

EXISTENCE OF A MILD SOLUTION FOR IMPULSIVE NEUTRAL FRACTIONAL DIFFERENTIAL EQUATIONS WITH NONLOCAL CONDITIONS

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Abstract. In the present work, we investigate the existence of a mild solution of the fractional order differential equation with impulsive conditions in a Banach space. We establish the existence of a mild solution by using some fixed point theorems and resolvent operator theory. We present an example for showing the effectiveness of the main theory.

Mathematics subject classification (2010): 34K37, 34K40, 34K45, 35R11, 45J05, 45K05.

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REFERENCES

- [1] H.M. AHMED, *Fractional neutral evolution equations with nonlocal conditions*, Adv. Difference Equ., 2013 (2013), 117.
- [2] P. BALASUBRAMANIAM, V. VEMBARASAN, T. SENTHILKUMAR, *Approximate controllability of impulsive fractional integro-differential systems with nonlocal conditions in Hilbert space*, Numer. Funct. Anal. Optim., 35 (2014), 177–197.
- [3] E. BAZHLEKOVA, *Fractional evolution equations in Banach spaces*, Ph.D. Thesis, Eindhoven University of Technology, 2001.
- [4] M. BENCHOHRA, J. HENDERSON, S.K. NTOUYAS, *Impulsive differential equations and inclusions*, Contemporary Mathematics and Its Applications, Vol.2, Hindawi Publishing Corporation, New York, 2006.
- [5] L. BYSZEWSKI, *Theorems about the existence and uniqueness of solutions of a semilinear evolution nonlocal Cauchy problem*, J. Math. Anal. Appl., 162 (1991), 497–505.
- [6] L. BYSZEWSKI, V. LAKSHMIKANTHAM, *Theorem about the existence and uniqueness of a solution of a nonlocal abstract Cauchy problem in a Banach space*, Applied Anal., 40 (1990), 11–19.
- [7] A. CHADHA, D.N. PANDEY, *Existence results for an impulsive neutral fractional integrodifferential equation with infinite delay*, Int. J. Diff. Equ., 2014 (2014), pp-10.
- [8] A. CHADHA, D.N. PANDEY, *Existence of a mild solution for an impulsive neutral fractional integro-differential equation with nonlocal conditions*, J. Fract. Cal. Appl., 6 (2015), 5–20.
- [9] K. EZZINBIA, X. FU, K. HILAL, *Existence and regularity in the α -norm for some neutral partial differential equations with nonlocal conditions*, Nonlinear Analysis: TMA, 67 (2007), 1613–1622.
- [10] K. EZZINBIA, X. FU, *Existence and regularity of solutions for some neutral partial differential equations with nonlocal conditions*, Nonlinear Analysis: TMA, 57 (2004), 1029–1041.
- [11] S. FARAHI, T. GUENDOUZI, *Approximate controllability of fractional neutral stochastic evolution equations with nonlocal conditions*, Results. Math., 2014 (2014), pp-21.
- [12] M. FEČKAN, Y. ZHOU, J.-R. WANG, *On the concept and existence of solution for impulsive fractional differential equations*, Commun Nonlinear Sci Numer Simulat, 17 (2012), 3050–3060.
- [13] M. FEČKAN, Y. ZHOU, J.-R. WANG, *Response to “Comments on the concept of existence of solution for impulsive fractional differential equations”*, Commun Nonlinear Sci Numer Simulat, 19 (2014), 4213–4215.

