

SOME RESULTS ON WEIGHTED CRITICAL QUASILINEAR PROBLEMS

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Abstract. In this paper, we are concerned with a kind of quasilinear elliptic problems, which involves the Caffarelli-Kohn-Nirenberg inequality and critical exponents. By employing variational methods and analytical techniques, the existence of sign-changing solutions to the problem is proved.

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