

FORCED OSCILLATION OF SECOND-ORDER NONLINEAR FUNCTIONAL DYNAMIC EQUATIONS ON TIME SCALES

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Abstract. In this paper, we establish some new oscillation criteria for the second-order nonlinear functional dynamic equation with forced term

$$(r(t)x^\Delta(t))^\Delta \pm p(t)f(x(\tau(t))) = e(t),$$

on a time scale \mathbb{T} . No restriction is imposed on the forcing term $e(t)$ to satisfy the Kartsatos condition. $p(t)$ and $r(t)$ are real-valued rd-continuous functions defined on \mathbb{T} . There are many cases have been taken into consideration: (i) $p(t) > 0$, $\tau(t) \leq t (\geq t)$ and $\tau(t) \leq \sigma(t) (\geq \sigma(t))$ (ii) $p(t)$ changes its sign, $\tau(t) \leq t (\geq t)$, $\tau: \mathbb{T} \rightarrow \mathbb{T}$ is a strictly increasing differentiable function and $\lim_{t \rightarrow \infty} \tau(t) = \infty$. Our results not only generalize and extend some existing results but also can be applied to the oscillation problems that are not covered in literature. Finally, we give some examples to illustrate our main results.

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