

PRACTICAL STABILITY OF CAPUTO FRACTIONAL DIFFERENTIAL EQUATIONS BY LYAPUNOV FUNCTIONS

RAVI AGARWAL, S. HRISTOVA AND D. O'REGAN

Abstract. The practical stability of a nonlinear nonautonomous Caputo fractional differential equation is studied using Lyapunov like functions. The novelty of this paper is based on the new definition of the derivative of a Lyapunov like function along the given fractional differential equation. Comparison results using this definition for scalar fractional differential equations are presented. Several sufficient conditions for practical stability, practical quasi stability, strongly practical stability of the zero solution and the corresponding uniform types of practical stability are established.

Mathematics subject classification (2010): 34A34, 34A08, 34D20.

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