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GENERATORS WITH A CLOSURE RELATION

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Abstract. Assume that a block operator of the form $\begin{pmatrix} & A_1 \\ A_2 & 0 \end{pmatrix}$, acting on the Banach space $X_1 \times X_2$, generates a contraction C_0-semigroup. We show that the operator A_S defined by $A_S x = A_1 \begin{pmatrix} x \\ SA_2 x \end{pmatrix}$ with the natural domain generates a contraction semigroup on X_1. Here, S is a boundedly invertible operator for which $\varepsilon I - S^{-1}$ is dissipative for some $\varepsilon > 0$. With this result the existence and uniqueness of solutions of the heat equation can be derived from the wave equation.

1. Introduction

The question whether an (unbounded) operator is the generator a C_0-semigroup appears naturally for abstract differential equations in the discussion of well-posedness. In this paper we relate the well-posedness of two abstract differential equations.

Starting with an abstract Cauchy problem (ACP) on the space $X_1 \times X_2$,

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = A_{ext} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \qquad x(0) = x_0, \tag{ACP-1}$$

for an operator A_{ext} of the form

$$A_{ext} = \begin{pmatrix} & A_1 \\ A_2 & 0 \end{pmatrix}, \quad \begin{array}{l} A_1 : D(A_1) \subset X_1 \times X_2 \to X_1, \\ A_2 : D(A_2) \subset X_1 \to X_2, \end{array} \tag{1.1}$$

we set $A_S x_1 = A_1 \begin{pmatrix} x_1 \\ SA_2 x_1 \end{pmatrix}$ where S is a bounded operator, and define the ACP

$$\dot{x} = A_S x, \qquad x(0) = x_0 \in X_1. \tag{ACP-2}$$

The question is whether (ACP-2) is well-posed when (ACP-1) is assumed to be well-posed.

The idea comes from port-based modeling, see e.g. [5, 8]. There, A_{ext} defines a structure relating the variables $(f_1, f_2)^T$ and $(e_1, e_2)^T$, by $f = A_{ext} e$. Now, adding the closure relation $e_2 = Sf_2$, where S maps from X_2 to X_2, yields the structure A_S, as depicted in Figure 1. There, the operator S is seen as adding dissipation.

The form (1.1) appears in the context of port-Hamiltonian systems, see [3, 8], but is applicable in wider settings, see [9]. Motivated by this, we will study well-posedness

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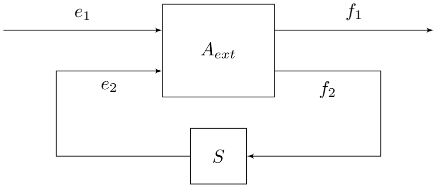

Figure 1: *Interconnection structure*

in terms of operators generating contraction semigroups. Hence, we want to know whether the operator A_S will generate a contraction C_0-semigroup if this holds for the initial system of A_{ext}. The case of X_1 and X_2 being Hilbert spaces has already been solved and can be found in [3, 8, 9]. Our aim is to generalize the result, including the conditions on S, to arbitrary Banach spaces.

A natural application is given by the heat equation for the space L^1. We conclude existence and uniqueness of its solutions from the undamped wave equation. Motivated by the example we give further results concerning the analyticity of the semigroup generated by A_S.

1.1. Semi-inner-products

In this section we collect some facts we are going to need.

The following notion was introduced by Lumer in 1961, see [6]. From now on, X will be a Banach space.

DEFINITION 1. For a Banach space X, a mapping $[\cdot,\cdot] : X \times X \to \mathbb{C}$ is called *semi-inner-product, SIP*, if for all $x, y, z \in X$ and $\lambda \in \mathbb{C}$

- $[x + \lambda z, y] = [x, y] + \lambda [z, y]$ *(linearity in first component)*,
- $[x, x] = \|x\|^2$ *(positive definiteness)*,
- $|[x, y]|^2 \leqslant [x, x][y, y]$ *(Cauchy-Schwarz inequality)*.

