Operators
and
Matrices
Volume 6, Number 2 (2012), 347–356
doi:10.7153/oam-06-25

JORDAN STRUCTURES IN BANACH SPACES

CHO-HO CHU

Abstract. We explain how Jordan algebraic structures in Banach spaces arise from the geometry of symmetric manifolds and discuss some applications.

1. Introduction

Since the introduction of Jordan algebras in quantum formalism by P. Jordan, J. von Neumann and E. Wigner [15], many unexpected applications in Lie algebra, geometry and analysis have been found (cf. [4, 13, 26, 27]). We discuss the geometric aspect of Jordan algebraic structures in Banach spaces, which has its origin in the study of infinite dimensional symmetric manifolds. Indeed, since É. Cartan's seminal work, Lie theory has been an important tool in the study of Rimannian symmetric spaces and their classification. It was found relatively recently that Jordan algebras and Jordan triple systems can be used to describe a large class of symmetric spaces which is also accessible in infinite dimension.

In finite dimensions, it is well known [2, 12] that the Hermitian symmetric spaces of non-compact type, which form a subclass of Riemannian symmetric spaces, can be realized as bounded symmetric domains in spaces of several complex variables, of which the open unit disc in $\mathbb{C}$ is the simplest example. The concept of a bounded symmetric domain as well as É. Cartan's classification of these domains [2] can be extended to infinite dimension in the following way.

Let D be a domain in a complex Banach space V, that is, D is a nonempty open connected set in V. We call D a *symmetric domain* if each point $a \in D$ admits a symmetry $s_a : D \longrightarrow D$ (which is necessarily unique). A symmetry s_a *at* a is defined to be a biholomorphic map such that s_a^2 is the identity map and a is an isolated fixed point of s_a. A symmetric domain is said to be *irreducible* if it is not (biholomorphic to) a Cartesian product of symmetric domains. Cartan's classification can be very briefly described by saying that every (finite dimensional) irreducible bounded symmetric domain is biholomorphic to the open unit ball in one of the six types of complex vector spaces of matrices over $\mathbb{C}$ or the Cayley algebra $\mathbb{O}$. These spaces are finite dimensional JB*-triples. An infinite dimensional extension of this classification is the following result due to Kaup [20].

Mathematics subject classification (2010): 17C65, 32M15, 57C35, 58B20, 46L05, 46L70.
Keywords and phrases: Symmetric domain; Jordan triple systems; Möbius transformation; Bergmann operator; Denjoy-Wolff Theorem.
An ICM Satellite conference talk given in Bangalore, India on 11 August 2010.

Paper OaM-06-25

THEOREM 1.1. *Let D be a bounded symmetric domain in a complex Banach space. Then D is biholomorphic to the open unit ball of a JB*-triple.*

A precursor and a finite dimensional version of the above result is due to Loos [25, 30]. Our objective is to discuss the main ingredients in the proof of the above theorem and various ramifications.

2. JB*-triples

We continue to use the above notation. A JB*-triple is a complex Banach space equipped with a Jordan triple product. How is this Jordan structure constructed in the above theorem? A crucial and fundamental device is the seminal result of H. Cartan [3], which states that the automorphism group $\mathrm{Aut}\,D$ of biholomorphic self-maps on a bounded domain $D \subset \mathbb{C}^n$ carries the structure of a Lie group, and its infinite dimensional generalization due to Upmeier [32] and Vigué [34]. If D is symmetric but possibly infinite dimensional, then the Lie group $\mathrm{Aut}\,D$ induces a JB*-structure on the tangent space T_aD at any chosen point $a \in D$ and moreover, D is biholomorphic to the open unit ball of T_aD.

For completeness, we recall that a mapping $f : D_1 \longrightarrow D_2$ between open sets D_i in complex Banach spaces V_i $(i = 1,2)$ is called *holomorphic* if it has a (Fréchet) *derivative* $f'(a)$ at every point $a \in D$, where $f'(a) : V_1 \longrightarrow V_2$ is a bounded linear map satisfying

$$\lim_{z \to a} \frac{\|f(z) - f(a) - f'(a)(z-a)\|}{\|z-a\|} = 0.$$

The mapping f is called *biholomorphic* if it is bijective, holomorphic and its inverse f^{-1} is also holomorphic. The biholomorphic self-maps on a domain D form a group $\mathrm{Aut}\,D$ under function composition, called the *automorphism group* of D.

