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ESSENTIAL NORM OF INTEGRAL OPERATORS ON MORREY TYPE SPACES

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Abstract. In this paper, we investigate the essential norm of two classes of integral operators on Morrey type spaces H^2_K. As an application, we characterize the compactness of these operators.

1. Introduction

Let $\mathbb{D}$ denote the open unit disk in the complex plane $\mathbb{C}$ and $\partial\mathbb{D}$ be its boundary. As usual, $H(\mathbb{D})$ denotes the class of all analytic functions on $\mathbb{D}$. Let $0 < p < \infty$. The Hardy space H^p consists of all $f \in H(\mathbb{D})$ such that

$$\|f\|_{H^p}^p = \sup_{0<r<1} \frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p d\theta < \infty.$$

As usual, H^∞ denote the space of bounded analytic function. We say that an $f \in H^2$ belongs to the $BMOA$ space, if

$$\|f\|_*^2 = \sup_{I\subseteq\partial\mathbb{D}} \frac{1}{|I|} \int_I |f(\zeta) - f_I|^2 \frac{d\zeta}{2\pi} < \infty.$$

Here

$$f_I = \frac{1}{|I|} \int_I f(\zeta) \frac{d\zeta}{2\pi}, \; I \subseteq \partial\mathbb{D}.$$

Under the norm $\|f\|_{BMOA} = |f(0)| + \|f\|_*$, $BMOA$ is a Banach space. From [6], we know that $\|f\|_*$ is comparable with $\sup_{w\in\mathbb{D}} \|f\circ\sigma_w - f(w)\|_{H^2}$, where $\sigma_w(z) = \frac{w-z}{1-\overline{w}z}$ is a Möbius transformation of $\mathbb{D}$. We say that an $f \in H(\mathbb{D})$ belongs to the $VMOA$ space, if

$$\lim_{|w|\to 1} \|f\circ\sigma_w - f(w)\|_{H^2} = 0.$$

Let $K : [0,\infty) \to [0,\infty)$ be a right-continuous and nondecreasing function. An $f \in H^2$ is said to belong to the Morrey type space, denoted by H^2_K, if (see [29])

$$\|f\|_{H^2_K}^2 = \sup_{I\subseteq\partial\mathbb{D}} \frac{1}{K(|I|)} \int_I |f(\zeta) - f_I|^2 \frac{d\zeta}{2\pi} < \infty.$$

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When $K(t)=t$, H_K^2 is just the $BMOA$ space. When $K(t)=t^{\lambda}$ $(0<\lambda<1)$, the space H_K^2 gives the classical Morrey space $\mathscr{L}^{2,\lambda}$, which was first studied by Wu and Xie in [28] in the case of the unit disk. H_K^2 was recently introduced by Wulan and Zhou in [29]. Morrey space was first studied in [16] by Morrey for solutions of partial differential equations.

Let $g\in H(\mathbb{D})$. In [17], Pommerenke introduced an integral operator as follows.

$$J_g f(z)=\int_0^z f(\xi)g'(\xi)d\xi, \qquad f\in H(\mathbb{D}), \qquad z\in\mathbb{D}.$$

The operator J_g is called the Volterra type operator, which can be seen as a generalization of the classical Cesàro operator (see [3, 19]). Pommerenke showed that J_g is bounded on H^2 if and only if $g\in BMOA$. Aleman and Siskakis studied the operator J_g on Hardy spaces and weighted Bergman spaces in [1, 2]. Similarly, the companion integral operator was defined by

$$I_g f(z)=\int_0^z f'(\xi)g(\xi)d\xi, \qquad f\in H(\mathbb{D}), \qquad z\in\mathbb{D}.$$

The multiplication operator M_g is defined by

$$M_g f(z)=f(z)g(z), \qquad f\in H(\mathbb{D}), \qquad z\in\mathbb{D}.$$

It is easy to see that

$$J_g f+I_g f+f(0)g(0)=M_g f.$$

Recently, the boundedness, compactness and essential norm of the operators J_g and I_g between some spaces of holomorphic functions, as well as their extensions on the unit ball in $\mathbb{C}^n$, were investigated, for example, in [1, 2, 7, 8, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 27, 30, 31, 32] (see also the related references therein).

