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ASYMPTOTIC STABILITY AND STABILITY SWITCHES IN A LINEAR INTEGRO–DIFFERENTIAL SYSTEM

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Abstract. This paper is concerned with the stability problem of a linear integro-differential system with distributed delay in the diagonal terms. We establish some explicit conditions for the zero solution of the system to be asymptotically stable. In particular, as the delay parameter increases monotonously under certain conditions, the zero solution switches finite times from stability to instability to stability, and becomes unstable eventually.

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