Let U be the open unit disc in $\mathbb{C}$ and let $H(D,U)$ denote the vector space of holomorphic maps from a *bounded* domain D in a complex Banach space V to U. Then D is endowed with the *Carathéodory metric*

$$d(z,w) = \sup\left\{ \tanh^{-1}\left|\frac{f(z)-f(w)}{1-f(z)\overline{f(w)}}\right| : f \in H(D,U)\right\} \qquad (z,w \in D).$$

Fix a point $a \in D$. Let $r > 0$ so that the open ball $B(a,4r) = \{z \in D : d(z,a) < 4r\}$ is contained in D. Define a metric ρ on $\mathrm{Aut}\,D$ by

$$\rho(f,g) = \sup\{d(f(z),g(z)) : z \in B(a,r/2)\} \qquad (f,g \in \mathrm{Aut}\,D).$$

Note that the metric ρ is defined by the ball $B(a,r/2)$ of radius $r/2$. Equipped with the topology induced by the metric ρ, the automorphism group $\mathrm{Aut}\,D$ becomes a real Lie group whose Lie algebra $\mathfrak{aut}\,D$ is formed by the complete holomorphic vector fields on D, with the usual Lie brackets of vector fields [32, 34]. A *holomorphic vector field* X on D is a holomorphic selection of a tangent vector at each point in D. Identifying each tangent space with the ambient Banach space V, we can, and will, view X as a holomorphic map $X : D \longrightarrow V$. A holomorphic vector field X gives rise to a flow of local biholomorphic transformations of D by elementary theory of differential equations: for every $z_0 \in D$, there is a holomorphic map $g : T \times \Omega \longrightarrow D$ defined on the product of convex domains $T \subset \mathbb{C}$ and $\Omega \subset D$, with $0 \in T$ and $z_0 \in \Omega$, such that

$$\frac{\partial g(t,z)}{\partial t} = X(g(t,z)), \quad g(0,z_0) = z_0.$$

The vector field X is called *complete* if T can be chosen to contain the whole real line $\mathbb{R}$ and $\Omega = D$, in which case we write

$$\exp X(z) := g(1,z) \qquad (z \in D).$$

The map $\exp X : D \longrightarrow D$ is biholomorphic and we have the exponential mapping $\exp : X \in \mathfrak{aut} D \mapsto \exp X \in \operatorname{Aut} D$. The real Lie algebra $\mathfrak{aut} D$ is a Banach Lie algebra in the norm

$$\|X\| = \sup\{\|X(z)\| : z \in B(a, r/2)\} \qquad (X \in \mathfrak{aut} D).$$

Now let D be a bounded symmetric domain in a complex Banach space V and fix a point $a \in D$ as before. The symmetry $s_a \in \operatorname{Aut} D$ induces an involutive automorphism $\theta = Ad\, s_a : \mathfrak{aut} D \longrightarrow \mathfrak{aut} D$ via differentiation. The involution θ has eigenvalues ± 1 and gives an eigenspace decomposition

$$\mathfrak{aut} D = \mathfrak{k} \oplus \mathfrak{p}$$

where $\mathfrak{k}$ is the 1-eigenspace and $\mathfrak{p}$ the (-1)-eigenspace of θ. The eigenspace $\mathfrak{k}$ is the Lie algebra of the isotropy group $K = \{g \in \operatorname{Aut} D : g(a) = a\}$ whereas the eigenspace $\mathfrak{p}$ is *real* linear isomorphic to the tangent space V via the evaluation map

$$X \in \mathfrak{p} \mapsto X(a) \in V.$$

The real vector space $\mathfrak{p}$ admits a complex structure, that is, there is a map

$$J : \mathfrak{p} \longrightarrow \mathfrak{p}$$

which satisfies $J^2 = -id$ and is defined by $(JX)(a) = iX(a) \in V$. Morevoer, we have $J\theta = \theta J$ and

$$[JX, Y](a) = i[X, Y](a) \qquad (X \in \mathfrak{p}, Y \in \mathfrak{aut} D).$$

With the complex structure J, the eigenspace $\mathfrak{p}$ becomes a complex vector space and is complex linear isomorphic to V. We can now construct the Jordan triple product on V via the Lie triple product.

