpha \to \mathscr{B}^\beta$ is bounded, then the following statements are equivalent:*

(a) *The operator $D^n_{\varphi,u}: H^\infty_\alpha \to \mathscr{B}^\beta$ is compact;*

(b) *The operator $D^n_{\varphi,u}: H^\infty_{\alpha,0} \to \mathscr{B}^\beta$ is compact;*

(c)

$$\lim_{|\varphi(w)|\to 1}\|D^n_{\varphi,u}f_{\varphi(w)}\|_{\mathscr{B}^\beta}=0 \quad and \quad \lim_{|\varphi(w)|\to 1}\|D^n_{\varphi,u}g_{\varphi(w)}\|_{\mathscr{B}^\beta}=0;$$

(d)

$$\lim_{|\varphi(z)|\to 1}\frac{(1-|z|^2)^\beta|u'(z)|}{(1-|\varphi(z)|^2)^{\alpha+n}}=0 \quad and \quad \lim_{|\varphi(z)|\to 1}\frac{(1-|z|^2)^\beta|u(z)\varphi'(z)|}{(1-|\varphi(z)|^2)^{1+\alpha+n}}=0.$$

Proof. (a) $\Longrightarrow$ (b). This implication is clear.

(b) $\Longrightarrow$ (c). Assume that $D^n_{\varphi,u}: H^\infty_{\alpha,0} \to \mathscr{B}^\beta$ is compact. Let $\{w_k\}_{k\in\mathbb{N}}$ be a sequence in $\mathbb{D}$ such that $\lim_{k\to\infty}|\varphi(w_k)|=1$ (if such a sequence does not exist then the limits in (c) automatically hold). Since the sequences $\{f_{\varphi(w_k)}\}$ and $\{g_{\varphi(w_k)}\}$ are bounded in $H^\infty_{\alpha,0}$ and converge to 0 uniformly on compact subsets of $\mathbb{D}$, by Lemma 2, we get

$$\|D^n_{\varphi,u}f_{\varphi(w_k)}\|_{\mathscr{B}^\beta}\to 0 \text{ and } \|D^n_{\varphi,u}g_{\varphi(w_k)}\|_{\mathscr{B}^\beta}\to 0 \tag{14}$$

as $k\to\infty$, which means that (c) holds.

(c) $\Longrightarrow$ (d). Suppose that the limits in (c) are 0. Using the inequality (9), we get

$$\begin{aligned}&\frac{(1-|w|^2)^\beta|u(w)\varphi'(w)|}{(1-|\varphi(w)|^2)^{1+\alpha+n}}\\ &\leqslant \frac{(\alpha+1+n)\|D^n_{\varphi,u}f_{\varphi(w)}\|_{\mathscr{B}^\beta}+(\alpha+1)\|D^n_{\varphi,u}g_{\varphi(w)}\|_{\mathscr{B}^\beta}}{\prod_{j=1}^{n+1}(\alpha+j)|\varphi(w)|^{n+1}}\to 0\end{aligned} \tag{15}$$

as $|\varphi(w)|\to 1$. Moreover, using (7), we deduce

$$\begin{aligned}&\frac{(1-|w|^2)^\beta|u'(w)|}{(1-|\varphi(w)|^2)^{\alpha+n}}\\ &\leqslant \frac{\|D^n_{\varphi,u}f_{\varphi(w)}\|_{\mathscr{B}^\beta}}{\prod_{j=1}^{n}(\alpha+j)|\varphi(w)|^n}+\frac{(\alpha+n+1)(1-|w|^2)^\beta|u(w)\varphi'(w)||\varphi(w)|}{(1-|\varphi(w)|^2)^{1+\alpha+n}}\to 0,\end{aligned} \tag{16}$$

as $|\varphi(w)| \to 1$. The desired result follows.

(d) $\Longrightarrow$ (a). Assume that (d) holds. By (d), we have that for any $\varepsilon > 0$, there is a constant δ, $0 < \delta < 1$, such that

$$\frac{(1-|z|^2)^\beta |u'(z)|}{(1-|\varphi(z)|^2)^{\alpha+n}} < \varepsilon \quad \text{and} \quad \frac{(1-|z|^2)^\beta |u(z)\varphi'(z)|}{(1-|\varphi(z)|^2)^{\alpha+1+n}} < \varepsilon, \tag{17}$$

whenever $\delta < |\varphi(z)| < 1$.

