

## EXISTENCE OF SOLUTIONS TO NONLINEAR BOUNDARY VALUE PROBLEMS

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*Abstract.* In this paper we provide sufficient conditions for the existence of solutions to nonlinear boundary value problems. We do so by applying a general abstract strategy for solving nonlinear equations with a linear component. We apply this to general systems by first isolating a linear periodic system and using the general theory of periodic solutions to find conditions on the additional nonlinear components to guarantee solutions.

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