LEMMA 2.1. *Let D be a bounded symmetric domain in a complex Banach space V. Then V has the structure of a Jordan triple, that is, there is a triple product $\{\cdot,\cdot,\cdot\} : V^3 \longrightarrow V$ which is complex linear in the outer variables but conjugate linear in the middle variable, and satisfies the Jordan triple identity*

$$\{u,v,\{x,y,z\}\} = \{\{u,v,x\},y,z\} - \{x,\{v,u,y\},z\} + \{x,y,\{u,v,z\}\}$$

for all $u,v,x,y,z \in V$.

Proof. Let $X \in \mathfrak{p} \mapsto X(a) \in V$ be the linear isomorphism given above. For $x, y, z \in V$ with $x = X(a), y = Y(a)$ and $z = Z(a)$, the desired triple product is defined by

$$\{x,y,z\} = -\frac{1}{4}[[X,Y],Z](a) + \frac{1}{4}[[JX,Y],JZ](a). \qquad \square$$

The concept of a *Jordan triple* was introduced by Meyberg [29] in order to extend Koecher's construction [23, 24] of Lie algebras from Jordan algebras. This construction was also discovered independently by Kantor [17, 18] and Tits [31], now called the *Tits-Kantor-Koecher construction* which lies in the background of Lemma 2.1. More details of the construction can be seen in [7, Example 4.8].

A *Jordan algebra* [14, 27] is a commutative, but not necessarily associative, algebra satisfying the Jordan identity

$$(ab)a^2 = a(ba^2).$$

A complex Jordan algebra $\mathscr{A}$ with an involution $*$ is a Jordan triple in the canonical Jordan triple product defined by

$$\{a,b,c\} = (ab^*)c + a(b^*c) - b^*(ac) \qquad (a,b,c \in \mathscr{A}).$$

Given a Jordan triple V and $z \in V$, one can define the *box operator* $z \square z : V \longrightarrow V$ by

$$z \square z(x) = \{z,z,x\} \qquad (x \in V).$$

The box operators are fundamental in the geometry of symmetric manifolds.

In Lemma 2.1, the Banach space V, which is linearly isomorphic to the tangent space T_aD, together with the Jordan triple product $\{\cdot,\cdot,\cdot\}$, need not be a JB*-triple in its original norm. We need to renorm V to a JB*-triple.

DEFINITION 2.2. A complex Banach space V is called a *JB*-triple* if it is a Jordan triple with a continuous Jordan triple product satisfying

(i) $z \square z$ is a hemitian operator on V, that is, $\|\exp it(z \square z)\| = 1$ for all $t \in \mathbb{R}$;

(ii) $z \square z$ has non-negative spectrum;

(iii) $\|z \square z\| = \|z\|^2$

for all $z \in V$.

A JB*-triple is called a *JBW*-triple* if, as a Banach space, it has a predual which is necessarily unique.

LEMMA 2.3. *Let D be a bounded symmetric domain in a complex Banach space V. Then D is biholomorphic to the open unit ball of a JB*-triple $V' = (V, \|\cdot\|_a)$*

Proof. View V as the tangent space T_aD at a chosen point $a \in D$, as before. Define the Carathéodory norm on V by

$$\|v\|_a = \sup\{|f'(a)(v)| : f \in H(D,U), f(a) = 0\} \qquad (v \in V). \tag{1}$$

Then $(V, \|\cdot\|_a)$ is the desired JB*-triple. $\square$

REMARK 2.4. If, in the preceding lemma, D happens to be the open unit ball of V, then by choosing $a = 0$, the norm $\|\cdot\|_a$ is the original norm and V itself is a JB*-triple.

To achieve all the results above, one very important key is the fact that each vector field $X \in \mathfrak{p}$ is a polynomial vector field of degree 2 and has the form

$$X(z) = X(a) - \{z, X(a), z\} \qquad (z \in D). \tag{2}$$

Complete proofs for what has been asserted all above can be found in the original work of Kaup [19, 20] or [4, 33]. The converse of Theorem 1.1 is the following result in [20].