Let $(f_k)_{k\in\mathbb{N}}$ be a sequence in H_α^∞ with $\sup_{k\in\mathbb{N}} \|f_k\|_{H_\alpha^\infty} \leqslant M$ and $f_k \to 0$ uniformly on compact subsets of $\mathbb{D}$ as $k \to \infty$. In light of Lemma 2, we only need to show that $\|D_{\varphi,u}^n f_k\|_{\mathscr{B}^\beta} \to 0$ as $k \to \infty$. Using (17), for $|\varphi(w)| > r$, we have

$$\begin{aligned}
&(1-|w|^2)^\beta |(D_{\varphi,u}^n f_k)'(w)| \\
&\leqslant (1-|w|^2)^\beta |u'(w) f_k^{(n)}(\varphi(w))| + (1-|w|^2)^\beta |u(w) f_k^{(n+1)}(\varphi(w))\varphi'(w)| \\
&\leqslant C\|f_k\|_{H_\alpha^\infty} \left(\frac{(1-|w|^2)^\beta |u'(w)|}{(1-|\varphi(w)|^2)^{\alpha+n}} + \frac{(1-|w|^2)^\beta |u(w)\varphi'(w)|}{(1-|\varphi(w)|^2)^{1+\alpha+n}} \right) < 2MC\varepsilon.
\end{aligned}$$

By Cauchy's estimate, if f_k is a sequence which converges on compact subset of $\mathbb{D}$ to zero, then the sequence $f_k^{(n)}$ also converges on compact subset of $\mathbb{D}$ to zero as $k \to \infty$. Hence, for $|\varphi(w)| \leqslant r$, we have

$$(1-|w|^2)^\beta |(D_{\varphi,u}^n f_k)'(w)| < \varepsilon, \ \text{ as } \ k \to \infty.$$

Since $|u(0) f_k^{(n)}(\varphi(0))| \to 0$ as $k \to \infty$, we obtain that

$$\begin{aligned}
&\|D_{\varphi,u}^n f_k\|_{\mathscr{B}^\beta} = |u(0) f_k^{(n)}(\varphi(0))| + \sup_{w\in\mathbb{D}} (1-|w|^2)^\beta |(D_{\varphi,u}^n f_k)'(w)| \\
&= \sup_{|\varphi(w)|>r} (1-|w|^2)^\beta |(D_{\varphi,u}^n f_k)'(w)| + \sup_{|\varphi(w)|\leqslant r} (1-|w|^2)^\beta |(D_{\varphi,u}^n f_k)'(w)| \to 0,
\end{aligned}$$

as $k \to \infty$. Hence the operator $D_{\varphi,u}^n : H_\alpha^\infty \to \mathscr{B}^\beta$ is compact by Lemma 2. The proof of this theorem is finished. □

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REFERENCES

[1] F. COLONNA AND S. LI, *Weighted composition operators from the Besov spaces to the Bloch spaces*, Bull. Malaysian Math. Sci. Soc., to appear.

[2] F. COLONNA AND S. LI, *Weighted composition operators from Hardy spaces into logarithmic Bloch spaces*, J. Funct. Spaces Appl., Vol. 2012, Article ID 454820, 20 pages.

[3] C. C. COWEN AND B. D. MACCLUER, *Composition Operators on Spaces of Analytic Functions*, Studies in Advanced Mathematics, CRC Press, Boca Raton, 1995.

[4] X. FU AND X. ZHU, *Weighted composition operators on some weighted spaces in the unit ball*, Abstr. Appl. Anal. **2008** (2008), Article ID 605807, 8 pages.

[5] D. GU, *Weighted composition operators from generalized weighted Bergman spaces to weighted-type space*, J. Inequal. Appl. **2008** (2008), Article ID 619525, 14 pages.

[6] W. HE AND L. JIANG, *Composition operator on Bers-type spaces*, Acta Math. Sci. **22B**, 3 (2002), 404–412.

[7] R. A. HIBSCHWEILER AND N. PORTNOY, *Composition followed by differentiation between Bergman and Hardy spaces*, Rocky Mountain J. Math. **35**, 3 (2005), 843–855.

[8] S. LI AND S. STEVIĆ, *Weighted composition operators from Bergman-type spaces into Bloch spaces*, Proc. Indian Acad. Sci. Math. Sci. **117** (2007), 371–385.

[9] S. LI AND S. STEVIĆ, *Composition followed by differentiation between Bloch type spaces*, J. Comput. Anal. Appl. **9** (2007), 195–205.

[10] S. LI AND S. STEVIĆ, *Generalized composition operators on Zygmund spaces and Bloch type spaces*, J. Math. Anal. Appl. **338** (2008), 1282–1295.

[11] S. LI AND S. STEVIĆ, *Weighted composition operators from Zygmund spaces into Bloch spaces*, Appl. Math. Comput. **206**, 2 (2008), 825–831.

[12] S. LI AND S. STEVIĆ, *Composition followed by differentiation from mixed-norm spaces to α-Bloch spaces*, Sb. Math. **199**, 12 (2008), 1847–1857.

[13] S. LI AND S. STEVIĆ, *Composition followed by differentiation between H^∞ and α-Bloch spaces*, Houston J. Math. **35** (2009), 327–340.