In [10], the authors studied the operators J_g, I_g and M_g on the Morrey space $\mathscr{L}^{2,\lambda}$ $(0<\lambda<1)$. Motivated by [10], Qian and the second author of this paper studied he boundedness of J_g, I_g and M_g on the Morrey type space H_K^2 in [18]. We proved that, under some conditions posed on K, I_g is bounded on H_K^2 if and only if $g\in H^\infty$, as well as J_g is bounded on H_K^2 if and only if $g\in BMOA$.

In this paper, we continue our study of these operators on the Morrey type space H_K^2 and study the essential norm of J_g and I_g on H_K^2. As an application, we get the characterization of the compactness of J_g, I_g and M_g on H_K^2.

Recall that the essential norm of a bounded linear operator $T:X\to Y$ is its distance to the set of compact operators S mapping X to Y, that is,

$$\|T\|_{e,X\to Y}=\inf\{\|T-S\|_{X\to Y}: S \text{ is compact}\},$$

where X and Y are two Banach spaces and $\|\cdot\|_{X\to Y}$ is the operator norm.

For our aim, we need some constraints on K in the rest of this paper. By [4], we may assume that K is defined on $[0,1]$ and extend its domain to $[0,\infty)$ by setting $K(t)=K(1)$ for $t>1$. We also assume that $K(t)\approx K(2t)$. The symbol $f\approx g$ means that $f \lesssim g \lesssim f$. We say that $f \lesssim g$ if there exists a constant C such that $f \leqslant Cg$. Finally, we assume that

$$\int_0^1 \frac{\varphi_K(s)}{s} ds < \infty, \tag{1}$$

$$\int_1^\infty \frac{\varphi_K(s)}{s^2} ds < \infty, \tag{2}$$

where

$$\varphi_K(s) = \sup_{0 \leqslant t \leqslant 1} K(st)/K(t), \quad 0 < s < \infty.$$

2. Main results and proofs

In this section, we give our main results and proofs. For this purpose, we need some auxiliary results.

LEMMA 1. [29] *Let K satisfy the conditions* (1) *and* (2)*. Then the following statements are equivalent.*

(1) $f \in H_K^2$;

(2)

$$\sup_{I \subseteq \partial\mathbb{D}} \frac{1}{K(|I|)} \int_{S(I)} |f'(z)|^2 (1-|z|^2) dA(z) < \infty,$$

where

$$S(I) = \left\{ z \in \mathbb{D} : 1-|I| \leqslant |z| < 1 \ \text{and} \ \frac{z}{|z|} \in I \right\};$$

(3)

$$\sup_{a \in \mathbb{D}} \frac{1-|a|^2}{K(1-|a|^2)} \int_{\mathbb{D}} |f'(z)|^2 (1-|\sigma_a(z)|^2) dA(z) < \infty.$$

LEMMA 2. [18] *Let K satisfy the conditions* (1) *and* (2)*. Suppose that $f \in H_K^2$, then*

$$|f(z)| \lesssim \frac{\|f\|_{H_K^2} \sqrt{K(1-|z|^2)}}{\sqrt{(1-|z|^2)}}, \quad z \in \mathbb{D}.$$

LEMMA 3. ([9, Lemma 3]) *Suppose $g \in BMOA$. Then*

$$dist(g, VMOA) \approx \limsup_{r \to 1} \|g - g_r\|_{BMOA} \approx \limsup_{|a| \to 1} \|g \circ \sigma_a - g(a)\|_{H^2}.$$

Here $g_r(z) = g(rz)$ with $0 < r < 1$.

LEMMA 4. *Suppose $g \in BMOA$ and K satisfy the conditions* (1) *and* (2)*. Then $J_{g_r} : H_K^2 \to H_K^2$ is compact.*

Proof. Let $\{f_n\}$ be a sequence such that $\|f_n\|_{H^2_K} \leqslant 1$ and $f_n \to 0$ uinformly on compact subsets of $\mathbb{D}$ as $n \to \infty$. To prove that $J_{g_r} : H^2_K \to H^2_K$ is compact, we only need to show that $\lim_{n\to\infty} \|J_{g_r} f_n\|_{H^2_K} = 0$. Notice that $g_r(z) = g(rz)$, we have

$$|g_r'(z)| \leqslant |g'(rz)| \lesssim \frac{\|g\|_{BMOA}}{1-|rz|^2} \lesssim \frac{\|g\|_{BMOA}}{1-r^2}, \quad z \in \mathbb{D}.$$