LEMMA 1. *The following assertions hold*

i. Every Banach space X has a SIP, i.e. X is a SIP space.

ii. For SIP spaces $(X, [\cdot,\cdot]_X)$, $(Y, [\cdot,\cdot]_Y)$, the mapping defined by

$$\left[\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \right]_{X \times Y} := [x_1, x_2]_X + [y_1, y_2]_Y \tag{1.2}$$

is a SIP for $X \times Y$ equipped with the Euclidean norm

$$\left\| \begin{pmatrix} x \\ y \end{pmatrix} \right\|_{X \times Y} = \sqrt{\|x\|^2 + \|y\|^2}, \qquad x \in X, y \in Y. \tag{1.3}$$

Proof. i. relies on the Hahn-Banach theorem, see [6]. For ii. one simply checks the definition of a SIP. □

As an example, let us consider L^p spaces, see [1, page 90].

EXAMPLE 1. For the space $L^p[0,1]$, $p \geqslant 1$,

$$[f,g] = \int_0^1 f(s)\tilde{g}(s)\, ds, \qquad f,g \in L^p[0,1],$$

where

$$\tilde{g}(s) := \begin{cases} \overline{g(s)}|g(s)|^{p-2}\|g\|_{L^p}^{2-p}, & g(s) \neq 0 \\ 0 & \text{otherwise} \end{cases},$$

defines a SIP.

DEFINITION 2. Let X be a Banach space. An operator $A : D(A) \subset X \to X$ is called *dissipative*, if there exists a SIP, such that

$$\Re[Ax,x] \leqslant 0 \qquad \forall x \in D(A). \tag{1.4}$$

In the literature the notion of *dissipativity* for general Banach spaces is often introduced in a different way (see e.g. [1]). We remark that this definition is equivalent. For instance, (1.4) implies that for all $\lambda > 0$, $x \in X$,

$$\lambda\|x\|^2 = \lambda\Re[x,x] = \Re[(\lambda\,\mathrm{I}-A)x,x] + \Re[Ax,x] \leqslant \|(\lambda\,\mathrm{I}-A)x\| \cdot \|x\|,$$

where we used (1.4) and Cauchy-Schwarz in the last inequality. The converse employs the Banach-Alaoglu Theorem and can be found in Proposition II.3.23 in [1]. There, (1.4) is formulated as

$$\forall x \in D(A)\ \exists j(x) \in \mathscr{J}(x) := \left\{x' \in X' : \langle x,x'\rangle = \|x'\|^2 = \|x\|^2\right\} \text{ such that}$$
$$\Re\langle Ax, j(x)\rangle \leqslant 0,$$

(where X' denotes the dual of X, $\langle\cdot,\cdot\rangle$ the duality brackets). $\mathscr{J}(x)$ is called the *duality set of* x. Note that any *selection* $j : X \to X' : x \mapsto j(x) \in \mathscr{J}(x)$ defines a SIP $[\cdot,\cdot] = \langle\cdot, j(\cdot)\rangle$ and, vice versa, every SIP $[\cdot,\cdot]$ yields a selection $j(x) = [\cdot,x] \in \mathscr{J}(x)$ for all $x \in X$.

The following theorem is a standard result in semigroup theory and can be found in [1, Section II.3.b] or [7, Theorem 3.1] (in the latter dissipativity is defined via SIPs).

THEOREM 1.1. (Lumer-Phillips) *For the linear operator A on the Banach space X the following assertions are equivalent*

i. A generates a contraction C_0-semigroup,

ii. A is densely defined, dissipative and there exists some $\lambda > 0$ such that

$$\mathrm{ran}(\lambda\,\mathrm{I}-A) = X. \tag{1.5}$$

In this case A is dissipative w.r.t. any SIP on X, and (1.5) holds for every $\lambda > 0$. If X is reflexive, $D(A)$ is automatically dense from the other assumptions in ii.

