

## THE ADMB-KDV EQUATION IN ANISOTROPIC SOBOLEV SPACES

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*Abstract.* Considered herein is the anisotropic dissipation-modified Boussinesq-KdV equation which is a two-dimensional version of the KdV equation. It is established that the Cauchy problem associated to this equation is locally well-posed in anisotropic Sobolev spaces  $H^{s_1, s_2}(\mathbb{R}^2)$ , for all  $s_1 > -1/2$  and  $s_2 \geq 0$ . A global existence result for this equation will be obtained under suitable conditions.

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