EXISTENCE AND ASYMPTOTIC BEHAVIOR OF POSITIVE SOLUTIONS FOR A CLASS OF $(p(x),q(x))$– LAPLACIAN SYSTEMS

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Abstract. In this paper, our main purpose is to establish the existence of positive solution of the following system

$$\begin{cases} -\Delta_{p(x)}u = u^{\alpha(x)} + \lambda^{p(x)}v^{m(x)}, & x \in \Omega \\ -\Delta_{q(x)}v = v^{\beta(x)} + \theta^{q(x)}u^{n(x)}, & x \in \Omega \\ u = v = 0, & x \in \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with C^2 boundary, $p(x), q(x)$ are functions which satisfy some conditions, $-\Delta_{p(x)}u = -\text{div}(|\nabla u|^{p(x)-2}\nabla u)$ is called $p(x)$-Laplacian. We give the existence results of positive solutions and consider the asymptotic behavior of the solutions near the boundary. The approach is based on the sub- and super-solution method.

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