Since K satisfies (2), by Lemma 2.2 of [5], there exists a small enough $c > 0$ such that $\varphi_K(t) \lesssim t^{1-c}$, $t \geqslant 1$. We have (see the proof of Theorem 1 of [18])

$$\sup_{a\in\mathbb{D}} \frac{(1-|a|^2)^2 K(1-|z|^2)}{K(1-|a|^2)|1-\overline{a}z|^2} \lesssim 1.$$

Thus, by Lemma 1 we have

$$\begin{aligned}
&\|J_{g_r} f_n\|_{H^2_K} \\
&\approx \sup_{a\in\mathbb{D}} \left(\frac{1-|a|^2}{K(1-|a|^2)} \int_{\mathbb{D}} |f_n(z)|^2 |g_r'(z)|^2 \left(1-|\sigma_a(z)|^2\right) dA(z) \right)^{\frac{1}{2}} \\
&\lesssim \frac{\|g\|_{BMOA}}{1-r^2} \sup_{a\in\mathbb{D}} \left(\frac{1-|a|^2}{K(1-|a|^2)} \int_{\mathbb{D}} |f_n(z)|^2 \left(1-|\sigma_a(z)|^2\right) dA(z) \right)^{\frac{1}{2}} \\
&\lesssim \frac{\|g\|_{BMOA}}{1-r^2} \left(\int_{\mathbb{D}} |f_n(z)|^2 \frac{1-|z|^2}{K(1-|z|^2)} \left(\sup_{a\in\mathbb{D}} \frac{(1-|a|^2)^2 K(1-|z|^2)}{K(1-|a|^2)|1-\overline{a}z|^2} \right) dA(z) \right)^{\frac{1}{2}} \\
&\lesssim \frac{\|g\|_{BMOA}}{1-r^2} \left(\int_{\mathbb{D}} |f_n(z)|^2 \frac{1-|z|^2}{K(1-|z|^2)} dA(z) \right)^{\frac{1}{2}}.
\end{aligned}$$

Note that

$$\|f_n\|_{H^2_K} \leqslant 1 \quad \text{and} \quad |f_n(z)|^2 \frac{1-|z|^2}{K(1-|z|^2)} \lesssim 1,$$

by Lemma 2, the proof is finished by the Dominated Convergence Theorem. □

LEMMA 5. [26] *Let X, Y be two Banach spaces of analytic functions on $\mathbb{D}$. Suppose that*

(1) The point evaluation functionals on X are continuous.

(2) The closed unit ball of X is a compact subset of X in the topology of uniform convergence on compact sets.

(3) $T : X \to Y$ is continuous when X and Y are given the topology of uniform convergence on compact sets.

Then, T is a compact operator if and only if given a bounded sequence $\{f_n\}$ in X such that $f_n \to 0$ uniformly on compact sets, then the sequence $\{Tf_n\}$ converges to zero in the norm of Y.

THEOREM 1. *Suppose* $g \in H(\mathbb{D})$ *and* K *satisfy the conditions* (1) *and* (2)*. If* I_g *is bounded on* H_K^2*, then*

$$\|I_g\|_e \approx \sup_{z\in\mathbb{D}} |g(z)| = \|g\|_\infty.$$

Proof. From the result in [18], we know that $\|I_g\| \lesssim \sup_{z\in\mathbb{D}} |g(z)|$. Hence

$$\|I_g\|_e = \inf_S \|I_g - S\| \leqslant \|I_g\| \lesssim \sup_{z\in\mathbb{D}} |g(z)| = \|g\|_\infty.$$

On the other hand, we choose the sequence $\{w_n\} \subset \mathbb{D}$ such that $|w_n| \to 1$. Define

$$f_n(z) = \sqrt{\frac{K(1-|w_n|^2)}{1-|w_n|^2}}\,(\sigma_{w_n}(z) - w_n), \quad z \in \mathbb{D}.$$

By the proof of Theorem 1 of [18], we see that $\|f_n\|_{H_K^2} \lesssim 1$. Moreover, f_n converges to zero uniformly on compact subsets of $\mathbb{D}$. Then $\|Sf_n\|_{H_K^2} \to 0$ as $n \to \infty$ for any compact operator S on H_K^2 by Lemma 5. Hence

$$\begin{aligned}
\|I_g - S\| &\gtrsim \limsup_{n\to\infty} \|(I_g - S)f_n\|_{H_K^2} \\
&\geqslant \limsup_{n\to\infty} (\|I_g f_n\|_{H_K^2} - \|Sf_n\|_{H_K^2}) \\
&= \limsup_{n\to\infty} \|I_g f_n\|_{H_K^2}.
\end{aligned}$$

