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STABILITY OF POSITIVE SOLUTIONS TO $p\&2$–LAPLACE TYPE EQUATIONS

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Abstract. In this article, we first show the existence of a positive solution to

$$\begin{cases} -\Delta_p u - \alpha\Delta u = \lambda(u - f(u)) \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases}$$

by the method of lower and upper solutions and then under certain conditions on f, we show the stability of positive solution.

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