2. Main result

THEOREM 2.1. *Let $A_1 : D(A_1) \subset X_1 \times X_2 \to X_1$ and $A_2 : D(A_2) \subset X_1 \to X_2$ be operators such that*

$$A_{ext} := \begin{pmatrix} & A_1 & \\ A_2 & & 0 \end{pmatrix},$$
$$D(A_{ext}) = \{(x_1,x_2) \in X_1 \times X_2 : x_1 \in D(A_2) \wedge (x_1,x_2) \in D(A_1)\}$$

generates a contraction C_0-semigroup on $X_1 \times X_2$ equipped with the Euclidean norm, see (1.3). Let $S \in \mathscr{B}(X_2)$ be a boundedly invertible satisfying

$$\Re[x,Sx]_2 \geqslant m_2\|x\|_2^2 \qquad \forall x \in X_2, \tag{2.1}$$

for some $m_2 > 0$ and some SIP $[\cdot,\cdot]_2$ on X_2. Then

$$A_S x = A_1 \begin{pmatrix} x \\ SA_2x \end{pmatrix},$$

defined on $D(A_S) = \{x \in X_1 : (x, SA_2x) \in D(A_{ext})\}$ generates a contraction semigroup on X_1 provided that $D(A_S)$ is dense or that X_1 is reflexive.

Proof. By the Lumer-Phillips Theorem, the proof consists of two steps. First we show that A_S is dissipative. Let $[\cdot,\cdot]_1$ be a SIP on X_1. Then, let $[\cdot,\cdot]_{X_1\times X_2}$ be the SIP defined in (1.2) with respect to $[\cdot,\cdot]_1$ and $[\cdot,\cdot]_2$. For $x \in D(A_S)$ we get

$$\begin{aligned}
[A_Sx,x]_1 &= \left[A_1 \begin{pmatrix} x \\ SA_2x \end{pmatrix}, x\right]_1 \\
&= \left[A_1 \begin{pmatrix} x \\ SA_2x \end{pmatrix}, x\right]_1 + \left[A_2x, SA_2x\right]_2 - \left[A_2x, SA_2x\right]_2 \\
&= \left[A_{ext} \begin{pmatrix} x \\ SA_2x \end{pmatrix}, \begin{pmatrix} x \\ SA_2x \end{pmatrix}\right]_{X_1\times X_2} - \left[A_2x, SA_2x\right]_2
\end{aligned} \tag{2.2}$$

The second term is less or equal zero by the assumption (2.1). By Theorem 1.1, A_{ext} is dissipative w.r.t. any SIP on $X_1 \times X_2$. Together this yields

$$\Re[A_Sx,x]_1 \leqslant 0.$$

Hence, A_S is dissipative.

To show the range condition (1.5), let $\lambda \in \mathbb{R}$ and consider

$$P = \begin{pmatrix} 0 & 0 \\ 0 & \lambda\,\mathrm{I} - S^{-1} \end{pmatrix} \in \mathscr{B}(X_1 \times X_2).$$

$A_{ext} + P$ is a bounded perturbation of a generator, hence, it also generates a semigroup, see [1, Theorem III.1.3]. By (2.1) we have for $x = (x_1,x_2)^T \in X_1 \times X_2$ that

$$\Re[Px,x]_{X_1\times X_2} = \Re[(\lambda\,\mathrm{I} - S^{-1})x_2, x_2]_2 \leqslant \left(\lambda - \frac{m_2}{\|S\|^2}\right)\|x_2\|^2.$$

Thus, P is dissipative if $\lambda \in (0, m_2/\|S\|^2]$, and then, $A_{ext}+P$ generates a contraction semigroup by the Lumer-Phillips Theorem. Particularly, the range of $\lambda \mathrm{I} - A_{ext} - P$ equals $X_1 \times X_2$. Hence, for any pair $(g,0) \in X_1 \times X_2$ there exists $(x_1,x_2) \in X_1 \times X_2$ such that

$$(\lambda \mathrm{I} - A_{ext} - P)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} g \\ 0 \end{pmatrix}. \tag{2.3}$$

By the structure of A_{ext}, the second component reads

$$\lambda x_2 - A_2 x_1 + S^{-1}x_2 - \lambda x_2 = 0,$$

which implies $x_2 = SA_2x_1$. Inserting in the first component of (2.3) gives

$$\lambda x_1 - A_1 \begin{pmatrix} x_1 \\ SA_2x_1 \end{pmatrix} = g,$$

which is $(\lambda \mathrm{I} - A_S)x_1 = g$. Thus, $\operatorname{ran}(\lambda \mathrm{I} - A_S) = X_1$.