THEOREM 2.5. *Let V be a JB*-triple. Then its open unit ball is a bounded symmetric domain.*

Proof. Let D be the open unit ball of V. Evidently $s_0(z) = -z$ is a symmetry at the origin 0. To see that every point $a \in D$ has a symmetry, we only need to '*move*' the symmetry s_0 to a via the Möbius transformation $g_a : D \longrightarrow D$ defined by

$$g_a(x) = a + B(a,a)^{1/2}(id + x\square a)^{-1}(x) \qquad (x \in D)$$

where $B(a,a) : V \longrightarrow V$ is the Bergman operator defined by

$$B(a,a)(v) = v - 2\{a,a,v\} + \{a,\{a,v,a\},a\} \qquad (v \in V).$$

The symmetry s_a at a is given by $s_a = g_a \circ s_0 \circ g_a^{-1}$. $\square$

3. Applications

The importance of JB*-triples in the geometry of Hermitian symmetric spaces cannot be overemphasized. On the other hand, JB*-triples also play an important role in analysis. In the vista of functional analysis, they form an important class of Banach spaces, including Hilbert spaces, C*-algebras, spaces of operators between Hilbert spaces and some exceptional Jordan Banach algebras. Further, the construction of the Jordan triple product from a bounded symmetric domain reveals that *every* complex Banach space V admits a Jordan algebraic structure! Indeed, let D be the open unit ball of V and let

$$\mathfrak{aut}\, D = \mathfrak{k} \oplus \mathfrak{p}$$

be the eigenspace decomposition induced by the symmetry $s_0(z) = -z$ at the origin $0 \in D$. Although D need not be a symmetric domain, the vector fields $X \in \mathfrak{p}$ are still polynomials of degree 2. Let

$$V_s = \{X(0) : X \in \mathfrak{aut}\, D\} = \{X(0) : X \in \mathfrak{p}\}$$

which is a closed subspace of V and the open unit ball $D_s := D \cap V_s$ of V_s is a symmetric domain. Therefore V_s is a JB*-triple and is called the *symmetric part* of V [21]. Of course, V itself is a JB*-triple if, and only if, $V = V_s$.

The fact that every complex Banach space V contains a subspace V_s which is a JB*-triple is a remarkable phenomenon. This suggests that V_s is an interesting object of study.

EXAMPLE 3.1. A Banach algebra $\mathscr{A}$ with identity $\mathbf{1}$ is said to satisfy the *von Neumann inequality* if for each $a \in \mathscr{A}$ with $\|a\| \leqslant 1$, we have

$$\|p(a)\| \leqslant \sup\{|p(\alpha)| : |\alpha| = 1\}$$

for every polynomial p with complex coefficients. This property can be characterized by the the symmetric part $\mathscr{A}_s$ of $\mathscr{A}$. In fact, $\mathscr{A}$ satisfies the von Neumann inequality if, and only if, $\mathbf{1} \in \mathscr{A}_s$. The necessity has been shown in [1] and the sufficiency was shown in [10].

From the viewpoint of JB*-triples, many results in operator algebras can be seen simply through a geometric perspective. A C*-algebra $\mathscr{A}$ is a JB*-triple in the Jordan triple product

$$\{a,b,c\} = \frac{1}{2}(ab^*c + cb^*a) \qquad (a,b,c \in \mathscr{A}).$$

The complex Banach space $B(H,K)$ of bounded linear operators between Hilbert spaces H and K also forms a JB*-triple with the triple product

$$\{a,b,c\} = \frac{1}{2}(ab^*c + cb^*a) \qquad (a,b,c \in B(H,K)).$$

An important feature of JB*-triples is that their norm and triple product determine each other.

THEOREM 3.2. *Let $\varphi : V \longrightarrow W$ be a linear bijection between JB*-triples V and W. Then φ is an isometry if, and only if, φ is a triple homomorphism, that is, φ preserves the triple product:*

$$\varphi\{a,b,c\} = \{\varphi(a), \varphi(b), \varphi(c)\} \qquad (a,b,c \in V).$$

Proof. Let φ be an isometry. By the polarization

$$2\{x,y,z\} = \{x+z, y, x+z\} - \{x,y,x\} - \{z,y,z\},$$

it suffices to show

$$\varphi\{z,a,z\} = \{\varphi(z),\varphi(a),\varphi(z)\} \qquad (a,z \in V)$$

where, we may assume $\|z\| < 1$.

Consider V and W as the tangent space at 0 of the open unit balls D_V and D_W respectively, which are symmetric domains. Let $\mathfrak{aut}\, D_W = \mathfrak{k} \oplus \mathfrak{p}$ be the eigenspace decomposition induced by the symmetry $s_0(w) = -w$ at $0 \in D_W$.

