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LETTER TO THE EDITOR

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Abstract. In the note some remarks are given concerning certain results of the article [M. A. Sarigöl, *A remark on* $\varphi-|\bar{N},q_n;\delta|_k$ *summability*, this journal, **12**, 1 (2018), 55–58.]

1. Introduction

Let $\sum a_n$ be a given infinite series with partial sums (s_n). Let (p_n) be a sequence of positive numbers for which there holds

$$P_n=\sum_{\nu=0}^{n} p_\nu \longrightarrow \infty (n\to\infty), \qquad (P_{-i}=p_{-i}=0, \quad i\geqslant 1). \tag{1.1}$$

The sequence-to-sequence transformation

$$\sigma_n=\frac{1}{P_n}\sum_{\nu=0}^{n} p_\nu s_\nu \tag{1.2}$$

defines the sequence (σ_n) of the Riesz mean or simply the $(\bar{N},p_n)$ mean of the sequence (s_n) generated by the sequence of coefficients (p_n) [3]. The series $\sum a_n$ is said to be summable $|\bar{N},p_n;\delta|_k$, $k\geqslant 1$ and $\delta\geqslant 0$ if [2]

$$\sum_{n\geqslant 1}\left(\frac{P_n}{p_n}\right)^{(\delta+1)k-1}|\sigma_n-\sigma_{n-1}|^k \tag{1.3}$$

converges. In turn, setting $\delta=0$, we obtain the $|\bar{N},p_n|_k$, $k\geqslant 1$ summability, see [1].

Let (φ_n) be any sequence of positive reals. The series $\sum a_n$ is summable in the sense $\varphi-|\bar{N},p_n;\delta|_k$, $k\geqslant 1$ and $\delta\geqslant 0$ when [7]

$$\sum_{n\geqslant 1}\varphi_n^{(\delta+1)k-1}|\sigma_n-\sigma_{n-1}|^k<\infty. \tag{1.4}$$

If we specialize $\delta=0$ and $\varphi_n=P_n/p_n$, then $\varphi-|\bar{N},p_n;\delta|_k$ summability reduces to $|\bar{N},p_n|_k$ summability.

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2. Known results

In [4] Özarslan proved the following theorems to obtain equivalence between two general summability methods.

THEOREM 2.1. *Let* $k \geqslant 1$ *and* $0 \leqslant \delta < 1/k$ *while* $(\varphi_n), (p_n), (q_n)$ *be positive sequences. Assume that*

$$\sum_{n=\nu+1}^{m+1} \frac{\varphi_n^{(\delta+1)k-1} q_n^k}{Q_n^k Q_{n-1}} = \mathscr{O}\left(\varphi_\nu^{(\delta+1)k-1} \frac{q_\nu^{k-1}}{Q_\nu^k}\right) \tag{2.1}$$

as $m \to \infty$. *In order that every* $\varphi - |\overline{N}, p_n; \delta|_k$ *summable series be* $\varphi - |\overline{N}, q_n; \delta|_k$ *summable it is necessary that there holds*

$$\frac{q_n P_n}{Q_n p_n} = \mathscr{O}(1). \tag{2.2}$$

Moreover, when

$$\frac{p_n Q_n}{P_n q_n} = \mathscr{O}(1), \tag{2.3}$$

then (2.2) *is also sufficient for the asserted conclusion.*

THEOREM 2.2. *Let* (p_n) *and* (q_n) *be positive sequences satisfying the constraint* (2.1), $k \geqslant 1$ *and* $0 \leqslant \delta < 1/k$. *In order that* $\varphi - |\overline{N}, p_n; \delta|_k$ *is equivalent to* $\varphi - |\overline{N}, q_n; \delta|_k$ *summability it is necessary and sufficient that both* (2.2) *and* (2.3) *hold.*

3. Remarks upon results

Firstly, reference [5] (listed also in [6]) concerns Fourier series which are not mentioned in [6]. Also, it should be noticed that in [4], Özarslan has not tried to establish both necessary and sufficient conditions in order that every $\varphi - |\overline{N}, p_n; \delta|_k$ summable series should be $\varphi - |\overline{N}, q_n; \delta|_k$ summable; the article [4] contains the equivalence of two general summability methods *only under some suitable conditions*.

So, there is no relevance between [6] and [4].

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