By assumption that either $D(A_S)$ is dense or X_1 is reflexive we conclude from Theorem 1.1 (Lumer-Phillips) that A_S generates a contraction semigroup. $\square$

REMARK 1.

1. Because of the boundedness of S^{-1}, condition (2.1) holds for all SIPs on X_2 if it holds for some SIP, see [7, Remark 2].

2. Note that since S is boundedly invertible, (2.1) is equivalent to

$$\exists \tilde{m} > 0\ \forall x \in X_2: \qquad \Re[S^{-1}x,x]_2 \leqslant \tilde{m}\|x\|^2 \Leftrightarrow \Re[(\tilde{m}\mathrm{I} - S^{-1}\mathrm{I})x,x]_2 \leqslant 0,$$

which means that $\tilde{m}\mathrm{I} - S^{-1}$ is dissipative.

3. For a boundedly invertible operator $B \in \mathscr{B}(X)$ on a Banach space X, B dissipative does not necessarily imply that B^{-1} is dissipative. In fact, by Lumer-Phillips this is equivalent to ask whether B^{-1} generates a contraction C_0-semigroup, if B does. The answer is negative in general, even in finite dimensions, see e.g. [2, Section 2]. However, on Hilbert spaces, the dissipativity of B^{-1} always follows from the one of B by the symmetry of the inner product.

4. For X_2 being a Hilbert space the assumptions on S are equivalent to

$$S \in \mathscr{B}(X_2) \text{ and } S + S^* \geqslant \varepsilon \mathrm{I} > 0.$$

We finish this part by showing that the converse of Theorem 2.1 does not hold in the sense that A_{ext} does not necessarily generate a contraction C_0-semigroup if A_S does. Looking at the proof, there is no reason to believe that the arguments in both parts (disspativity, range condition) could be reversed. For instance, let $S = \mathrm{I}$ and A_S be dissipative. Then, one gets that

$$\Re\left[A_{ext}\begin{pmatrix} x \\ SA_2x \end{pmatrix}, \begin{pmatrix} x \\ SA_2x \end{pmatrix}\right]_{X_1 \times X_2} \leqslant \|x\|^2_{X_1} \qquad \forall x \in D(A_S)$$

by reading the eq. (2.2) in reversed order. However, this won't give that A_{ext} is dissipative (and since A_{ext} should generate a semigroup, this should hold w.r.t. any SIP) in general. In fact, consider the matrix case

$$A_{ext} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \in \mathbb{R}^{2\times 2} \quad \Rightarrow \quad A_S = A_{\mathrm{I}} = 0,$$

with the Euclidean norm on $\mathbb{R}^2$. Clearly,

$$\left[A_{ext}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right] = \left[\begin{pmatrix} 0 \\ x_1 \end{pmatrix}, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right] = x_1 x_2.$$

Therefore, A_{ext} can not be dissipative, whereas $[A_S x, x] = 0$.

2.1. From wave to heat equation

We start with the undamped wave equation $\frac{\partial^2 w}{\partial t^2}(\xi,t) = \frac{\partial^2 w}{\partial \xi^2}(\xi,t)$ on $[0,1]$. The boundary conditions are chosen to be

$$\begin{cases} (K_1 - 1)\frac{\partial w}{\partial t}(1,t) = (K_1 + 1)\frac{\partial w}{\partial \xi}(1,t), \\ (1 - K_2)\frac{\partial w}{\partial t}(0,t) = (K_2 + 1)\frac{\partial w}{\partial \xi}(0,t), \end{cases} \forall t \geqslant 0, \text{with } |K_1|, |K_2| \leqslant 1. \tag{2.4}$$

This can be written as the following ACP on $L^p[0,1] \times L^p[0,1]$, $p \geqslant 1$,