Let $a \in V$. Since the evaluation map $Y \in \mathfrak{p} \mapsto Y(0) \in W$ is a linear isomorphism, there is a unique vector field $Y_{\varphi(a)} \in \mathfrak{p}$ such that $Y_{\varphi(a)}(0) = \varphi(a)$. Likewise, let X_a be the unique complete holomorphic vector field on D_V such that $X_a(0) = a$. Since $\varphi : D_V \longrightarrow D_W$ is a surjective linear isometry, $\varphi X_a \varphi^{-1}$ is a complete holomorphic vector filed on D_W satisfying

$$(\varphi X_a \varphi^{-1})(0) = \varphi(a).$$

Hence we have $\varphi X_a \varphi^{-1} = Y_{\varphi(a)}$. This gives, using (2),

$$\varphi(a) - \{w,\varphi(a),w\} = Y_{\varphi(a)}(w) = (\varphi X_a \varphi^{-1})(w) = \varphi(a) - \varphi\{\varphi^{-1}(w),a,\varphi^{-1}(w)\}$$
$$(w \in D_W)$$

and it follows that, letting $z = \varphi^{-1}(w) \in D_V$,

$$\varphi\{z,a,z\} = \{\varphi(z),\varphi(a),\varphi(z)\} \qquad (z \in D_V).$$

Conversely, let φ be a triple homomorphism and let $a \in V \backslash \{0\}$. Let $V(a)$ be the JB*-subtriple generated by a. Then $\varphi(V(a))$ is the JB*-subtriple $W(\varphi(a))$ generated by $\varphi(a)$ in W. Further, for each complex-valued triple homomorphism χ of $W(\varphi(a))$, the composite $\chi \circ \varphi$ is a complex-valued triple homomorphism of $V(a)$. It follows that

$$\|\varphi(a)\| = \sup\{|\chi(\varphi(a))| : \chi \text{ is a triple homomorphism of } W(\varphi(a))\} \leqslant \|a\|.$$

The same applies to the inverse φ^{-1} which concludes the proof. □

The above result in [20] not only subsumes Kadison's result [16] on isometries between C*-algebras, but also renders it a geometric perspective. The latter part of the proof of sufficiency in the preceding theorem makes use of the spectral theory in JB*-triples, applicable to *non-self-adjoint* operators in C*-algebras, namely, the closed subtriple $V(a)$ generated by *any* element a in a JB*-triple V is linearly isometric to the C*-algebra $C_0(\sigma(a))$ of continuous functions vanishing at infinity on the triple spectrum $\sigma(a) \subset [0,\infty)$ of a [20].

Cartan's classification of bounded symmetric domains and Theorem 1.1 can be viewed as generalizations of the Riemann mapping theorem. In this context, the open unit balls of JB*-triples are infinite dimensional generalization of the open unit disc U is the complex plane. It is therefore natural to study function theory on the open unit ball D of a JB*-triple V and expect fruitful applications of Jordan structures. Useful tools are the box operators $z\square z : V \longrightarrow V$, the Möbius transformations $g_a : D \longrightarrow D$ and the Bergman operators $B(b,c) : V \longrightarrow V$, where $a \in D$ and $b,c \in V$,

$$B(b,c)(z) = z - 2\{b,c,z\} + \{b,\{c,z,c\},b\} \qquad (z \in V).$$

We mention two examples to highlight the use of Jordan structures. We first note that the Bergamn operator $B(a,a)$ has positive spectrum and

$$g_a'(0) = B(a,a)^{1/2}.$$

In applications, it is essential to estimate the norm of the square roots of $B(a,a)$. The formula

$$\|B(a,a)^{-1/2}\| = \frac{1}{1-\|a\|^2}$$

has been derived in [22]. An alternative proof can be found in [4, Chapter 3]. If V is an abelian C*-algebra, we have

$$\|B(a,a)^{1/2}\| = 1 - \|a\|^2.$$

For a Hilbert space V with inner product $\langle \cdot,\cdot \rangle$, which is a JB*-triple in the triple product $2\{x,y,z\} = \langle x,y\rangle z + \langle z,y\rangle x$, we have

$$\|B(a,a)^{1/2}\|^2 = \|B(a,a)\| = \begin{cases} (1-\|a\|^2)^2 & \text{if } \dim V = 1 \\ 1-\|a\|^2 & \text{if } \dim V \geqslant 2 \end{cases}$$

(cf. [4, 8]). The following distortion theorem has been proved in [8].

