

EXISTENCE RESULTS FOR NONLINEAR SECOND-ORDER q -DIFFERENCE EQUATIONS WITH q -INTEGRAL BOUNDARY CONDITIONS

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Abstract. In this paper, we present some new existence and uniqueness results for the q -integral boundary value problem of nonlinear q -difference equation. Our results are based on Banach's contraction principle and Krasnowskii's fixed point theorem. An example is given to illustrate the advantage of our results.

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