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \xi} & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} := A_{ext} x, \qquad x(0) = x_0 \tag{2.5}$$

with

$$D(A_{ext}) = \left\{ \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \in (L^p[0,1])^2 : f_1, f_2 \text{ abs. continuous and} \right. \tag{2.6}$$
$$\left. \frac{\partial f_1}{\partial \xi}, \frac{\partial f_2}{\partial \xi} \in L^p[0,1], (Qf)_1(1) = K_1 (Qf)_2(1), (Qf)_2(0) = K_2 (Qf)_1(0) \right\},$$

where $Q = \frac{1}{\sqrt{2}}\left(\begin{smallmatrix} 1 & 1 \\ -1 & 1 \end{smallmatrix}\right)$. In the framework of Theorem 2.1 the operators A_1 and A_2 read

$$D(A_1) = \left\{ \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \in (L^p[0,1])^2 : f_1, f_2 \text{ abs. continuous and } \frac{\partial f_2}{\partial \xi} \in L^p[0,1], \right.$$
$$\left. (Qf)_1(1) = K_1 (Qf)_2(1), (Qf)_2(0) = K_2 (Qf)_1(0) \right\}, \quad A_1 \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \frac{\partial}{\partial \xi} f_2,$$

$$D(A_2) = \left\{ f \in L^p[0,1] : f \text{ abs. cont.}, \frac{\partial f}{\partial \xi} \in L^p[0,1] \right\}, \quad A_2 f = \frac{\partial}{\partial \xi} f.$$

By diagonalizing, $\mathscr{D} = Q A_{ext} Q^{-1}$, it is easy to show that A_{ext} generates a contraction C_0-semigroup (in the Euclidean norm). Furthermore, let $\xi \mapsto \lambda(\xi)$ be positive and continuously differentiable on $[0,1]$ and denote by S the induced multiplication operator. Then,

$$A_S f = A_1 \begin{pmatrix} f \\ SA_2 f \end{pmatrix} = \begin{pmatrix} 0 & \frac{\partial}{\partial \xi} \end{pmatrix} \begin{pmatrix} f \\ \lambda(\xi)\frac{\partial f}{\partial \xi} \end{pmatrix} = \frac{\partial}{\partial \xi}\left(\lambda(\xi)\frac{\partial f}{\partial \xi}\right), \tag{2.7}$$
$$D(A_S) = \left\{ f \in L^p[0,1] : (f, SA_2 f)^T \in D(A_{ext}) \right\}$$

By the assumptions on $\lambda(\xi)$, it follows easily that

$$D(A_S) = \left\{ f \in L^p[0,1] : f, \frac{\partial f}{\partial \xi} \text{ abs. continuous and } \frac{\partial f}{\partial \xi}, \frac{\partial^2 f}{\partial \xi^2} \in L^p[0,1], \right.$$
$$\left. (K_1+1)f(1) = (K_1-1)\lambda(1)\frac{\partial f}{\partial \xi}(1), (K_2+1)f(0) = (1-K_2)\lambda(0)\frac{\partial f}{\partial \xi}(0) \right\}$$

which is dense in $L^p[0,1]$. The operator A_S corresponds to the heat equation

$$\frac{\partial u}{\partial t}(\xi,t) = \frac{\partial}{\partial \xi}\left(\lambda(\xi)\frac{\partial u}{\partial \xi}(\xi,t)\right), \tag{2.8}$$

with the Robin boundary conditions

$$\begin{cases} (K_2+1)u(0,t) = (1-K_2)\lambda(0)\frac{\partial u}{\partial \xi}(0,t), \\ (K_1+1)u(1,t) = (K_1-1)\lambda(1)\frac{\partial u}{\partial \xi}(1,t), \end{cases} \quad \forall t \geqslant 0. \tag{2.9}$$

Hence, $\lambda(\xi)$ can represent the heat conduction coefficient. It remains to show that the assumptions on S are fulfilled. Clearly, S is a bounded operator which is boundedly invertible since there exist $\lambda_{min}, \lambda_{max}$ such that $0 < \lambda_{min} < \lambda(\xi) < \lambda_{max}$ for $\xi \in [0,1]$. To show (2.1) we use the SIP from Example 1,

$$[f, Sf] = \int_0^1 \lambda(s)^{p-1}|f(s)|^p \|Sf\|_{L^p}^{2-p}\, ds \geqslant \frac{1}{\lambda_{max}}\|Sf\|_{L^p}^2 \geqslant \frac{\lambda_{min}^2}{\lambda_{max}}\|f\|_{L^p}^2. \tag{2.10}$$

Thus, by Theorem 2.1, we conclude that A_S generates a contraction semigroup.