THEOREM 3.3. *Let D be the open unit ball of a JB*-triple V and let $f : D \longrightarrow V$ be a biholomorphic map onto $f(D)$ which is convex. Given that $f(0) = 0$ and $f'(0)$ is the identity map, we have, for $a \in D$,*

(i) $\dfrac{1}{(1+\|a\|)^2} \leqslant \|f'(a)\| \leqslant \dfrac{1}{(1-\|a\|)^2}$.

(ii) $\dfrac{(1-\|a\|)\|z\|}{(1+\|a\|)\|B(a,a)^{1/2}\|} \leqslant \|f'(a)(z)\| \leqslant \dfrac{\|z\|}{(1-\|a\|)^2} \qquad (z \in V)$.

Proof. We refer to [8] for details, but simply note that, for example, the Jordan technique is used in the following computation:

$$\begin{aligned} \|f'(a)(z)\| &\leqslant \frac{1+\|a\|}{1-\|a\|}\|z\|_a = \frac{1+\|a\|}{1-\|a\|}\|g_{-a}'(a)(z)\|_{g_{-a}(a)} \\ &= \frac{1+\|a\|}{1-\|a\|}\|B(a,a)^{-1/2}(z)\| \leqslant \frac{1+\|a\|}{1-\|a\|}\left(\frac{\|z\|}{1-\|a\|^2}\right) \\ &= \frac{\|z\|}{(1-\|a\|)^2} \end{aligned}$$

where $\|z\|_a$ is the Carathéodory norm defined in (1). If V is an abelian C*-algebra, the the lower bound in (ii) reduces to

$$\frac{\|z\|}{(1+\|a\|)^2}. \qquad \square$$

Our second example concerns the dynamics of a holomorphic map $f : D \longrightarrow D$ on a bounded symmetric domain D of any dimension. By Theorem 1.1, we regard D as the open unit ball of a JB*-triple V. If f has no fixed point and the image $f(D)$ is relatively compact, then it is easily seen that there is a sequence (z_k) in D converging to a boundary point $\xi \in \partial D$ with $\lim_k f(z_k) = \xi$. Further, if the sequence of operators

$$(1-\|z_k\|^2)B(z_k,z_k)^{-1/2} : V \longrightarrow V$$

converges uniformly to an operator $T \in L(V)$, which is the case if $\dim V < \infty$ or V is a Hilbert space, then for any $\lambda > 0$, the set

$$D(\xi,\lambda) := \{x \in D : \|B(x,x)^{-1/2}B(x,\xi)T\| < \lambda\}$$

is f-invariant, that is, $f(D(\xi,\lambda)) \subset D(\xi,\lambda)$. This result has been proved in [28]. When D is the open unit ball in a Hilbert space V, the following Denjoy-Wolff theorem for f has been proved in [9].

THEOREM 3.4. *Let D be the open unit ball of a Hilbert space and $f : D \longrightarrow D$ a holomorphic map without fixed point such that $f(D)$ is relatively compact. Then the iterates (f^n) of f converge to a constant function $g(\cdot) = \xi$, uniformly on any open ball strictly contained in D, where $\|\xi\| = 1$.*

Finally, we refer to [11] for applications of Jordan theory in harmonic analysis on symmetric cones. Some connections found recently between Jordan structures and harmonic analysis are described in [5, 6].

REFERENCES

[1] J. ARAZY, *Isometries of Banach algebras satisfying the von Neumann inequality*, Math. Scand. **74** (1994), 137–151.

[2] É. CARTAN, *Sur les domaines bornés homogènes de l'espace de n variables complexes*, Abh. Math. Semin. Univ. Hamburg **11** (1935), 116–162.

[3] H. CARTAN, *Les fonctions de deux variables complexes et le problème de la représentation analytique*, J. Math. Pures et Appl. **10** (1931), 1–114.

[4] C-H. CHU, *Jordan structures in geometry and analysis*, Cambridge Tracts in Math. **190**, Cambridge Univ. Press, Cambridge, 2012.

[5] C-H. CHU, *Matrix-valued harmonic functions on groups*, J. Reine Angew. Math. **552** (2002), 15–52.

[6] C-H. CHU, *Matrix convolution operators on groups*, Lecture Notes in Math. **1956**, Springer-Verlag, Heidelberg, 2008.

[7] C-H. CHU, *Jordan triples and Riemannian symmetric spaces*, Advances in Math. **219** (2008), 2029–2057.

[8] C-H. CHU, H. HAMADA, T. HONDA AND G. KOHR, *Distorsion theorems for convex mappings on homogeneous balls*, J. Math. Anal. Appl. **369** (2010), 437–442.

