

AN ECO-EPIDEMIOLOGICAL MODEL IN TWO COMPETING SPECIES

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Abstract. The population sizes of species are affected not only by ecological interactions, such as predation and competition, but also by infectious diseases. In this paper, we propose a model combining disease and competition, and try to understand how the disease affects the two competing species. We assume that only one of the species is susceptible to an SI type disease with mass action incidence, and that infected individuals do not reproduce but suffer additional disease induced death. We further assume that infection does not reduce the competitive ability of infectives. We show that if species 1 is a superior competitor without disease, then infection of species 1 can enable an inferior competitor to coexist, either as a stable equilibrium or as a limit cycle. If in the absence of the disease, the two species coexist, then the introduction of the disease is partially determined by the basic reproduction number. If the basic reproduction number is larger than 1, then our system is uniformly persistent and the unique coexisting endemic disease equilibrium is globally stable under certain conditions. Meanwhile, if species 1 is an inferior competitor without disease, then infection of species 1 can not change the outcomes under certain conditions.

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