

THE ANTIMAXIMUM PRINCIPLE AND THE EXISTENCE OF A SOLUTION FOR THE GENERALIZED p -LAPLACE EQUATIONS WITH INDEFINITE WEIGHT

MIEKO TANAKA

Abstract. This paper treats the antimaximum principle and the existence of a solution for quasilinear elliptic equation $-\operatorname{div}(a(x, |\nabla u|)\nabla u) = \lambda m(x)|u|^{p-2}u + h(x)$ in Ω under the Neumann boundary condition. Here, a map $a(x, |y|)y$ on $\overline{\Omega} \times \mathbb{R}^N$ is strictly monotone in the second variable and satisfies certain regularity conditions. This equation contains the p -Laplacian problem as a special case.

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