[9] C-H. CHU AND P. MELLON, *Iteration of compact holomorphic maps on a Hilbert ball*, Proc. Amer. Math. Soc. **125** (1997), 1771–1777.

[10] C-H. CHU AND P. MELLON, *Jordan structures in Banach spaces and symmetric manifolds*, Expos. Math. **16** (1998), 157–180.

[11] J. FARAUT AND A. KORANYI, *Analysis on symmetric cones*, Clarendon Press, Oxford, 1994.

[12] HARISH-CHANDRA, *Representations of semi-simple Lie groups* VI, Amer. J. Math. **78** (1956), 564–628.

[13] R. IORDĂNESCU, *Jordan structures in analysis, geometry and physics*, Editura Academiei Române, Bucharest, 2009.

[14] N. JACOBSON, *Structure and representations of Jordan algebras*, Amer. Math. Soc. Colloq. Publ. **39** (1968).

[15] P. JORDAN, J. VON NEUMANN AND E. WIGNER, *On an algebraic generalisation of the quantum mechanical formalism*, Ann. of Math. **36** (1934), 29–64.

[16] R. V. KADISON, *Isometries of operator algebras*, Ann. of Math. **54** (1951), 325–338.

[17] I. L. KANTOR, *Classification of irreducible transitive differential groups*, Dokl. Akad. Nauk SSSR **158** (1964), 1271–1274.

[18] I. L. KANTOR, *Transitive differential groups and invariant connections on homogeneous spaces*, Trudy Sem. Vecktor. Tenzor. anal. **13** (1966), 310–398.

[19] W. KAUP, *Algebraic characterization of symmetric complex Banach manifolds*, Math. Ann. **228** (1977), 39–64.

[20] W. KAUP, *A Riemann mapping theorem for bounded symmetric domains in complex Banach spaces*, Math. Z. **183** (1983), 503–529.

[21] W. KAUP AND H. UPMEIER, *Banach spaces with biholomorphically equivalent unit balls are isomorphic*, Proc. Amer. Math. Soc. **58** (1976), 129–133.

[22] W. KAUP, *Hermitian Jordan triple systems and their automorphisms of bounded symmetric domains*, in: Non-associative algebra and its applications, Oviedo, 1993 (ed. S. González), Kluwer Acad. Publ. Dordrecht (1994), 204–214.

[23] M. KOECHER, *Imbedding of Jordan algebras into Lie algebras* I, Bull. Amer. J. Math. **89** (1967), 787–816.

[24] M. KOECHER, *An elementary approach to bounded symmetric domains*, Lecture Notes, Rice University, 1969.

[25] O. LOOS, *Bounded symmetric domains and Jordan pairs*, Mathematical Lectures, University of California, Irvine, 1977.

[26] K. MCCRIMMON, *Jordan algebras and their applications*, Bull. Amer. Math. Soc. **84** (1978), 612–627.

[27] K. MCCRIMMON, *A taste of Jordan algebras*, Universitext, Springer-Verlag, Heidelberg, 2004.

[28] P. MELLON, *Holomorphic invariance on bounded symmetric domains*, J. Reine Angew. Math. **523** (2000), 199–223.

[29] K. MEYBERG, *Jordan-Tripelsysteme und die Koecher-Konstruktion von Lie-Algebren*, Math. Z. **115** (1970), 58–78.

[30] I. SATAKE, *Algebraic strutures of symmetric domains*, Princeton Univ. Press, Princeton, 1980.

[31] J. TITS, *Une classe d'algèbres de Lie en relation avec les algèbres de Jordan*, Indag. Math. **24** (1962), 530–535.

[32] H. UPMEIER, *Über die Automorphismengruppe von Banach-Mannigfaltigkeiten mit invarianter Metrik*, Math. Ann. **223** (1976), 279–288.

[33] H. UPMEIER, *Jordan algebras in analysis, operator theory and quantum mechanics*, CBMS, Amer. Math. Soc., 1987.

[34] J. P. VIGUÉ, *Le groupe des automorphismes analytiques d'un domaine borné d'un espace de Banach complexe. Application aux domaines bornés symétriques*, Ann. Sc. Ec. Norm. Sup. **9** (1976), 203–282.

(Received January 11, 2011)

Cho-Ho Chu
School of Mathematical Sciences
Queen Mary, University of London
London E1 4NS, U.K.
e-mail: c.chu@qmul.ac.uk