

ON A CONJECTURE FOR THREE-DIMENSIONAL COMPETITIVE LOTKA-VOLTERRA SYSTEMS WITH A HETEROCLINIC CYCLE

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Abstract. For three-dimensional competitive Lotka-Volterra systems, Zeeman (1993) identified 33 stable equivalence classes. In this paper we show that: in the case of a heteroclinic cycle on the boundary of the carrying simplex of three-dimensional competitive Lotka-Volterra systems (class 27 in Zeeman's classification), the conditions (a) there is a pair of purely imaginary eigenvalues at an interior equilibrium, (b) the first focal value vanishes, (c') the second focal value vanishes, and (c) the heteroclinic cycle is neutrally stable do *not* imply (d) the third focal value vanishes. In particular, the conditions (a), (b), (c'), and (c) do not imply that the interior equilibrium is a center. This proves a conjecture by Gyllenberg and Yan (2009).

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