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DETERMINATION OF A LINEAR DIFFERENTIAL EQUATION ON HALF–LINE AND ITS SPECTRAL DISTRIBUTION FUNCTION FROM THE OTHERS RELATED

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Abstract. Consider two problems with symmetrical boundary value problems and defined by for $j=1,2$ through: $-y''+q_j(x)y=s^2y$, $0<x<\infty$, $y'(0)-k_jy(0)=0$ where $k_j\in\{h_1,h_2\},h_1,h_2$ are different real numbers, $s\in\{\lambda(h_1),\mu(h_2)\}$, $\{\lambda(h_1),\mu(h_2)\}$ represents the same family of eigenvalues for both problems, $q_j(x)$ are continuous real valued functions. Their uniqueness is determined through their respective spectral distribution function R_j. The aim of the paper is to relate both previous problems in the following way. We will assume the uniqueness of the first problem and determine the uniqueness of the second problem by linking: both spectral distribution functions R_j, both boundary conditions $y'(0)-k_jy(0)=0$ and both potential q_j.

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REFERENCES

[1] R. A. ADAMS, *Sobolev Spaces*, Academic Press, New York, 1970.
[2] J. BOURGAIN, *Global Solutions of Non-linear Schrödinger Equations*, American Mathematical Society, Colloquium Publications, Volumen **46**, US, 1999.
[3] I. M. GELFAND AND B. M. LEVITAN, *On the Determination of a Differential Equation by its Spectral Function*, Izv. Akad. Nauk. SSSR. Ser. Mat., **15**, (1951), 309–360. (English traslation: Amer. Math. Soc. Traslations, Ser., **2**, I, 253–304.
[4] J. GINIBRE, *Introduction aux Équations de Schrödinger Nonlineáres*, Onze Éditions, Paris, 1998.
[5] J. L. JOURNE AND A. SOFFER, *Decay Estimates for Schrödinger Operators*, Comm. Pure Appl. Math., **44**, (1991), 573–604.
[6] T. KATO, *Nonlinear Schrödinger Equations*, Lectures Notes in Phys., **345**, (1989), 218–263.
[7] M. KEEL AND T. TAO, *End Point Strichartz Estimates*, Amer. J., **120**, (1988), 955–980.
[8] B. M. LEVITAN AND M. G. GASYMOV, *Determination of a Differential Equation by Two of its Spectra*, Russian Math. Surveys, **19**, (1964), 1–63.
[9] B. M. LEVITAN, *Inverse Sturm-Liouville Problems*, VNU Science Press BV, Utrech, The Netherlands, 1987.
[10] V. A. MARCHENKO, *Certain Problems of the Theory of Second-Order Differential Operators* (in Russian), DAN SSSR **72**, 3 (1950), 457–560.
[11] V. A. MARCHENKO, *Certain Problems of the Theory of One-Dimensional, Linear Differential Operators of Second-Order* (in Russian), I. Trudy Mosk. Mat. Obshch **I**, (1952), 327–420.
[12] V. A. MARCHENKO, *Sturm-Liouville Operators and Applications*, Operator Theory: Advances and Applications, **22**, OT 22, Birkhäuser Verlang Basel, Germany, 1986.
[13] R. G. NEWTON, *Analytic Properties of Radial Wave Functions*, J. Math. Phys., **1**, 4 (1960), 301–312, 319–347.
[14] R. G. NEWTON, *Scattering Theory of Waves and Particles*, Second Edition, Springer-Verlag, New York, Inc., US, 1982.

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[15] C. A. PILLET AND C. E. WAYNE, *Invariant Manifolds for a Class of Dispersive, Hamiltonian, Partial Differential Equations*, J. Differential Equations, **141**, (1997), 310–326.

[16] M. REED AND R. SIMON, *Fourier Analysis, Self-Adjointness. Methods of Modern Mathematical Analysis*, Vol. **2**, Academic Press, Inc., New York, 1975.

[17] A. SOFFER AND M. I. WEINSTEIN, *Multichannel Non-Linear Scattering for Non-Integrable Equations II. The Case of Anisotropic Potentials and Data*, J. Differential Equations, **98**, (1992), 376–390.

[18] W. A. STRAUSS, *Non-Linear Wave Equations*, CBMS-RCSM, **73**, Amer. Math. Soc., Providence, R.I., 1989.

[19] R. S. STRICHARTZ, *Restriction of Fourier Transform to Quadratic Surfaces and Decay of Solutions of Wave Equations*, Duke Math. J., **44**, (1977), 705–774.

[20] R. WEDER, *$Lp-Lp'$ Estimates for the Schrödinger Equation on the Line and Inverse Scattering for the Non Linear Schrödinger Equation with a Potential*, Journal Functional Analysis, **170**, (2000), 37–68.

[21] R. WEDER, *Center Manifold for Non-Integrable Non-Linear Schrödinger Equation on the Line*, Comm. Math. Phys., (2000), 343–356.

[22] R. WEDER, *The Time-Dependent Approach to Inverse Scattering*, Minicourse, Pan-American Advanced Studies Institute (PASI) on Inverse Problems, Mathematical Sciences Research Institute, Berkeley, October 29-November 2, 2001.

[23] R. WEDER, *The $Lp-Lp'$ Estimate for the Schrödinger Equation on the Half-Line*, Journal of Mathematical Analysis and Applications, **281**, (2003), 233–243.

[24] R. WEDER, *Scattering for the Forced Non-Linear Schrödinger Equation with a Potential on the Half-Line*, Mathematical Methods in The Applied Science, **28**, (2005), 1219–1236.

[25] J. WEIDMANN, *Spectral Theory of Ordinary Differential Operators*, Lectures Notes in Math., **1258**, Springer-Verlag, Berlin, 1987.