

MAXIMAL AND MINIMAL POSITIVE SOLUTIONS OF A NONLINEAR QUADRATIC INTEGRAL EQUATION

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Abstract. We are concerned here with the existence of at least one continuous positive solution of the nonlinear quadratic integral equation

$$x(t) = a(t) + \lambda \int_0^t k_1(t,s)f(s,x(s))ds \int_0^t k_2(t,s)g(s,x(s))ds, \quad t \in [0, T].$$

where f and g are L^1 -Carathéodory functions. The maximal and minimal solutions are also proved.

Mathematics subject classification (2010): 45G10, 45D05, 45M20.

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REFERENCES

- [1] I. K. ARGYROS, *Quadratic equations and applications to Chandrasekhar's and related equations*, Bull. Austral. Math. Soc., **32**(1985) 275–292.
- [2] I. K. ARGYROS, *On a class of quadratic integral equations with perturbations*, Funct. Approx., **20**(1992) 51–63.
- [3] J. BANAŠ, M. LECKO, W.G. EL-SAYED, *Existence theorems of some quadratic integral equation*, J. Math. Anal. Appl. **227** (1998)276–279.
- [4] J. BANAŠ, A. MARTINON, *Monotonic solutions of a quadratic integral equation of Volterra type*, Comput. Math. Appl., **47** (2004) 271–279.
- [5] J. BANAŠ, J. CABALLERO, J. ROCHA AND K. SADARANGANI, *Monotonic solutions of a class of quadratic integral equation of Volterra type*, Comp. and Math. with Applications, **49** (2005) 943–952.
- [6] J. BANAŠ, J. ROCHA MARTIN, K. SADARANGANI, *On the solutions of a quadratic integral equation of Hammerstien type*, Mathematecal and Computer Modelling, **43** (2006) 97–104.
- [7] J. BANAŠ, B. RZEPKA, *Monotonic solutions of a quadratic integral equations of fractional-order*, J. Math. Anal. Appl., **332** (2007) 1370–11378.
- [8] J. BANAŠ, B. RZEPKA, *Nondecreasing solutions of a quadratic singular Volterra integral equation*, Math. Comput. Modelling, **49** (2009) 488–496.
- [9] A. M. A. EL-SAYED, M. M .SALEH AND E. A. A. ZIADA, *Numerical and analytic solution for a nonlinear quadratic integral equation*, Math. Sci. Res. J., **12**(8) (2008) 183–191.
- [10] A. M. A. EL-SAYED, H. H. G. HASHEM, *Carathéodory type theorem for a nonlinear quadratic integral equation*, Math. Sci. Res. J., **12**(4) (2008)71–95.
- [11] A. M. A. EL-SAYED, H. H. G. HASHEM, AND E. A. A. ZIADA, *Picard and Adomian methods for quadratic integral equation*, Computational and Applied Mathematics, **29**(3) (2010) 447–463.
- [12] A. M. A. EL-SAYED, M. SH. MOHAMED, F. F. S. MOHAMED, *Existence of positive continuous solution of a quadratic integral equation of fractional orders*, Journal of Fractional Calculus and Applications, **1**(9) (2011) 1–7.

- [13] A. M. A. EL-SAYED, M. SH. MOHAMED, Y. M. Y. OMAR, *Existence of continuous solution of a quadratic integral equation of convolution type*, Journal of Fractional Calculus and Applications, **3**(S4) (2012) 1–9.
- [14] A. M. A. EL-SAYED, M. SH. MOHAMED, N. A. O. BUHALIMA, *Existence of a unique continuous solution for a quadratic integral equations*, Journal of Fractional Calculus and Applications, **3**(S22) (2012) 1–7.
- [15] A. M. A. EL-SAYED, H. H. G HASHEM AND Y. M. Y. OMAR, *Positive continuous solution of a quadratic integral equation of fractional orders*, Math. Sci. Lett., **2**(1)(2013) 19–27.
- [16] W. G. EL-SAYED, B. RZEPKA, *Nondecreasing solutions of a quadratic integral equation of Urysohn type*, Computers and Mathematics with Applications, **51** (2006) 1065–1074.
- [17] GEORGE H. PIMBLEY JR., *Positive solutions of a quadratic integral equation*, Archive for Rational Mechanics and Analysis, **24**(2)(1967) 107–127.
- [18] K. GOEBEL, W. A. KIRK, *Topics in metric fixed point theory*, Cambridge University Press, Cambridge (1990).
- [19] A. N. KOLMOGOROV AND S. V. FOMIN, *Introductory real Analysis*, Dover Publ. Inc. 1975.
- [20] V. LAKSHMIKANTHAM AND S. LEELA, *Differential and integral inequalities*, vol. **1**, New York-London, 1969.
- [21] K. MALEKNEJAD, P. TORABI, R. MOLLAPOURASL, *Fixed point method for solving nonlinear quadratic Volterra integral equations*, Computers & Mathematics with Applications, **62**(6) (2011) 2555–2566.
- [22] H.A.H. SALEM, *On the quadratic integral equations and their applications*, Computers & Mathematics with Applications, **62** (2011) 2931–2943.
- [23] E.A.A. ZIADA, *Adomian solution of a nonlinear quadratic integral equation*, Journal of the Egyptian Mathematical Society, **21** (2013) 52–56.