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FREDHOLM ALTERNATIVE FOR THE SECOND ORDER DIFFERENTIAL OPERATOR ASSOCIATED TO A CLASS OF BOUNDARY CONDITIONS

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Abstract. This work is concerned with the Fredholm property of the second order differential operator associated to a class of boundary conditions. Several sufficient conditions will be proved along with constructing the generalized inverse for such operator. The result is a basic tool to analysis the boundary value problems at resonance for nonlinear perturbation of such operators.

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