In addition, by Lemma 1,

$$\begin{aligned}
&\|I_g f_n\|_{H_K^2} \\
&\approx \sup_{a\in\mathbb{D}} \left(\frac{1-|a|^2}{K(1-|a|^2)} \int_{\mathbb{D}} |f_n'(z)|^2 |g(z)|^2 \left(1 - |\sigma_a(z)|^2\right) dA(z) \right)^{\frac{1}{2}} \\
&\geqslant \left(\frac{1-|w_n|^2}{K(1-|w_n|^2)} \int_{\mathbb{D}} \frac{K(1-|w_n|^2)(1-|w_n|^2)}{|1-\overline{w_n}z|^4} |g(z)|^2 \left(1 - |\sigma_{w_n}(z)|^2\right) dA(z) \right)^{\frac{1}{2}} \\
&= \left(\int_{\mathbb{D}} |\sigma_{w_n}'(z)|^2 |g(z)|^2 \left(1 - |\sigma_{w_n}(z)|^2\right) dA(z) \right)^{\frac{1}{2}} \\
&= \left(\int_{\mathbb{D}} |(g \circ \sigma_{w_n})(z)|^2 \left(1 - |z|^2\right) dA(z) \right)^{\frac{1}{2}} \\
&\gtrsim |g(w_n)|.
\end{aligned}$$

Since $w_n \in \mathbb{D}$ is arbitrary, we have

$$\|I_g\|_e = \inf_S \|I_g - S\| \gtrsim \limsup_{n\to\infty} \|I_g f_n\|_{H_K^2} \gtrsim \sup_{z\in\mathbb{D}} |g(z)| = \|g\|_\infty.$$

The proof is completed. □

THEOREM 2. *Suppose K satisfy the conditions* (1) *and* (2) *and $J_g : H_K^2 \to H_K^2$ is bounded. Then $J_g : H_K^2 \to H_K^2$ satisfies*

$$\|J_g\|_e \approx dist(g, VMOA) \approx \limsup_{|a|\to 1} \|g\circ\sigma_a - g(a)\|_{H^2}.$$

Proof. Let $\{I_n\}$ be the subarc sequence of $\partial\mathbb{D}$ such that $\lim_{n\to\infty}|I_n| = 0$. Denote $w_n = (1-|I_n|)\zeta_n$, where ζ_n is the center of I_n, $n = 1, 2, \ldots$. Then

$$1-|w_n|^2 \approx |1-\overline{w_n}z| \approx |I_n|, \quad z\in S(I_n).$$

Thus, by $K(2t)\approx K(t)$ and nondecreasing of K, we know that

$$K(1-|w_n|^2) \approx K(|I_n|), \quad z\in S(I_n).$$

Take

$$h_n(z) = \frac{(1-|w_n|^2)\sqrt{K(1-|w_n|^2)}}{(1-\overline{w_n}z)^{\frac{3}{2}}}, \quad z\in\mathbb{D}.$$

Then $h_n \to 0$ uniformly on the compact subsets of $\mathbb{D}$ as $n\to\infty$ and $\|h_n\|_{H_K^2} \lesssim 1$ by the proof of Theorem 2 of [18]. Thus, for any compact operator S on H_K^2, by Lemma 5 we have

$$\lim_{n\to\infty} \|Sh_n\|_{H_K^2} = 0.$$

Therefore

$$\begin{aligned}
\|J_g - S\| &\gtrsim \limsup_{n\to\infty}\left(\|J_g h_n\|_{H_K^2} - \|Sh_n\|_{H_K^2}\right)\\
&= \limsup_{n\to\infty}\|J_g h_n\|_{H_K^2}\\
&\approx \limsup_{n\to\infty}\left(\frac{1}{K(|I_n|)}\int_{S(I_n)} |(J_g h_n)'(z)|^2(1-|z|^2)\,dA(z)\right)^{\frac{1}{2}}\\
&= \limsup_{n\to\infty}\left(\frac{1}{K(|I_n|)}\int_{S(I_n)} |h_n(z)|^2|g'(z)|^2(1-|z|^2)\,dA(z)\right)^{\frac{1}{2}}\\
&\approx \limsup_{n\to\infty}\sqrt{\frac{1}{|I_n|}\int_{S(I_n)} |g'(z)|^2(1-|z|^2)\,dA(z)}.
\end{aligned}$$

