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BOUNDARY BLOW–UP RATES OF LARGE SOLUTIONS FOR QUASILINEAR ELLIPTIC EQUATIONS WITH CONVENTION TERMS

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Abstract. We use Karamata regular variation theory to study the exact asymptotic behavior of large solutions near the boundary to a class of quasilinear elliptic equations with convection terms

$$\begin{cases} \Delta_p u \pm |\nabla u|^{q(p-1)} = b(x)f(u), & x \in \Omega, \\ u(x) = +\infty, & x \in \partial\Omega, \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^{\mathbb{N}}$. The weight function $b(x)$ is a non-negative continuous function in the domain, $f(u) \in C^2[0,+\infty)$ is increasing on $[0,\infty)$, and regularly varying at infinity with index $\rho > p-1$.

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