2.2. Further results

Motivated by the example in Subsection 2.1, one might ask when A_S is even generating an analytic semigroup. Without further assumptions on the operator A_{ext} this does not seem to work in general. However, the following theorem gives an answer.

THEOREM 2.2. *Assume that A_{ext} from Theorem 2.1 has the form*

$$A_{ext} = \begin{pmatrix} 0 & A_{12} \\ A_{21} & 0 \end{pmatrix}, \tag{2.11}$$
$$D(A_{ext}) = \{(x_1, x_2) \in X_1 \times X_2 : x_1 \in D(A_{21}), x_2 \in D(A_{12})\}$$

and that $\mathscr{A} := \left(\begin{smallmatrix} I & 0 \\ 0 & S \end{smallmatrix}\right) A_{ext}$ *generates a* C_0*-group, where* $S \in \mathscr{B}(X_2)$*. Then,*

$$A_S = A_{12} S A_{21},$$

with $D(A_S) = \{x \in X_1 : x \in D(A_{21}), SA_{21}x \in D(A_{12})\}$ *generates an analytic semigroup of angle* $\frac{\pi}{2}$*.*

Proof. It is a fact that if $\mathscr{A}$ generates a C_0-group, it follows that $\mathscr{A}^2$ generates an analytic C_0-semigroup of angle $\frac{\pi}{2}$, see [1, Corollary II.4.9]. Therefore, the result follows by considering the upper left entry of

$$\mathscr{A}^2 = \begin{pmatrix} 0 & A_{12} \\ SA_{21} & 0 \end{pmatrix} \begin{pmatrix} 0 & A_{12} \\ SA_{21} & 0 \end{pmatrix} = \begin{pmatrix} A_{12}SA_{21} & 0 \\ 0 & A_{21}SA_{12} \end{pmatrix},$$

where

$$D(\mathscr{A}^2) = \{(x_1, x_2) \in D(A_{21}) \times D(A_{12}) : SA_{21}x_1 \in D(A_{12}), SA_{12}x_2 \in D(A_{21}\}. \quad \square$$

REMARK 2. Given that A_{ext} generates a C_0-group, the assumption in Theorem 2.2, that $\mathscr{A}$ generates a C_0-group, can be checked by means of (multiplicative) perturbation results for generators, see e.g. [4].

In the following we note that the group generation is not surprising in the view of the assumptions in Theorem 2.1

THEOREM 2.3. (Lemma 5.1 in [9]) *Let* A_{ext}*, given in the form (2.11), generate a* C_0*-semigroup* $T(t)$ *with constants* M, ω *such that* $\|T(t)\| \leqslant Me^{t\omega}$ *for all* $t > 0$*. Then,* A_{ext} *can be extended to a* C_0*-group which satisfies* $\|T(t)\| \leqslant Me^{|t|\omega}$*. In particular, if* A_{ext} *generates a contraction semigroup, then* A_{ext} *generates a group of isometries.*

With the results of this subsection we are able to continue the discussion of the example of the wave and heat equation in Section 2.1. To conclude the analyticity of the semigroup generated by A_S, (2.7), it remains to check that $\mathscr{A} = \left(\begin{smallmatrix} I & 0 \\ 0 & S \end{smallmatrix}\right) A_{ext}$ generates a C_0-group. By Proposition 2.3, it even suffices to show that $\mathscr{A}$ generates a C_0-semigroup. In fact, by diagonalizing and using the specific assumptions on S (the multiplication operator induced by λ), this is not hard to deduce (see also [5, Chapters 12 and 13]).

2.3. Remarks and outlook

One might question the use of SIPs instead of employing the more common dissipativity definition only relying on the norm. The reason is that the condition on S and the proof happens to be natural in the view of the Hilbert space result.

Discussing more general S (and at the same time restricting the form of A_{ext}) as $S = i\mathrm{I}$, like it is done in [9, Section 4] for Hilbert spaces, might be possible as well as adaptions to nonlinear S.

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