Since $\{I_n\}$ is arbitrary, we have

$$\|J_g\|_e \gtrsim \limsup_{|I|\to 0}\sqrt{\frac{1}{|I|}\int_{S(I)} |g'(z)|^2(1-|z|^2)\,dA(z)}.$$

By the boundedness of $J_g : H_K^2 \to H_K^2$, we see that $g\in BMOA$. It follows from the proof of Lemma 3.4 of [19], for $g\in BMOA$,

$$\limsup_{|a|\to 1}\|g\circ\sigma_a - g(a)\|_{H^2} \approx \limsup_{|I|\to 0}\sqrt{\frac{1}{|I|}\int_{S(I)} |g'(z)|^2(1-|z|^2)\,dA(z)}.$$

Hence

$$\|J_g\|_e \gtrsim \limsup_{|a|\to 1} \|g\circ\sigma_a - g(a)\|_{H^2} \approx dist(g, VMOA).$$

On the other hand, by Lemma 4, $J_{g_r}: H_K^2 \to H_K^2$ is compact. Combining this with Theorem 2 in [18] and the linearity of J_g respect to g, we get

$$\|J_g\|_e \leqslant \|J_g - J_{g_r}\| = \|J_{g-g_r}\| \approx \|g-g_r\|_{BMOA}.$$

Hence, by Lemma 3 we obtain

$$\begin{aligned}\|J_g\|_e &\lesssim \limsup_{|r|\to 1} \|g-g_r\|_{BMOA} \approx \limsup_{|a|\to 1} \|g\circ\sigma_a - g(a)\|_{H^2}\\ &\approx dist(g, VMOA).\end{aligned}$$

The proof is complete. □

From Theorems 1 and 2, we immediately get the following result.

COROLLARY 1. *Suppose $g \in H(\mathbb{D})$ and K satisfy the conditions (1) and (2). Then*

(i) I_g is compact on H_K^2 if and only if $g = 0$.

(ii) J_g is compact on H_K^2 if and only if $g \in VMOA$.

REMARK 1. If $K(t) = t^\lambda$ $(0 < \lambda < 1)$, then K satisfies our conditions and H_K^2 is just the Morrey space $\mathscr{L}^{2,\lambda}$. Hence our results generalize the results in [10]. If $K(t) = t$, H_K^2 is the $BMOA$ space. However K does not satisfy the condition (2). Hence, our results do not include the case of $BMOA$ space. In [19], Siskakis and Zhao proved that $J_g: BMOA \to BMOA$ is compact if and only if

$$\lim_{|I|\to 0} \frac{(\log \frac{2}{|I|})^2}{|I|} \int_{S(I)} |g'(z)|^2 (1-|z|^2)\, dA(z) = 0.$$

THEOREM 3. *Suppose that $g \in H(\mathbb{D})$ and K satisfy the conditions (1) and (2). Then M_g is compact on H_K^2 if and only if $g = 0$.*

Proof. If $g = 0$, it is obvious that M_g is a compact operator on H_K^2.

Suppose M_g is compact on H_K^2. For any $w_n \in \mathbb{D}$ such that $|w_n| \to 1$, take the function h_n defined in Theorem 2. By the compactness of M_g we have $\lim_{n\to\infty} \|M_g h_n\|_{H_K^2} = 0$. By Lemma 2, we obtain

$$\begin{aligned}|M_g h_n(z)| &= \left| \frac{(1-|w_n|^2)\sqrt{K(1-|w_n|^2)}}{(1-\overline{w_n}z)^{\frac{3}{2}}} g(z) \right|\\ &\lesssim \frac{\|M_g h_n\|_{H_K^2} \sqrt{K(1-|z|^2)}}{\sqrt{(1-|z|^2)}}.\end{aligned}$$

Taking $z = w_n$, we get $|g(w_n)| \lesssim \|M_g h_n\|_{H_K^2}$. Let $n \to \infty$. We get $\lim_{n\to\infty} g(w_n) = 0$. Since $\{w_n\}$ is arbitrary sequence of $\mathbb{D}$ such that $|w_n| \to 1$, by the Maximum Modulus Principle we deduce that $g = 0$. The proof is complete. □

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