

APPROXIMATION SCHEMES FOR MONOTONE SYSTEMS OF NONLINEAR SECOND ORDER PARTIAL DIFFERENTIAL EQUATIONS: CONVERGENCE RESULT AND ERROR ESTIMATE

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Abstract. We consider approximation schemes for monotone systems of fully nonlinear second order partial differential equations. We first prove a general convergence result for monotone, consistent and regular schemes. This result is a generalization to the well known framework of Barles-Souganidis, in the case of scalar nonlinear equation. Our second main result provides the convergence rate of approximation schemes for weakly coupled systems of Hamilton-Jacobi-Bellman equations. Examples including finite difference schemes and Semi-Lagrangian schemes are discussed.

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