[14] Z. LOU, *Composition operators on Bloch type spaces*, Analysis (Munich) **23** (2003), 81–95.

[15] K. MADIGAN AND A. MATHESON, *Compact composition operators on the Bloch space*, Trans. Amer. Math. Soc. **347** (1995), 2679–2687.

[16] S. OHNO, K. STROETHOFF AND R. ZHAO, *Weighted composition operators between Bloch-type spaces*, Rocky Mountain J. Math. **33** (2003), 191–215.

[17] S. STEVIĆ, *Weighted composition operators between mixed norm spaces and H_α^∞ spaces in the unit ball*, J. Inequal. Appl. **2007** (2007), Article ID 28629, 9 pages.

[18] S. STEVIĆ, *Generalized composition operators between mixed norm space and some weighted spaces*, Numer. Funct. Anal. Optimization **29** (2008), 959–978.

[19] S. STEVIĆ, *Norm and essential norm of composition followed by differentiation from α-Bloch spaces to H_μ^∞*, Appl. Math. Comput. **207** (2009), 225–229.

[20] S. STEVIĆ, *Products of composition and differentiation operators on the weighted Bergman space*, Bull. Belg. Math. Soc. Simon Stevin **16** (2009), 623–635.

[21] S. STEVIĆ, *Weighted differentiation composition operators from mixed-norm spaces to weighted-type spaces*, Appl. Math. Comput. **211** (2009), 222–233.

[22] S. STEVIĆ, *Weighted differentiation composition operators from mixed-norm spaces to the nth weighted-type space on the unit disk*, Abstr. Appl. Anal. **2010** (2010), Article ID 246287, 15 pages.

[23] S. STEVIĆ, *Weighted iterated radial composition operators between some spaces of holomorphic functions on the unit ball*, Abstr. Appl. Anal. **2010** (2010), Article ID 801264, 14 pages.

[24] S. STEVIĆ, *Composition followed by differentiation from H^∞ and the Bloch space to nth weighted-type spaces on the unit disk*, Appl. Math. Comput. **216** (2010), 3450–3458.

[25] S. STEVIĆ, *Weighted differentiation composition operators from H^∞ and Bloch spaces to nth weigthed-type spaces on the unit disk*, Appl. Math. Comput. **216** (2010), 3634–3641.

[26] S. STEVIĆ, *On a product-type operator from Bloch spaces to weighted-type spaces on the unit ball*, Appl. Math. Comput. **217** (2011), 5930–5935.

[27] S. STEVIĆ, *Characterizations of composition followed by differentiation between Bloch-type spaces*, Appl. Math. Comput. **218** (2011), 4312–4316.

[28] S. STEVIĆ AND A. K. SHARMA, *Iterated differentiation followed by composition from Bloch-type spaces to weighted BMOA spaces*, Appl. Math. Comput. **218** (2011), 3574–3580.
[29] S. STEVIĆ, A. K. SHARMA AND A. BHAT, *Essential norm of products of multiplication composition and differentiation operators on weighted Bergman spaces*, Appl. Math. Comput. **218** (2011), 2386–2397.
[30] M. WANG AND Y. LIU, *Weighted composition operator between Bers-type spaces*, Acta Math Sci **27A**, 4 (2007), 665–671.
[31] Y. WU AND H. WULAN, *Products of differentiation and composition operators on the Bloch space*, Collect. Math. **63** (2012), 93–107.
[32] W. YANG, *Weighted composition operators from Bloch-type spaces to weighted-type spaces*, Ars Combin. **92** (2009), 415–423.
[33] W. YANG, *Products of composition and differentiation operators from $\mathcal{Q}_K(p,q)$ spaces to Bloch-type spaces*, Abstr. Appl. Anal. **2009** (2009), Article ID 741920, 14 pages.
[34] K. ZHU, *Bloch type spaces of analytic functions*, Rocky Mountain J. Math. **23**, 3 (1993), 1143–1177.
[35] X. ZHU, *Products of differentiation, composition and multiplication from Bergman type spaces to Bers type space*, Integ. Tran. Spec. Funct. **18**, 3 (2007), 223–271.
[36] X. ZHU, *Generalized weighted composition operators from Bloch-type spaces to weighted Bergman spaces*, Indian J. Math. **49** (2007), 139–149.
[37] X. ZHU, *Generalized weighted composition operators on weighted Bergman spaces*, Numer. Funct. Anal. Opt. **30** (2009), 881-893.
[38] X. ZHU, *Weighted composition operators from $F(p,q,s)$ spaces to H^∞_μ spaces*, Abstr. Appl. Anal. **2009** (2009), Article ID 290978, 12 pages.
[39] X. ZHU, *Generalized weighted composition operators on Bloch-type spaces*, Ars Combin., to appear.

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