

SOLOW MULTI-CAPITAL GROWTH MODEL DESCRIBED BY A SYSTEM OF DIFFERENTIAL EQUATIONS ON TIME SCALES

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Abstract. In this paper we derive a system of differential equations on time scales of the Solow type corresponding to a production function depending on several capitals. A sufficient condition for the exponential stability of the steady-state solution with positive coordinates is proved. The obtained results are applied to the case of the Cobb-Douglas type production function.

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