

## THREE POSITIVE SOLUTIONS OF STURM-LIOUVILLE BOUNDARY VALUE PROBLEMS FOR FRACTIONAL DIFFERENTIAL EQUATIONS

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*Abstract.* We establish the results on the existence of three positive solutions to Sturm-Liouville boundary value problems of the singular fractional differential equation with the nonlinearity depending on  $D_{0+}^{\mu} u$

$$\begin{cases} D_{0+}^{\alpha} u(t) + f(t, u(t), D_{0+}^{\mu} u(t)) = 0, & t \in (0, 1), 1 < \alpha < 2, \\ a \lim_{t \rightarrow 0} I_{0+}^{2-\alpha} u(t) - b \lim_{t \rightarrow 0} [I_{0+}^{2-\alpha} u(t)]' = \int_0^1 g(s, u(s), D_{0+}^{\mu} u(s)) ds, \\ c D_{0+}^{\mu} u(1) + du(1) = \int_0^1 h(s, u(s), D_{0+}^{\mu} u(s)) ds. \end{cases}$$

Our analysis relies on the well known five functional fixed point theorems. An example is given to illustrate the efficiency of the main theorems.

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