

EXISTENCE OF POSITIVE SOLUTION FOR A SINGULAR SYSTEM INVOLVING GENERAL QUASILINEAR OPERATORS

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Abstract. In this paper we study a result of existence of positive solution the following class of singular system:

$$(P) \quad \begin{cases} -\operatorname{div}(a_1(|\nabla u|^{p_1})|\nabla u|^{p_1-2}\nabla u) = h_1(x)v^{-\gamma_1} + k_1(x)v^{\alpha_1} & \text{in } \Omega, \\ -\operatorname{div}(a_2(|\nabla v|^{p_2})|\nabla v|^{p_2-2}\nabla v) = h_2(x)u^{-\gamma_2} + k_2(x)u^{\alpha_2} & \text{in } \Omega, \\ u, v > 0 & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded smooth domain in \mathbb{R}^N with $N \geq 3$, $2 \leq p_1, p_2 < N$. For $i = 1, 2$, $\alpha_i, \gamma_i \in (0, p_i - 1)$ and h_i and k_i are continuous functions. The hypotheses on the functions $a_i : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ allow to show that (P) includes a large class of systems. We use topological arguments to show the main result.

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