

NON-SIMULTANEOUS BLOW-UP FOR A SEMILINEAR PARABOLIC SYSTEM WITH LOCALIZED REACTION TERMS

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Abstract. In this paper, we study positive blow-up solutions of the semilinear parabolic system with localized reactions $u_t = \Delta u + v^r + u^p(0,t)$, $v_t = \Delta v + u^s + v^q(0,t)$ in the ball $B = \{x \in \mathbb{R}^N : |x| < R\}$, under the homogeneous Dirichlet boundary condition. It is shown that non-simultaneous blow-up may occur according to the value of p , q , r , and s ($p, q, r, s > 1$). We also investigate blow-up rates of all total blow-up solutions when simultaneous blow-up occurs.

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