

OSCILLATION CRITERIA OF CERTAIN THIRD ORDER NEUTRAL DIFFERENTIAL EQUATIONS

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Abstract. This paper deals with the oscillation of the following third order neutral delay differential equation

$$(r(t)|z''(t)|^{\alpha-1}z''(t))' + q(t)|x(\tau(t))|^{\alpha-1}x(\tau(t)) = 0,$$

where $t \geq t_0$, $\alpha > 0$ and $z(t) = x(t) + p(t)x(\delta(t))$. We will establish some new sufficient conditions which insure that any solution of this equation oscillates or converges to zero. Two examples are also provided to illustrate the relevance of the main results.

Mathematics subject classification (2010): 34K10, 34K11.

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