- [14] T.E. GOVINDAN, *An existence result for the Cauchy problem for stochastic systems with heredity*, Diff. Int. Equ., 15 (2002), 103–113.
- [15] A. GRANAS, J. DUGUNDJI, *Fixed Point Theory*, Springer-Verlag, New York, 2003.
- [16] K. KARTHIKEYAN, A. ANGURAJ, K. MALAR, J.J. TRUJILLO, *Existence of mild and classical solutions for nonlocal impulsive integrodifferential equations in Banach spaces with measure of noncompactness*, Int. J. Diff. Equ., 2014 (2014), pp-10.
- [17] A.A. KILBAS, H.M. SRIVASTAVA, J.J. TRUJILLO, *Theory and Applications of Fractional Differential Equations*, Elsevier, Amsterdam, 2006.
- [18] V.B. KOLMANOVSKII, V.R. NOSOV, *Stability of neutral-type functional differential equations*, Nonlinear Analysis: TMA, 6 (1982), 873–910.
- [19] V. LAKSHMIKANTHAM, D. BAINOV, P.S. SIMEONOV, *Theory of Impulsive Differential Equations*, World Scientific, Singapore-London, 1989.
- [20] F. LI, G.M. N'GUEREKATA, *An existence result for neutral delay integrodifferential equations with fractional order and nonlocal conditions*, Abstr. Appl. Anal., Volume 2011 (2011), Article ID 952782, 20 pages.
- [21] F. LI, J. LIANG, H.-K. XU, *Existence of mild solutions for fractional integrodifferential equations of Sobolev type with nonlocal conditions*, J. Math. Anal. Appl., 391 (2012), 510–525.
- [22] F. LI, *Nonlocal Cauchy problem for delay fractional integrodifferential equations of neutral type*, Adv. Diff. Equ., 2012 (2012), pp-23.
- [23] Y. LIU, B. AHMAD, *A study of impulsive multiterm fractional differential equations with single and multiple base points and applications*, The Scientific World Journal, 2014 (2014), pp-28.
- [24] K.S. MILLER, B. ROSS, *An Introduction to the Fractional Calculus and Fractional Differential Equations*, John Wiley and Sons, Inc. New York, 1993.
- [25] M.G. MOPHOU, *Existence and uniqueness of mild solutions to impulsive fractional differential equations*, Nonlinear Analysis: TMA, 72 (2010), 1604–1615.
- [26] A. PAZY, *Semi-groups of Linear operator and Applications of Partial Differential Equations*, Springer verlag, 1983.
- [27] I. PODLUBNY, *Fractional Differential Equations*, Academic press, New York (1993).
- [28] J. PRUSS, *Evolutionary Integral Equations and Applications*, in Monographs Math., Vol. 87, Birkhauser-Verlag, 1993.
- [29] S.G. SAMKO, A.A. KILBAS, O.I. MARICHEV, *Fractional Integrals and Derivatives: Theory and Applications*, Gordon and Breach Science Publisher, Yverdon, 1993.
- [30] X.-B. SHU, Q. WANG, *The existence and uniqueness of mild solutions for fractional differential equations with nonlocal conditions of order $1 < \alpha < 2$* , Comp. Math. Appl., 64 (2012), 2100–2110.
- [31] X.-B. SHU, Y. LAI, Y. CHEN, *The existence of mild solutions for impulsive fractional partial differential equations*, Nonlinear Analysis: TMA, 74 (2011), 2003–2011.
- [32] G. TR. STAMOV, I.M. STAMOVA, *Almost periodic solutions for impulsive fractional differential equations*, Dynamical Systems: An International Journal, 29 (2014), 119–132.
- [33] V. VIJAYAKUMAR, C. RAVICHANDRAN, R. MURUGESU, J.J. TRUJILLO, *Controllability results for a class of fractional semilinear integro-differential inclusions via resolvent operators*, Applied Math. Comp., 247 (2014), 152–161.
- [34] G. WANG, B. AHMAD, L. ZHANG, J.J. NIETO, *Comments on the concept of existence of solution for impulsive fractional differential equations*, Commun Nonlinear Sci Numer Simulat, 19 (2014) 401–403.
- [35] G. WANG, S. LIU, D. BALEANU, L. ZHANG, *A new impulsive multi-orders fractional differential equation involving multipoint fractional integral boundary conditions*, Abstr. Appl. Anal., Volume 2014 (2014), Article ID 932747, 10 pages.
- [36] Z. YAN, H. ZHANG, *Existence of solutions to impulsive fractional partial neutral stochastic integro-differential inclusions with state-dependent delay*, Elect. J. Diff. Equ., 2013 (2013), 1–21.
- [37] Z. YAN, X. JIA, *Impulsive problems for fractional partial neutral functional integro-differential inclusions with infinite delay and analytic resolvent operators*, Mediterr. J. Math., 2013 (2013), 1–36.
- [38] X. ZHANG, X. HUANG, Z. LIU, *The Existence and uniqueness of mild solutions for impulsive fractional equations with nonlocal conditions and infinite delay*, Nonlinear Analysis: Hybrid Systems, 4 (2010), 775–781.
- [39] L. ZHU, G. LI, *Existence results of semilinear differential equations with nonlocal initial conditions in Banach spaces*, Nonlinear Analysis: TMA, 74 (2011), 5133–5140.

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