Differential
Equations
& Applications
Volume 3, Number 1 (2011), 11–19

STABILITY OF FUNCTIONAL DIFFERENTIAL EQUATIONS WITH OSCILLATING COEFFICIENTS AND DISTRIBUTED DELAYS

MICHAEL I. GIL'

Abstract. We consider the scalar equation

$$\dot{x}(t) + \sum_{j=1}^{m} a_j(t) \int_0^h x(t-s)dr_j(s) = 0 \ \ (h = const > 0,\, \dot{x} = dx/dt),$$

where $r_j(s)$ are nondecreasing functions. Besides, we do not require that $a_j(t)$ are positive for all $t \geqslant 0$. So the function

$$z + \sum_{j=1}^{m} a_j(t) \int_0^h e^{-zs} dr_j(s)$$

can have zeros in the right-hand plane for some $t \geqslant 0$. It is proved that the considered equation is exponentially stable, provided $a_j(t) = b_j + c_j(t)$, where b_j are positive constants, such that all the zeros of the function $z + \sum_{j=1}^{m} b_j \int_0^h e^{-zs} dr_j(s)$ are in the open left-hand plane, and the integrals $\int_0^t c_j(s)ds$ $(j = 1, \dots, m)$ are sufficiently small for all $t > 0$.

Mathematics subject classification (2010): 34K20.
Keywords and phrases: functional differential equation, linear equation, exponential stability.

REFERENCES

[1] N. V. AZBELEV, P.M. SIMONOV, *Stability of Differential Equations with Aftereffect*, Stability Control Theory Methods Appl., **20**, Taylor & Francis, London, 2003.
[2] L. BEREZANSKY, E. BRAVERMAN, *On exponential stability of linear differential equations with several delays*, J. Math. Anal. Appl., **324** (2006), 1336–1355.
[3] L. BEREZANSKY, E. BRAVERMAN, *On stability of some linear and nonlinear delay differential equations*, J. Math. Anal. Appl., **314** (2006), 391–411.
[4] M. I. GIL', *The Aizerman-Myshkis problem for functional-differential equations with causal nonlinearities*, Funct. Differ. Equ., **11**, 1-2 (2005), 175–185.
[5] A. IVANOV, E. LIZ, AND S. TROFIMCHUK, *Halanay inequality, Yorke 3/2 stability criterion, and differential equations with maxima*, Tohoku Math. J., **54**, 2 (2002), 277–295.
[6] V. KOLMANOVSKII, A. MYSHKIS, *Introduction to the Theory and Applications of Functional Differential Equations*, Math. Appl., **463**, Kluwer, Dordrecht, 1999.
[7] T. KRISZTIN, *On stability properties for one-dimensional functional-differential equations*, Funkcial. Ekvac., **34** (1991), 241–256.
[8] J. C. LILLO, *Oscillatory solutions of the equation* $y'(x) = m(x)y(x-n(x))$, J. Differential Equations, **6** (1969), 1–35.
[9] E. LIZ, V. TKACHENKO, AND S. TROFIMCHUK, *A global stability criterion for scalar functional differential equations*, SIAM J. Math. Anal., **35** (2003), 596–622.
[10] A. D. MYSHKIS, *On solutions of linear homogeneous differential equations of the first order of stable type with a retarded argument*, Mat. Sbornik N.S., **28** (1951), 15–54.
[11] J. W. H. SO, J. S. YU, AND M. P. CHEN, *Asymptotic stability for scalar delay differential equations*, Funkcial. Ekvac., **39** (1996), 1–17.
[12] X. WANG, L. LIAO, *Asymptotic behavior of solutions of neutral differential equations with positive and negative coefficients*, J. Math. Anal. Appl., **279** (2003), 326–338.

Paper DEA-03-02

[13] T. YONEYAMA, *On the stability for the delay-differential equation* $\dot{x}(t) = -a(t)f(x(t-r(t)))$, J. Math. Anal. Appl., **120** (1986), 271–275.

[14] T. YONEYAMA, *On the 3/2 stability theorem for one-dimensional delay-differential equations*, J. Math. Anal. Appl., **125** (1987), 161–173.

[15] T. YONEYAMA, *The 3/2 stability theorem for one-dimensional delay-differential equations with unbounded delay*, J. Math. Anal. Appl., **165**, 1 (1992), 133–143.

[16] Z. ZHANG, Z. WANG, *Asymptotic behavior of solutions of neutral differential equations with positive and negative coefficients*, Ann. Differential Equations, **17**, 3 (2001), 295–305.